A Course Module

on

ELECTRICAL CIRCUIT ANALYSIS (20A02301T)

Prepared by

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SREE RAMA ENGINEERING COLLEGE

Approved by AICTE, New Delhi – Affiliated to JNTUA, Ananthapuramu
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Rami Reddy Nagar, Karakambadi road,Tirupati-517507

ELECTRICAL AND ELECTRONICS ENGINEERING

Course Code	ELECTRICAL CIRCUIT ANALYSIS		L	T	P	C
20A02301T			3	0	0	3
Pre-requisite	Fundamentals of Electrical Circuits	Semester	III			

Course Objectives:

- To know the analysis of three phase balanced and unbalanced circuits and to measure active and reactive powers in three phase circuits.
- Knowing how to determine the transient response of R-L, R-C, R-L-C series circuits for D.C and A.C excitations.
- To know the applications of Fourier transforms to electrical circuits excited by non sinusoidal sources.
- Study of Different types of filters, equalizers.

Course Outcomes (CO):

- Understand the analysis of three phase balanced and unbalanced circuits and to measure active and reactive powers in three phase circuits.
- To get knowledge about how to determine the transient response of R-L, R-C, R-L-C series circuits for D.C and A.C excitations.
- Applications of Fourier transforms to electrical circuits excited by non-sinusoidal sources are known.
- To design filters and equalizers.

UNIT - I	**Locus Diagrams & Resonance**	8 Hrs

Series R-L, R-C, R-L-C and Parallel Combination with Variation of Various Parameters - Resonance-Series, Parallel Circuits, Frequency Response, Concept of Bandwidth and Q Factor.

UNIT - II	**Two Port Networks**	9 Hrs

Two Port Network Parameters – Impedance – Admittance - Transmission and Hybrid Parameters and their Relations - Concept of Transformed Network - Two Port Network Parameters Using Transformed Variables.

UNIT - III	**Transient Analysis**	12 Hrs

D.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for D.C Excitation - Initial Conditions in network - Initial Conditions in elements - Solution Method Using Differential Equation and Laplace Transforms - Response of R-L & R-C Networks to Pulse Excitation.
A.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for Sinusoidal Excitations - Solution Method Using Differential Equations and Laplace Transforms.

UNIT - IV	**Fourier Transforms**	10 Hrs

Fourier Theorem - Trigonometric Form and Exponential Form of Fourier series – Conditions of Symmetry - Line Spectra and Phase Angle Spectra - Analysis of Electrical Circuits to Non Sinusoidal Periodic Waveforms. Fourier Integrals and Fourier Transforms – Properties of Fourier Transforms and Application to Electrical Circuits.

UNIT - V	**Filters**	9 Hrs

Filters – Low Pass – High Pass, Band Pass and Band Stop– RC, RL filters– derived filters and composite filters design – Attenuators – Principle of Equalizers – Series and Shunt Equalizers – L Type - T type and Bridged – T and Lattice Equalizers.

Textbooks:

1. William Hayt, Jack E. Kemmerly and Jamie Phillips, “Engineering Circuit Analysis”, Mc Graw Hill, 9th Edition, 2019.
2. A. Chakrabarti, “Circuit Theory: Analysis & Synthesis”, Dhanpat Rai & Sons, 2008.

Reference Books:

1. M.E. Van Valkenberg, “Network Analysis”, 3rd Edition, Prentice Hall (India), 1980.
2. V. Del Toro, “Electrical Engineering Fundamentals”, Prentice Hall International, 2009.
3. Charles K. Alexander and Matthew. N. O. Sadiku, “Fundamentals of Electric Circuits” Mc Graw Hill, 5th Edition, 2013.
4. MahamoodNahvi and Joseph Edminister, “Electric Circuits” Schaum’s Series, 6th Edition, 2013.
5. John Bird, Routledge, “Electrical Circuit Theory and Technology”, Taylor & Francis, 5th Edition, 2014.

Online Learning Resources:

- https://onlinecourses.nptel.ac.in/noc21_ee99/preview
- https://onlinecourses.nptel.ac.in/noc21_ee14/preview

Electrical circuit Analysis

unit-1 : Locus Diagrams and Resonance

Series R-L, R-C, R-L-C and Parallel Combination with variation of various parameters - Resonance - series, Parallel circuits, Frequency Response, concept of bandwidth and Q Factor

Introduction :

For a particular circuit like R-L, R-C, R-L-C. If any one of the element is variable then depending upon the value of the variable element circuit characteristic changes then circuit parameters like voltage, current and power consumed by the element is also changes.

Def of Locus diagram:

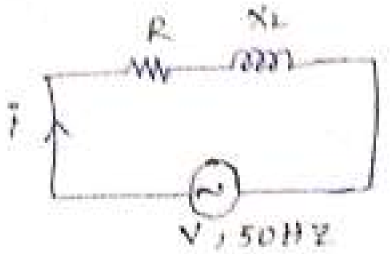
"It is defined as the Locus of the current obtained for various values of the variable element."

classification

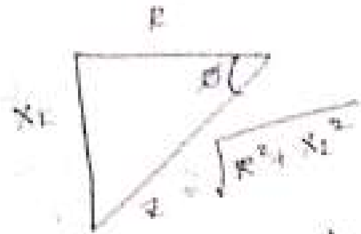
Locus diagrams are classified into two types

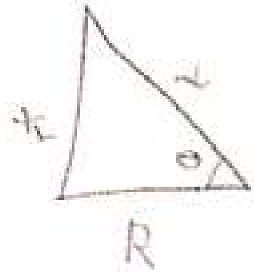
1. series R-L, R-C and R-L-C circuits.
2. Parallel combination of circuits.

Locus diagram of R-L circuit :



Impedenic diagram





According to KVL

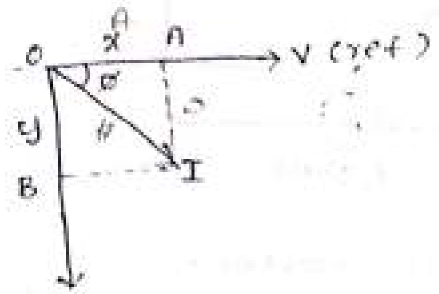
$V = I(R + jX_L)$

$V = IZ \quad , \; Z = R + jX_L$

$I = \frac{V}{Z} \longrightarrow (1)$

Now, phasor diagram for R-L circuit is

$(x = R, \; y =)$



From the phasor diagram

$\cos\phi = \frac{x}{I}$

$x = I\cos\phi \longrightarrow (2)$

$\sin\phi = \frac{OB}{I} = \frac{y}{-I}$

$y = -I\sin\phi \longrightarrow (3)$

$x^2 + y^2 = I^2\cos^2\phi + I^2\sin^2\phi$

$x^2 + y^2 = I^2(\cos^2\phi + \sin^2\phi)$

$$x^2+y^2 = I^2$$

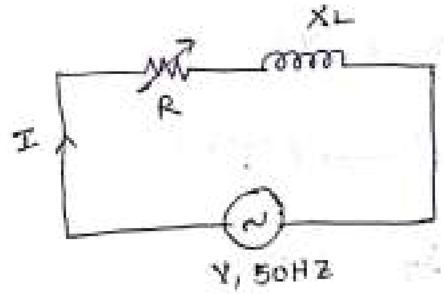
$$x^2+y^2 = \left(\frac{V}{Z}\right)^2$$

$$x^2+y^2 = \frac{V^2}{Z^2}$$

$$x^2+y^2 = \frac{V^2}{\left(\sqrt{R^2+X_L^2}\right)^2}$$

$$\boxed{\therefore x^2+y^2 = \frac{V^2}{R^2+X_L^2}} \longrightarrow (4)$$

case(i):- Variable R, constant X_L



Here X_L is constant

$$y = -I\sin\phi$$

$$= -\frac{V}{Z} \times \frac{X_L}{Z}$$

$$= \frac{-VX_L}{R^2+X_L^2}$$

$$y = -X_L \times \frac{V}{R^2+X_L^2} \longrightarrow (5)$$

$$\frac{V}{R^2+X_L^2} = \frac{-y}{X_L} \longrightarrow (6)$$

Substitute Eqn (6) in Eqn (4)

$$x^2+y^2 = V \times \frac{y}{R^2+X_L^2}$$

$$x^2+y^2 = V \times \frac{-y}{X_L}$$

$$x^2+y^2+2.y.\frac{V}{2X_L}+\left(\frac{V}{2X_L}\right)^2-\left(\frac{V}{2X_L}\right)^2=0$$

$$x^2+\left(y^2+\frac{V}{2X_L}\right)^2=\left(\frac{V}{2X_L}\right)^2$$

$$\therefore x^2+\left(y+\frac{V}{2X_L}\right)^2=\left(\frac{V}{2X_L}\right)^2 \longrightarrow (7)$$

Now we know the circle equation

$$(x-x_1)^2+(y-y_1)^2=r^2 \longrightarrow (8)$$

comparing eqn's (7) & (8), we get.

$x_1 = 0$,

$y_1 = \frac{-V}{2X_L}$

$r = \frac{V}{2X_L}$

$\therefore$ centre $=(x_1, y_1) = \left(0, \frac{-V}{2X_L}\right)$

radius $(r) = \frac{V}{2X_L}$

Construction of locus diagram :-

$\cos\phi = \frac{R}{Z}$ & $I = \frac{V}{Z} \Rightarrow I = \frac{V}{\sqrt{R^2+X_L^2}}$

Suppose if $\phi = 30^\circ$ then $\cos 30^\circ = 0.866$

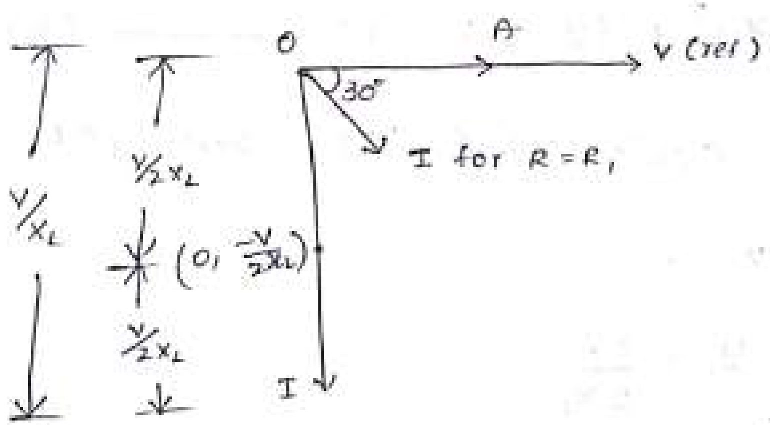
$\phi = 45^\circ$ then $\cos 45^\circ = 0.707$

$\phi = 60^\circ$ then $\cos\phi = 0.5$

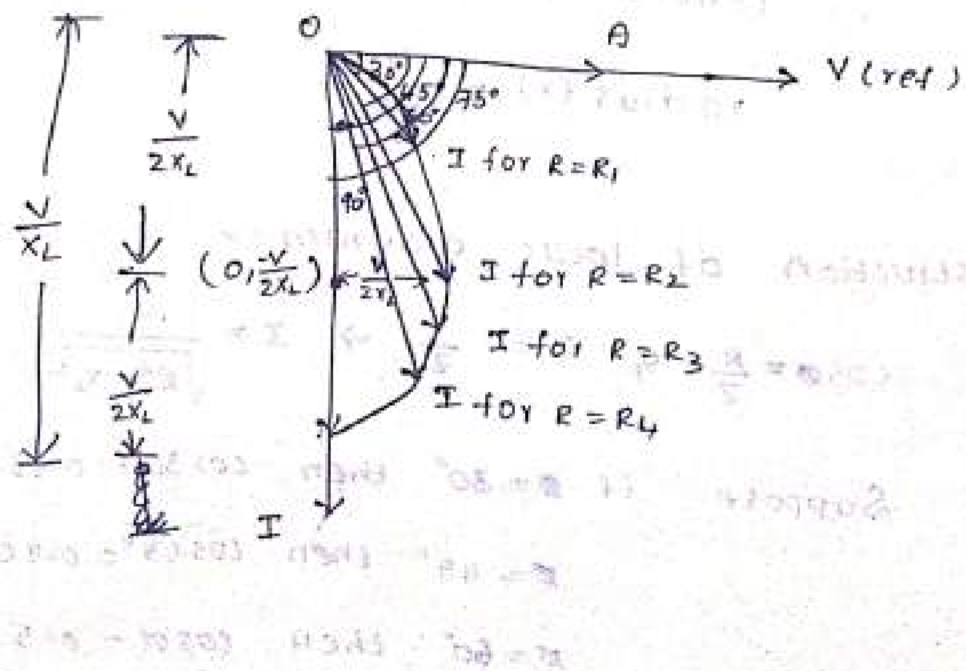
$\phi = 75°$ then $\cos 75° = 0.25$

$\phi = 90°$ then $\cos 90° = 0$

If $\cos 30° = 0.866$ and compared due to other angle it is high value and we know that R is directly proportional to $\cos\phi$ and 'R' value is high then if we put high value of 'R' in current eqn that means then the magnitude of current is less. at 30° magnitude of current is less.



If angle increases 'R' value decreases then current values increases.



Case (ii): constant R, variable X_L

We know

$$x^2 + y^2 = \frac{V^2}{R^2+X_L^2} \longrightarrow (4).$$

Here constant 'R'.

$$x = I\cos\phi$$

$$= \frac{V}{\sqrt{R^2+X_L^2}} \times \frac{R}{\sqrt{R^2+X_L^2}}$$

$$x = \frac{V \cdot R}{R^2+X_L^2}$$

$$\frac{V}{R^2+X_L^2} = \frac{x}{R} \longrightarrow (5)$$

Sub Eqn (5) in Eqn (4)

$$x^2 + y^2 = \frac{Vx}{R}$$

$$x^2 + y^2 - Vx/R = 0$$

$$x^2 + y^2 - 2x \cdot \frac{V}{2R} = 0$$

$$x^2 + y^2 - 2x \cdot \frac{V}{2R} + \left(\frac{V}{2R}\right)^2 - \left(\frac{V}{2R}\right)^2 = 0$$

$$\left(x - \frac{V}{2R}\right)^2 + y^2 - \left(\frac{V}{2R}\right)^2 = 0$$

$$\left(x - \frac{V}{2R}\right)^2 + y^2 = \left(\frac{V}{2R}\right)^2 \longrightarrow (6)$$

Now we know the circle equation

$$(x - x_1)^2 + (y - y_1)^2 = r^2 \longrightarrow (7)$$

comparing Eqn's (6) & (7), we get

$$\text{centre} = (x_1, y_1) = \left(\frac{V}{2R}, 0\right)$$

$$\text{Radius } (r) = \frac{V}{2R}.$$

Here also one of the element in the centre is zero. So locus diagram is a semicircle.

<u>Construction of locus diagram</u> :-

$$\sin\phi = \frac{X_L}{\sqrt{R^2 + X_L^2}}, \quad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

Case (i):- Constant R, Variable X_L

$$\text{Centre} = \left(\frac{V}{2R}, 0\right)$$

$$\text{radius} = \frac{V}{2R}$$

here variable is X_L, so

$$\sin\phi = \frac{X_L}{Z} \qquad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$\phi = 30°$, $\sin 30 = 0.5$

$\phi = 45°$, $\sin 45° = 1/\sqrt{2} = 0.707$

$\phi = 60°$, $\sin 60° = 0.866$

$\phi = 75°$, $\sin 75° = 0.965$

$\phi = 90°$, $\sin 90° = 1$

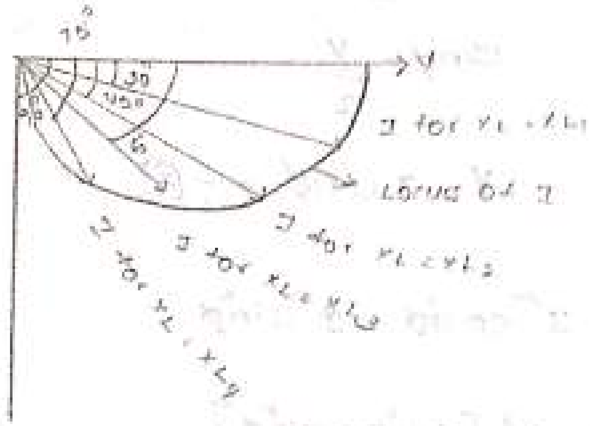
$\phi = 30°$, X_L is small, I is large

$\phi = 45°$, X_L is high, I is small

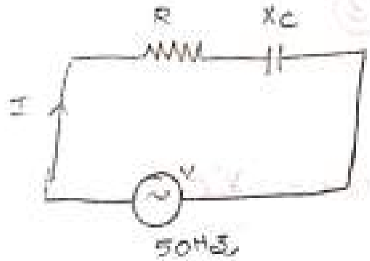
$\phi = 60°$, X_L is very high, I is very small

$\phi = 75°$, X_L is vv high, I is vv small

$\phi = 90°$,



* Locus diagram of R-C circuit:-

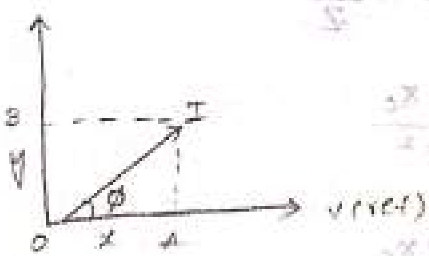


$$V = I(R + jX_C)$$

$$V = IZ$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_C^2}}$$

Phasor diagram:-



$$\cos\phi = \frac{OA}{I} = \frac{x}{I}$$

$$x = I\cos\phi \quad \text{—} \; ①$$

$$\sin\phi = \frac{OB}{I}$$

$$\sin\phi = \frac{y}{I}$$

$$y = I\sin\phi \quad \text{— (2)}$$

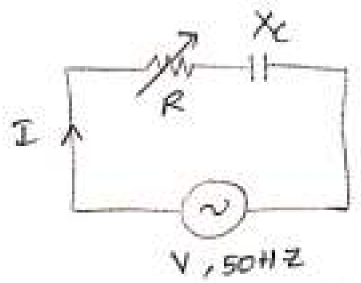
$$x^2 + y^2 = I^2\cos^2\phi + I^2\sin^2\phi$$

$$= I^2[\cos^2\phi + \sin^2\phi]$$

$$= I^2(1)$$

$$\boxed{x^2 + y^2 = \frac{V^2}{R^2 + X_C^2}} \quad \text{— (3)}$$

Case(i) :- Variable R constant X_C :-



Here X_C is constant

$$y = I\sin\phi$$

$$y = \frac{V}{Z} \times \frac{X_C}{Z}$$

$$y = \frac{V X_C}{Z^2}$$

$$y = \frac{V X_C}{R^2 + X_C^2}$$

$$y = X_C \times \frac{V}{R^2 + X_C^2} \longrightarrow (4)$$

$$\frac{V}{R^2+X_c^2} = \frac{y}{X_c} \longrightarrow (5)$$

sub eqn (5) in eqn (3)

$$x^2+y^2 = V \times \frac{y}{X_c}$$

$$x^2+y^2 - V \times \frac{y}{X_c} = 0$$

$$x^2+y^2 - 2V \times \frac{y}{2X_c} + \left(\frac{V}{2X_c}\right)^2 - \left(\frac{V}{2X_c}\right)^2 = 0$$

$$x^2+y^2 - 2y \times \frac{V}{2X_c} + \left(\frac{V}{2X_c}\right)^2 - \left(\frac{V}{2X_c}\right)^2 = 0$$

$$x^2 + \left(y - \left(\frac{V}{2X_c}\right)\right)^2 = \left(\frac{V}{2X_c}\right)^2 \longrightarrow (6)$$

Now we know the circle eqn

$$(x-x_1)^2 + (y-y_1)^2 = r^2 \longrightarrow (7)$$

comparing eqn's (6) & (7)

$$x_1 = 0$$

$$y_1 = \frac{V}{2X_c}$$

$$r = \frac{V}{2X_c}$$

$\therefore$ centre $= (x_1, y_1) = \left(0, \frac{V}{2X_c}\right)$

radius $(r) = \frac{V}{2X_c}$

Construction of locus diagram:-

$$\cos\phi = \frac{R}{Z}, \quad I = \frac{V}{Z} \Rightarrow I = \frac{V}{\sqrt{R^2+X_c^2}}$$

Suppose

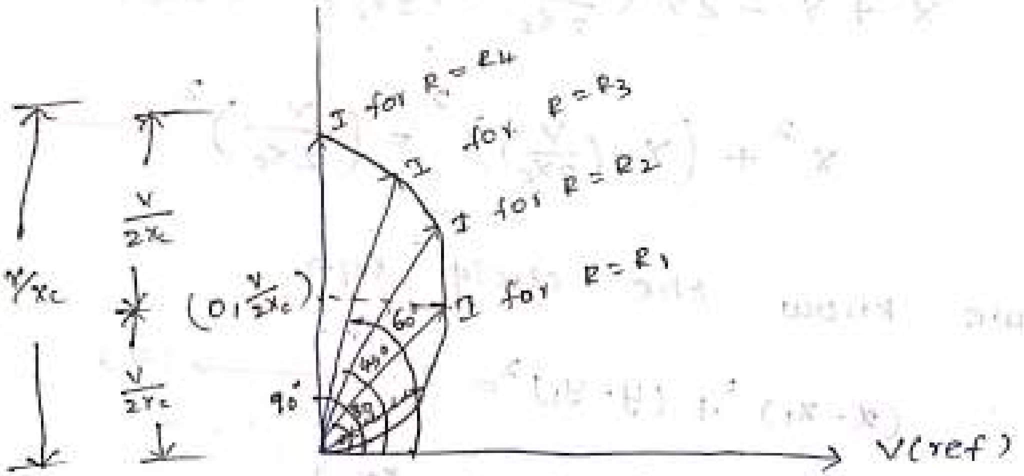
$\phi = 30°$, Then $\cos\phi = 0.866$

$\phi = 45°$, Then $\cos\phi = 0.707$

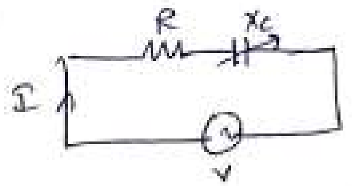
$\phi = 60°$, Then $\cos\phi = 0.5$

$\phi = 75°$, Then $\cos\phi = 0.25$

$\phi = 90°$, Then $\cos\phi = 0°$



Case(ii) :- constant R, variable X_c



$x = I\cos\phi$

$$x = \frac{V}{Z} \times \frac{R}{Z}$$

$$x = \frac{VR}{R^2 + X_c^2}$$

$$\frac{V}{R^2 + X_c^2} = \frac{x}{R} \longrightarrow (8)$$

Sub eqn (8) in eqn (3)

$$x^2+y^2 = V x \frac{x}{R}$$

$$x^2+y^2 - V \frac{x}{R} = 0$$

$$x^2+y^2 - 2x \frac{V}{2R} + \left(\frac{V}{2R}\right)^2 - \left(\frac{V}{2R}\right)^2 = 0$$

$$y^2 + \left(x - \frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2$$

comparing above eqn w.r.t.o $(x-x_1)^2 + (y-y_1)^2 = r^2$

$x_1 = \frac{V}{2R}$, $y_1 = 0$, $r = \frac{V}{2R}$, $\left(\frac{V}{2R}, 0\right)$, $\frac{V}{2R}$

construction of locus diagram

$$\sin\phi = \frac{X_C}{\sqrt{R^2+X_C^2}}, \quad I = \frac{V}{\sqrt{R^2+X_C^2}}$$

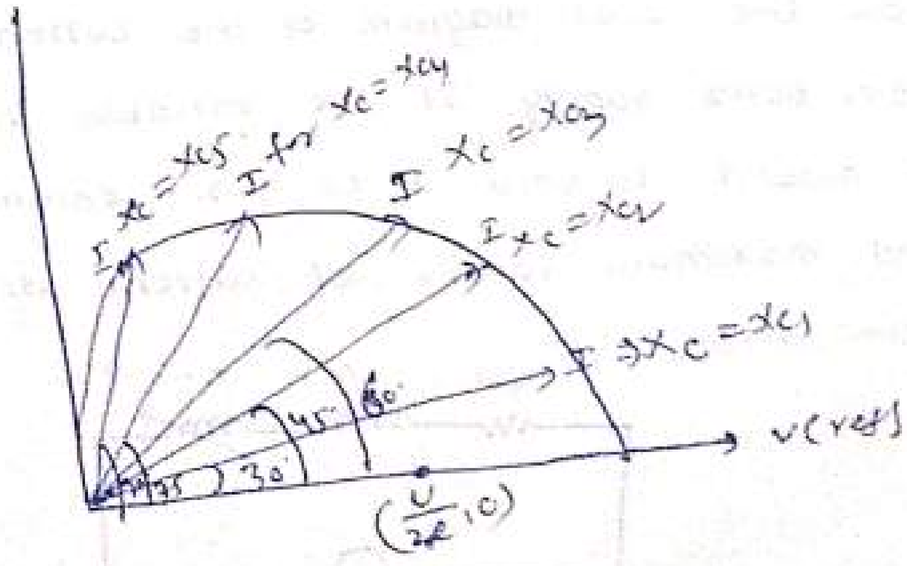
$\phi = 30°$ then $\sin\phi = 0.5$

$\phi = 45°$ then $\sin\phi = 0.707$

$\phi = 60°$ then $\sin\phi = 0.866$

$\phi = 75°$ then $\sin\phi = 0.965$

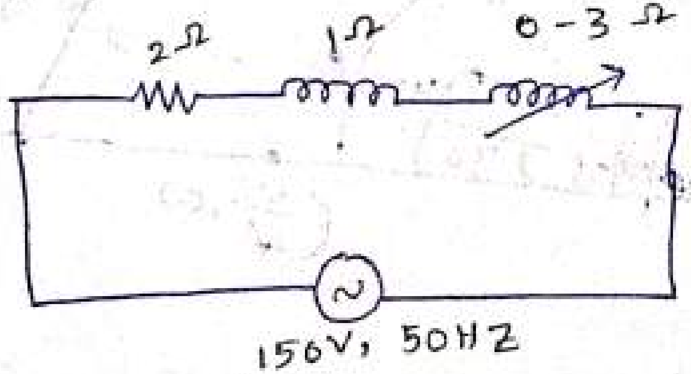
$\phi = 90°$ then $\sin\phi = 1$



PROBLEMS :-

1. A circuit consisting of a choke coil with resistance of 2Ω and reactance of 1Ω at 50 Hz. It is connected in series with other choke coil with negligible resistance and variable inductive reactance. Draw the locus diagram of the current drawn from a 150V, 50Hz supply. If the variable inductive reactance is allowed to vary 0 to 3Ω. Calculate the minimum and maximum values of current and corresponding power factor.

Sol:-



In this problem,

variable reactance (X_L) & constant R

$$\text{radius} = \frac{V}{2R} = \frac{150}{2 \times 2}$$

$$\boxed{\text{radius} = 37.5\ m}$$

$$\text{centre}\left(\frac{V}{2R}, 0\right) = (37.5, 0)$$

X_L is varies from 1 to 1+3

1 to 4 Ω

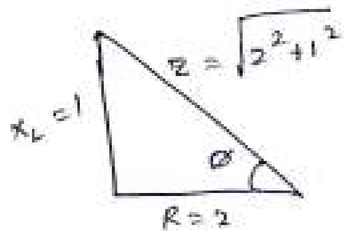
case(i):- Put $X_L = 0\,\Omega$

Total reactance (X_L) = $1+0 = 1\,\Omega$

$R = 2\,\Omega$

$$I = \frac{V}{\sqrt{R^2+X_L^2}} = \frac{150}{\sqrt{2^2+1^2}}$$

$$\boxed{I = 67.08\ A}$$



$$\tan\phi = \frac{X_L}{R}$$

$$\phi_{min} = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

$$\phi_{min} = 26.56^\circ$$

$$\cos\phi_{min} = \cos 26.56$$

$$\boxed{\cos\phi_{min} = 0.894}$$

case(ii):- $X_L = 3\Omega$

$R = 2\Omega$

Total $(X_L) = 1+3 = 4\Omega$

$$I = \frac{V}{\sqrt{R^2+X_L^2}} = \frac{150}{\sqrt{2^2+4^2}}$$

$I = 33.5\,A$

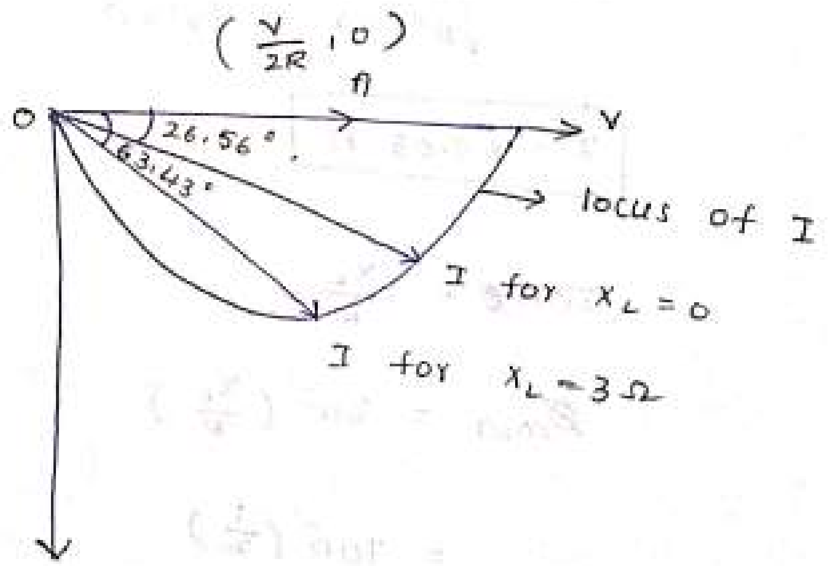
$$\phi_{max} = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \tan^{-1}\left(\frac{4}{2}\right)$$

$\phi_{max} = 63.43$

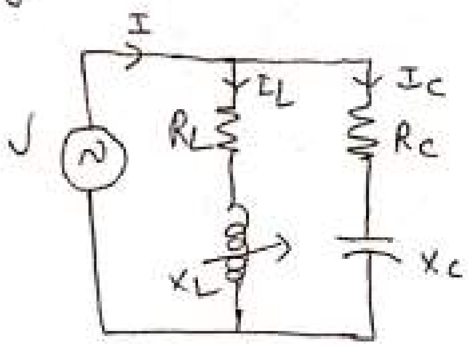
$\cos\phi_{max} = \cos(63.43)$

$$\boxed{\cos\phi_{max} = 0.447}$$



Parallel RLC circuit

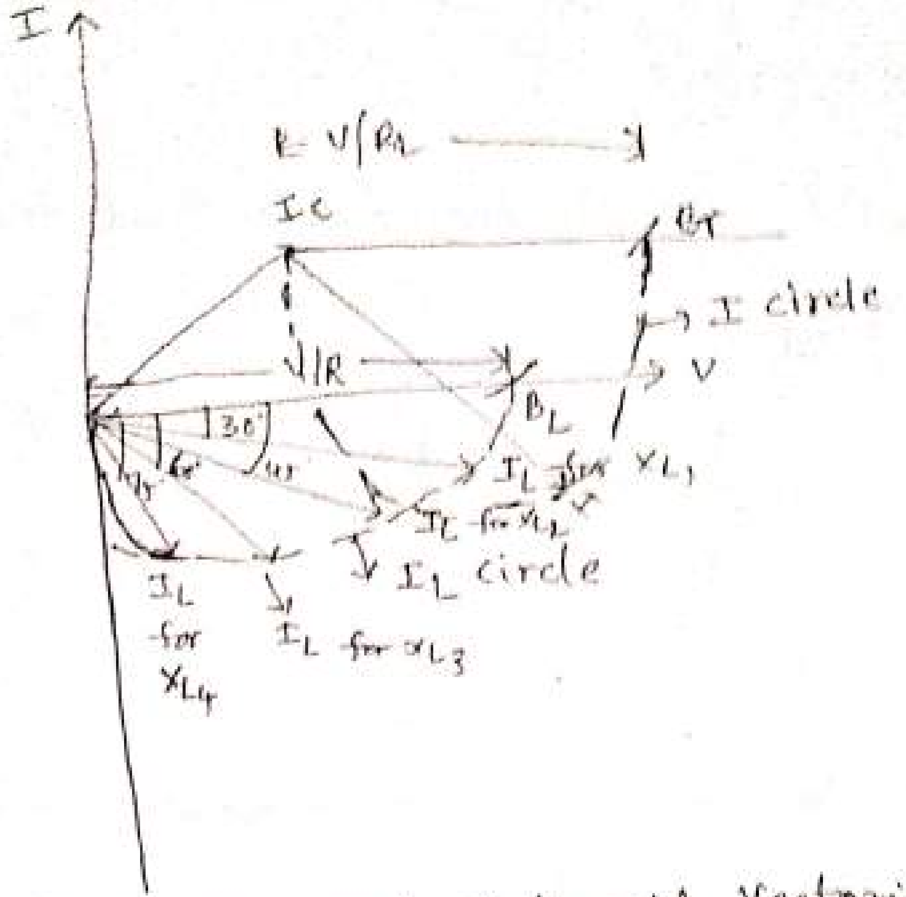
Parallel LC circuit along with internal resistances as shown in fig



In the above circuit, there are two branch currents i_L & i_C along with total current i

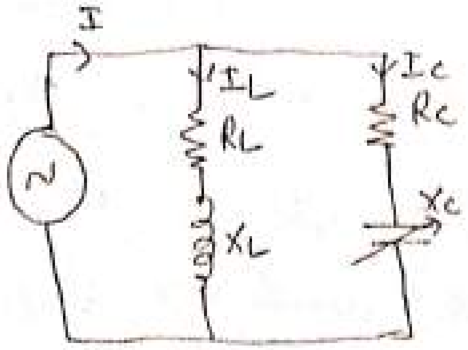
Case(1): Varying X_L:

- In this case, X_L is variable, X_C, R_L, R_C are fixed. and I_C is through capacitor is constant since R_C, R_L are fixed and it leads the voltage vector OV by an angle θ_C $\left[\theta_C = \tan^{-1}\frac{X_C}{R_C}\right]$
- The current I_L through the inductance is vector OI_L and its amplitude is maximum and is equal to $\frac{V}{R_L}$ when X_L is zero and it is in phase with applied voltage V.
- $I = \frac{V}{\sqrt{R^2+X_L^2}}$, $\sin\phi = \frac{X_L}{Z}$
- when X_L is increased from 0 to infinity and current is decreased from higher value to lower value. and its phase angle will be $\theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right)$ and it is same as series R-L circuit.



- To get Total current circle add Vectorially the current I_C and I_L.

Case (2): Varying X_C



- here current I_L through inductor is constant since R_L and X_L are fixed and it lags the voltage vector OV by an angle $\theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right)$

- The current I_C through the capacitance is the vector OI_C. its amplitude is maximum and equal to V/R_C when $X_C = 0$

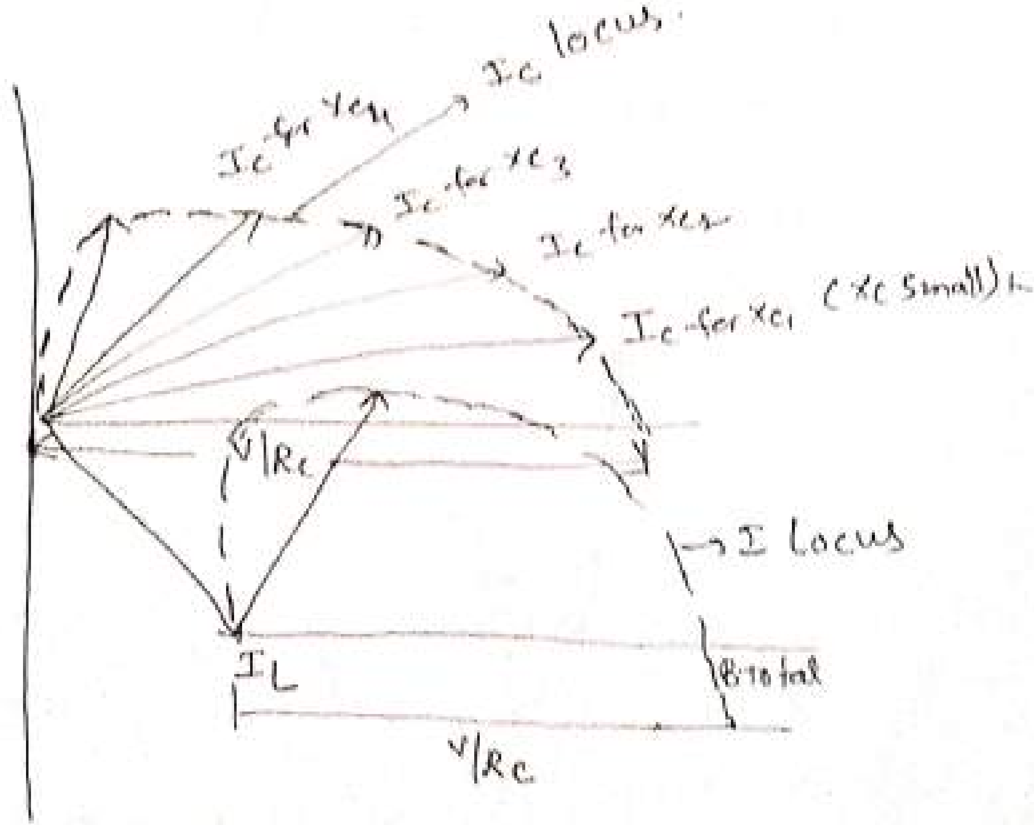
and it is in phase with applied voltage v

$\rightarrow$ $I = \frac{V}{\sqrt{R^2+X_C^2}}$ $\sin\theta = \frac{X_C}{Z}$ $\sin 30 = 0.5$, $\sin 45 = 0.707$, $\sin 60 = 0.866$

- when X_C is increased from 0 to infinity, its amplitude is decreases to lower value and phase will be lead by 90°

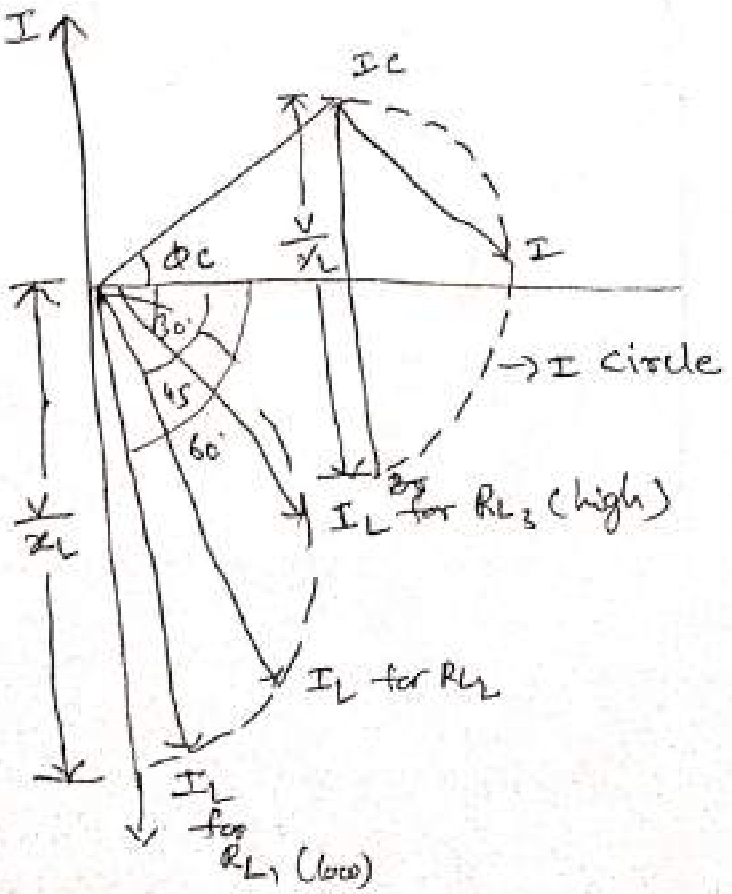
- Phase angle $\theta_C = \tan^{-1}\left(\frac{X_C}{R_C}\right)$

- The ~~cur~~ locus of current is a semicircle with diameter of length equal to $\frac{V}{R_C}$



Case(3): Varying R_L :

- The current I_C through capacitance is constant since R_C & C are fixed and it leads the voltage vector OV by an angle $\theta_C = Tan^{-1}\left(\frac{X_C}{R_C}\right)$.
- The current I_L through the inductance is OI_L. its amplitude is maximum and is equal to $\frac{V}{X_L}$ where R_L is zero. It's phase will be lagging the voltage by 90°
- when R_L is increased from '0' to infinity, its amplitude decreases to lower value $I = \frac{V}{\sqrt{R_L^2+X_L^2}}$ $\cos\theta = \frac{R_L}{Z}$
- phase angle is lagging the voltage 'V' by an angle $\theta_L = Tan^{-1}\left(\frac{X_L}{R_L}\right)$
- Locus of current is a semi circle with diameter is equal to V/R_L



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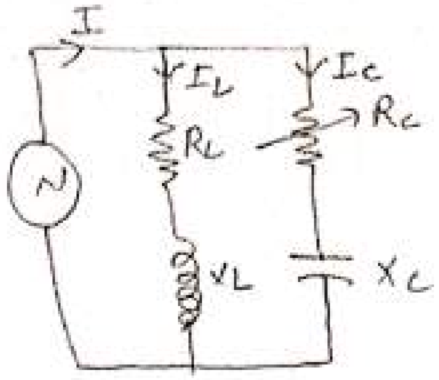
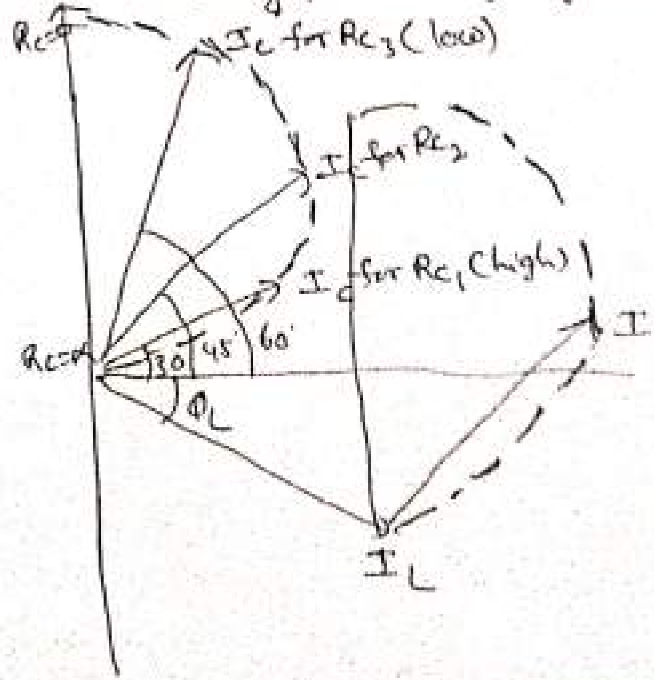
Case (4): Varying R_C:



- The current I_L through the inductor is constant since R_L & L are fixed and it lags the applied voltage vector OV by an angle $\theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right)$
- The current I_C through the capacitance is the vector OI_C. Its amplitude is maximum and is equal to $\frac{V}{X_C}$ when R_C is 0 and its phase will be leading the voltage by 90°
- when R_C is increased from 0 to infinity its amplitude is decreases to lower value (or 0) and it will be in phase with applied voltage 'V'.
- phase angle will be leading the voltage by an angle $\theta_c = \tan^{-1}\left(\frac{X_C}{R_C}\right)$
- $\cos\phi = \frac{R_C}{Z}$
- $I = \frac{V}{\sqrt{R_C^2 + X_C^2}}$



Introduction :-

Resonance :- Resonance is the phenomenon in which applied voltage and resultant current are always inphase with each other.

[or]

An Ac circuit is said to be in resonance if it exhibits unity power factor.

[or]

At resonance impedance becomes resistances only. i.e, $Z = R$.

[or]

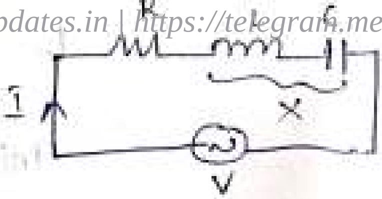
At resonance net reacten reactances become Zero.

Series Resonance :-

In Series RLC circuit, if any one of the parameter is Varied Such that applied Voltage & resultant Current are inphase with each other.

$Z = R + jX$

$= R + j(X_L - X_C)$



At resonance, $X = 0$

$X_L - X_C = 0$

$\boxed{X_L = X_C}$

$\boxed{Z = R}$

At resonance,

$X_L = X_C$

$\omega L = \frac{1}{\omega C}$

$2\pi f_r L = \frac{1}{2\pi f_r C}$

$f_r^2 = \frac{1}{4\pi^2 LC}$

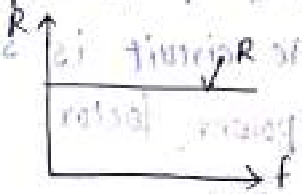
$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \ Hz}$ → Resonant frequency.

PF = 1 → power factor

Imp Graphical representation of Various elements in Series resonance

(1) Resistance :-

R is independent of frequency (f).



(2) Inductive reactance :-

$jX_L = j\omega L$

$jX_L = j2\pi fL$

$\boxed{X_L \propto f}$

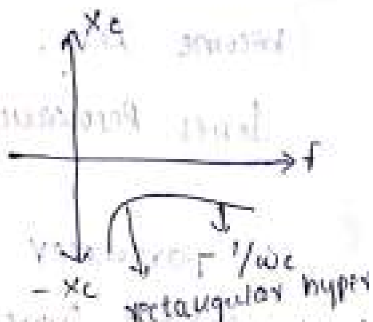
(3) capacitance :-

$-jX_C = -j \times \frac{1}{\omega C}$

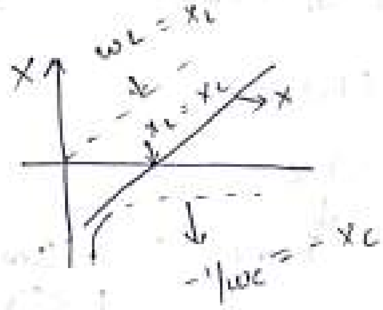
$= j(-1/\omega C)$

$= j\left(\frac{-1}{2\pi fC}\right)$

$\boxed{X_C \propto 1/f}$

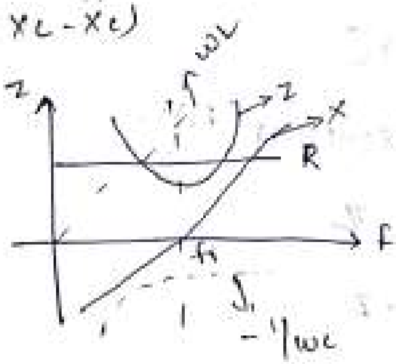


(4) Reactance (X):-



(5) Impedence:-

$Z = R + i(X_L - X_C)$



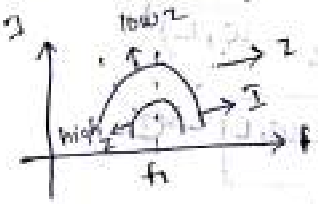
At resonant frequency, Z is minimum.

(6) Current:-

$I = V/Z$

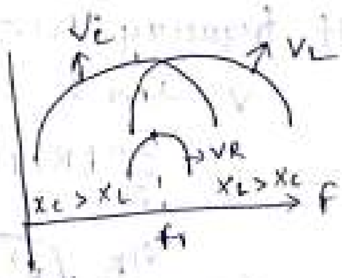
$I \propto 1/Z$

At resonant frequency, Z is minimum, I is maximum.



(7) Voltage:-

$V_R = iR$



Band width :-

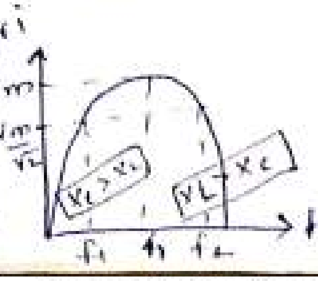
Bandwidth is the difference in frequencies at which voltage or current is equal to 70.7% of its maximum value. $\boxed{\text{Half power frequencies.}}$

B.W $= f_2 - f_1$

$f_1 \rightarrow$ lower cut off frequency

$f_2 \rightarrow$ Upper " " "

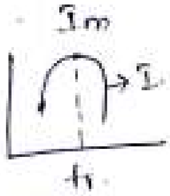
$0.707 V_m = 70.7\% V_m$



Imp
Relationship b/w Bandwidth & RLC :-

At resonant frequency (f_r),

we know, $V = I Z$
$\downarrow \quad \downarrow$
$V_{rms} \quad I_{rms}$



At f_r :-

$V = I_m R$

$$\boxed{I_m = \frac{V}{R}} \rightarrow (1)$$

At lower cut off frequency (f_1):-

we know, $V_{rms} = I_{rms} R$

$$V = \frac{I_m}{\sqrt{2}} \cdot Z$$

$$V = \frac{V}{\sqrt{2}R} (R + j(X_{C_1} - X_{L_1}))$$

$$\sqrt{2}R = \sqrt{R^2 + (X_{C_1} - X_{L_1})^2}$$

both sides on squarring

$$2R^2 = R^2 + \left(\frac{1}{\omega_1 C} - \omega_1 L\right)^2$$

$$R^2 = (1/\omega_1 C - \omega_1 L)^2$$

$$\boxed{R = \frac{1}{\omega_1 C} - \omega_1 L} \rightarrow (2)$$

At high cut off frequency (f_2):-

$$V = I_{rms} Z$$

$$= \frac{I_m}{\sqrt{2}} (R + j(X_{L_2} - X_{C_2}))$$

$$V = \frac{V/R}{\sqrt{2}} \sqrt{R^2 + (X_{L_2} - X_{C_2})^2}$$

$$\sqrt{2}R = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2}$$

Squarring on both sides.

$$2R^2 = R^2 + (X_{L_2} - X_{C_2})^2$$

$$R^2 = \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2$$

$$\boxed{R = \omega_2 L - \frac{1}{\omega_2 C}} \rightarrow (3)$$

Equating equ ② & ③

$$R = \omega_2 L - \frac{1}{\omega_2 C} = \frac{1}{\omega_1 C} - \omega_1 L$$

$$\omega_2 L + \omega_1 L = \frac{1}{\omega_1 C} + \frac{1}{\omega_2 C}$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left(\frac{\omega_2 + \omega_1}{\omega_1 \omega_2}\right)$$

$$L = \frac{1}{C}\left(\frac{1}{\omega_1 \omega_2}\right)$$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC}} \rightarrow ④$$

we know, $f_r = \frac{1}{2\pi\sqrt{LC}}$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\boxed{\omega_r^2 = \frac{1}{LC}} \rightarrow ⑤$$

Substitute equ ⑤ in equ ④

$$\omega_1 \omega_2 = \omega_r^2$$

$$\boxed{\omega_r = \sqrt{\omega_1 \omega_2}}$$

Hence $\boxed{f_r = \sqrt{f_1 f_2}}$

∴ Resonant frequency is the geometrical mean of f_1 & f_2.

Adding equ ② & ③

$$R + R = \frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C}$$

$$2R = \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) + L(\omega_2 - \omega_1)$$

$$2R = \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) + L(\omega_2 - \omega_1)$$

we know $\omega_1 \omega_2 = 1/LC$ (from equ 4).

$$2R = \frac{1}{C}\cdot\left(\frac{\omega_2 - \omega_1}{1/LC}\right) + L(\omega_2 - \omega_1)$$

$$2R = \frac{1}{\cancel{C}}(\omega_2 - \omega_1)L\cancel{C} + L(\omega_2 - \omega_1)$$

$$2R = (\omega_2 - \omega_1)(L+L)$$

$$2R = 2L(\omega_2 - \omega_1)$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$2\pi f_2 - 2\pi f_1 = R/L$$

$$2\pi (f_2 - f_1) = \frac{R}{L}$$

$$(f_2 - f_1) = \frac{R}{2\pi L}$$

$$\boxed{\text{Bandwidth} = \frac{R}{2\pi L}}$$

* Quality factor:- Quality factor (Q) is defined as the ratio of voltage across inductor (or) capacitor to be the Sup voltage.

It is the indication of quality of coil (L (or) ~~R L C~~

$$\therefore Q = \frac{V_L}{V}$$

At resonance,

$$V = V_R$$

$$Q = \frac{V_L}{V_R}$$

$$= \frac{I X_L}{I R}$$

$$\boxed{Q = \frac{X_L}{R}}$$

$$\boxed{Q = \frac{\omega L}{R}}$$

$$Q = \frac{V_C}{V}$$

At resonance,

$$V = V_R$$

$$Q = \frac{V_C}{V_R}$$

$$= \frac{I X_C}{I R}$$

$$\boxed{Q = \frac{X_C}{R}}$$

$$\boxed{Q = \frac{1}{\omega C R}}$$

Imp.
*** Quality factor = $\boxed{Q = 2\pi \times \frac{\text{Energy stored/cycle}}{\text{Energy dissipated/cycle}}}$

Relation ship b/w Bandwidth, Quality factor & Resonant frequency:-

we know, Bandwidth = $\frac{R \times f_r}{2\pi f_r L}$

$$= \frac{R \times f_r}{\omega_r L}$$

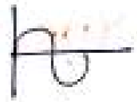
$$= \frac{R \times f_r}{X_L}$$

$$B\cdot\omega = \frac{f_r}{(X_L/R)} = \frac{f_r}{Q} \Rightarrow \boxed{B\cdot\omega = \frac{f_r}{Q}}$$

Half power frequencies in terms of resonance frequency:-

For Symmetrical wave,

$$f_r = \frac{f_1+f_2}{2} \rightarrow (1)$$



From eq (1),

$$f_1 = 2f_r - f_2 \rightarrow (2)$$

we know, Bandwidth $= f_2 - f_1 = \frac{R}{2\pi L} \rightarrow (3)$

Substitute (f_1) in eq (3)

$$f_2 - 2f_r + f_2 = \frac{R}{2\pi L}$$

$$2f_2 - 2f_r = \frac{R}{2\pi L}$$

$$2(f_2 - f_r) = \frac{R}{2\pi L}$$

$$\boxed{f_2 = \frac{R}{4\pi L} + f_r \; Hz} \rightarrow (4)$$

llly from eq (1),

$$f_2 = 2f_r - f_1 \rightarrow (5)$$

Substitute eq (5) in eq (3)

$$2f_r - f_1 - f_1 = \frac{R}{2\pi L}$$

$$2f_r - 2f_1 = \frac{R}{2\pi L}$$

$$2(f_r - f_1) = \frac{R}{2\pi L}$$

$$f_r - f_1 = \frac{R}{4\pi L}$$

$$\boxed{f_1 = f_r - \frac{R}{4\pi L}} \rightarrow (6)$$

Selectivity (S):- It is reciprocal of Quality factor (Q).

$$S = \frac{1}{Q}$$

$$= \frac{1}{\frac{R}{2\pi L}}$$

$$\boxed{S = \frac{2\pi L}{R}} \quad (\text{or}) \quad S = \frac{1}{Q}$$

$$= \frac{1}{f_r/B.\omega} = \frac{B.\omega}{f_r}$$

Voltage across L and C at resonance:-

$$V_L = I(jX_L)$$

at resonance $\Rightarrow I = V/R$.

$$V_L = \frac{V}{R} \cdot j(\omega_r L)$$

$$= j \cdot \frac{\omega_r L}{R} \cdot V$$

$$= j \cdot \frac{X_L}{R} \cdot V$$

$$\therefore \boxed{V_L = jQV}$$

$$V_C = I(-jX_C)$$

At resonance $\rightarrow I = V/R$

$$V_C = V/R(-jX_C) = -j \cdot \left(\frac{X_C}{R}\right)V$$

$$\therefore \boxed{V_C = -jQV}$$

Frequency for voltage across Inductor is maximum:-

we know,

$$V_L = IX_L \text{ (in magnitude)}$$

$$= \frac{V}{Z} X_L$$

$$= \frac{V\omega L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{V\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V^2\omega^2L^2C^2}{R^2\omega^2C^2 + (\omega^2LC - 1)^2} \rightarrow (1)$$

$$V_L^2 = \frac{V^2\omega^2L^2}{R^2 + (\omega L - 1/\omega C)^2}$$

$$V_c = \frac{V}{\omega C\sqrt{R^2+(\omega L - 1/\omega C)^2}}$$

$$V_c^2 = \frac{V^2}{\omega^2 C^2\left[R^2+(\omega L - 1/\omega C)^2\right]}$$

$$V_c^2 = \frac{V^2 \cdot \omega^2 C^2}{\omega^2 C^2\left(R^2\omega^2C^2+(\omega^2LC-1)^2\right)}$$

$$V_c^2 = \frac{V^2}{R^2\omega^2C^2+(\omega^2LC-1)^2}$$

$$\omega^4L^2C^2+1-2\omega^2LC$$

To get frequency, V_c is maximum.

$$\frac{dV_c^2}{d\omega}=0$$

$$\left(R^2\omega^2C^2+(\omega^2LC-1)^2\right)(0) - V^2\left(2\omega R^2C^2+4\omega^3L^2C^2-4\omega LC\right)=0$$

$$2\omega R^2C^2+4\omega^3L^2C^2-4\omega LC=0$$

$$2\omega C\left(R^2C+2\omega^2L^2C-2L\right)=0$$

$$R^2C+2\omega^2L^2C-2L=0$$

$$2\omega^2L^2C = 2L-R^2C$$

$$\omega^2 = \frac{2L}{2L^2C}-\frac{R^2C}{2L^2C}$$

$$\omega^2 = \frac{1}{LC}-\frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}$$

$$\boxed{f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}}$$

$f_1 = 71.18$ Hz
$f_2 = 72.1$ Hz
$f_r = 70.81$ Hz

1) A Series ckt with $R = 10\Omega$, $L = 0.1$ H, & $C = 50\ \mu F$, as an applied voltage $V = 50$ at an angle 0° with a variable frequency. find (a) Resonant frequency, (b) frequency at which voltage across inductor is maximum, (c) frequency at which voltage across capacitor is max. d) voltage across inductor at resonant.

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Q) An inductance of 0.5 H, Resistances of 5 Ω & Capacitances 8 μF are in series across a 220 V AC supply, (a) calculate the frequency at which ckt resonance, (b) find the current at resonance, bandwidth of power frequency & voltage across inductance & capacitances.

A) Given data,

$R = 5\,\Omega$,

$L = 0.5\,H$,

$C = 8\,\mu F = 8 \times 10^{-6} F$

(a) frequency at resonance :- $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.5 \times 0.8 \times 10^{-6}}}$

$= 79.58\ Hz$.

(b) $V = IZ$

at resonance $Z = R$,

$V = IR$

$I = V/R$

$= \frac{220}{5} = 44\,A$.

∴ Bandwidth $= B.W = \frac{R}{2\pi L}$

$= \frac{5}{2 \times 3.14 \times 0.5}$

$= 1.59\ Hz$.

half power frequencies $= f_1 = f_r - \frac{R}{4\pi L}$

$= 79.58 - \frac{5}{4 \times 3.14 \times 0.5}$

$= 78.79\ Hz$.

$f_2 = f_r + \frac{R}{4\pi L}$

$= 79.58 + \frac{5}{4 \times 3.14 \times 0.5} = 80.40\ Hz$.

Quality factor :- $Q = \frac{f_r}{B.W}$

$= \frac{79.58}{1.59}$

$= 50$

Voltage across Inductor (V_L) :-

we know, $Q = \frac{V_L}{V_R} = \frac{V_L}{V}$

$V_L = Q \times V$

$V_L = 50 \times 220$

$= 11 \text{ kV}$

Voltage across capacitor (V_C) :-

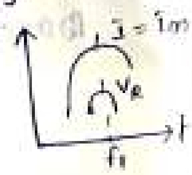
$Q = \frac{V_C}{V_R} = \frac{V_C}{V}$

$V_C = QV$

$= 11 \text{ kV}$.

Note :-

In Series resonance, current $I = I_m$ & voltage $V = V_R$



3) In series RLC ckt has quality factor of 5 at 50 Radians/sec. The current flowing through ckt at resonance is 10 amperes & applied voltage 100V. The total impedence of the ckt is 20Ω, find the ckt elements.

A) Given data,

Quality ~~packing~~ factor = 5

$\omega = 50$ rad/sec.

At resonance $I = I_m = 10A$

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$V = 100V$

$Z = 20\Omega$

we know,

$V = IZ$

At resonance, $V = I_m R$

$$R = \frac{V}{I_m} = \frac{100}{10} = 10\Omega$$

we know,

$$Q = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$L = \frac{QR}{\omega}$$

$$= \frac{5 \times 10}{50}$$

$$= 1\,H$$

& Also

we know,

$$Q = \frac{X_C}{R} = \frac{1}{\omega C R}$$

$$Q\omega R = \frac{1}{C}$$

$$C = \frac{1}{Q\omega R}$$

$$= \frac{1}{5 \times 50 \times 10}$$

$$= 400\,\mu F$$

4) In the series RLC ckt with $L = 0.5$ H, as an instantaneous voltage $v = 70.7 \sin(500t + 30°)$ v & instantaneous current $i = 1.5 \sin(500t)$ A. find the value's of R & C. At what frequency will the ckt be resonance.

A) Given data, $L = 0.5\,H$

$v = 70.7 \sin(500t + 30°)\,v$ (ω)

$i = 1.5 \sin(500t)\,A$

$V = 70.7 \angle 30° \Rightarrow V_m = 70.7$

$i = 1.5 \angle 0° \Rightarrow I_m = 1.5$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{70.7}{\sqrt{2}} = 49.9\,V \approx 50\,V$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{1.5}{\sqrt{2}} = 1.06\,A$$

we know,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{49.9\angle 30^\circ}{1.06\angle 0^\circ}$$

$$= 47.07\angle 30$$

$$= 47.07(\cos 30 + j\sin 30)$$

$$= 47.07\left(\sqrt{3}/2 + j\cdot 1/2\right)$$

$$= 40.76 + j\,23.53.$$

$$= R + jX$$

where $R = 40.76\,\Omega$, $X = 23.53\,\Omega$

we know,

$$X = X_L - X_C$$

$$23.53 = \omega L - \frac{1}{\omega C}$$

$$23.53 = 500(0.5) - \frac{1}{500 \times C}$$

$$23.53 = 250 - \frac{1}{500 \times C}$$

$$23.53 - 250 = -\frac{1}{500C}$$

$$-226.47 = -\frac{1}{500C}$$

$$C = \frac{1}{500 \times 226.47}$$

$$= \frac{1}{113235}$$

$$\boxed{C = 8.83\,\mu F}$$

(b) $$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{0.5 \times 8.83 \times 10^{-6}}} = 75.78\,Hz$$

5) A Series RLC ckt with $R = 25\Omega$, $L = 0.6H$, results in a leading phase angle of $60°$ at frequency of 40 Hz. find the value of the C & at what frequency the ckt will be resonant.

A) Given data,

$R = 25\Omega$.

$L = 0.6H$

leading phase angle $(\phi) = 60°$.

$f = 40\ Hz$.

here leading phase angle. so $Z = R + j(X_C - X_L)$.

then $\tan\phi = \frac{X_C - X_L}{R}$

$\tan 60° = \frac{\frac{1}{\omega C} - \omega L}{25}$

$2 \times 3.14 \times 40$

$\sqrt{3}(25) = \frac{1}{2\pi f C} - 2\pi f L$

$= \frac{1}{251.2C} - 251.2(0.6)$

$43.30 = \frac{1}{251.2C} - 150.72$

$43.3 + 150.72 = \frac{1}{251.2C}$

$194.02 = \frac{1}{251.2C}$

$C = \frac{1}{251.2(194.02)}$

$$\boxed{C = 2.05 \times 10^{-5} F}$$

Resonant frequency $= f_r = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2 \times 3.14\sqrt{0.6 \times 2.05 \times 10^{-5}}}$

$$\boxed{f_r = 45.40\ Hz}$$

Admittance :- [Y]

It is the reciprocal of impedence (z)

$$Y = \frac{1}{Z}$$

Impedence parameter	Admittance parameter
Z	$Y = 1/Z$
R	$1/R = G$ [conductance]
X_L	$\frac{1}{X_L} = B_L$ (susceptive Inductance)
X_C	$\frac{1}{X_C} = B_C$ (susceptive capacitance)
$Z = R + j(X_L - X_C)$	$Y = G + j(B_C - B_L)$.

Impedence :- $V = IZ$

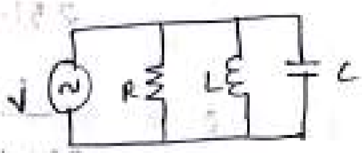
$I = V/Z$

$= V \cdot 1/Z$

$\boxed{I = V \cdot Y} \rightarrow$ Admittance.

Parallel Resonance:-

Consider parallel RLC ckt excited by alternating voltage as shown in figure.



Now express all the parameters in terms of ohms & ckt is redrawn as

Take $Y_1 = \frac{1}{R}$, $Y_2 = \frac{1}{jX_L} = \frac{-j}{X_L}$, $Y_3 = \frac{1}{-jX_C} = \frac{j}{X_C}$

The admittance $Y = Y_1 + Y_2 + Y_3$.

$$= \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

$$= \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$= \frac{1}{R} + j(B_C - B_L).$$

$$Y = G + j(B_C - B_L) \rightarrow (1)$$

$$Y = G + jB \rightarrow (1)$$

At resonance, $B = 0$

$$B_C - B_L = 0$$

$$B_C = B_L$$

$$\frac{1}{X_C} = \frac{1}{X_L}$$

$$\omega_r C = \frac{1}{\omega_r L}$$

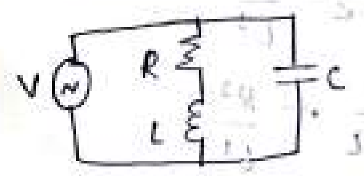
$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

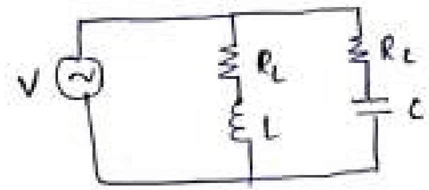
Resonant frequency $\Rightarrow f_r = \dfrac{1}{2\pi\sqrt{LC}}$ Hz.

1) Find the resonant frequency for the following ckt's.

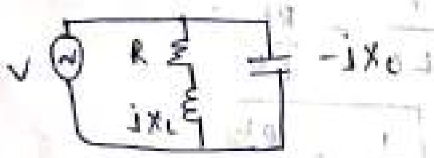
(A)



(B)



A)



$$Y_1 = \frac{1}{R + jX_L}$$

$$Y_2 = \frac{1}{-jX_C} = \frac{j}{X_C}$$

Total admittance : $Y = Y_1 + Y_2$.

$$Y = \frac{1}{R+jX_L} + \frac{j}{X_C}$$ [on Rationalization]

$$= \frac{R-jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$Y = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right) \rightarrow (1)$$

At resonance,

Imaginary part = 0.

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\omega_r C - \frac{\omega_r L}{R^2+(\omega_r L)^2} = 0$$

$$\omega_r C = \frac{\omega_r L}{R^2+(\omega_r L)^2}$$

$$R^2 + (\omega_r L)^2 = \frac{L}{C}$$

$$(\omega_r L)^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{L}{CL^2} - \frac{R^2}{L^2}$$

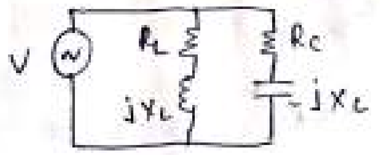
$$\omega_r^2 = \frac{1}{CL} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$\therefore$ Resonant frequency = $\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$

B)



$$Y_1 = \frac{1}{R_L + jX_L}, \quad Y_2 = \frac{1}{R_C - jX_C}$$

Total admittance :- $Y = Y_1 + Y_2$

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$= \frac{R_L}{R_L^2 + X_L^2} - j\frac{X_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + j\frac{X_C}{R_C^2 + X_C^2}$$

$$= \frac{R_L}{(R_L^2 + X_L^2)} + \frac{R_C}{(R_C^2 + X_C^2)} + j\left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right)$$

At resonance,

imaginary part $= 0$

$$\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} = 0$$

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$\frac{1}{\omega_r C\left(R_C^2 + \left(\frac{1}{\omega_r C}\right)^2\right)} = \frac{\omega_r L}{R_L^2 + (\omega_r L)^2}$$

$$\frac{1}{\omega_r C}\left(R_L^2 + (\omega_r L)^2\right) = \omega_r L\left(R_C^2 + \left(\frac{1}{\omega_r C}\right)^2\right)$$

$$\frac{\omega_r C}{\left(R_C^2 (\omega_r C)^2 + 1\right)} = \frac{\omega_r L}{R_L^2 + (\omega_r L)^2}$$

$$C\left(R_L^2 + (\omega_r L)^2\right) = L\left(R_C^2 \omega_r^2 C^2 + 1\right).$$

$$CR_L^2 + C\omega_r^2 L^2 = L R_C^2 \omega_r^2 C^2 + L$$

$$\text{(or)} \quad \omega_r^2 (CL^2 - LR_C^2 C^2) = L - CR_L^2$$

$$\omega_r^2 = \frac{L - CR_L^2}{CL^2 - LR_C^2 C^2}$$

$$\omega_r^2 = \frac{C\left[L/C - R_L^2\right]}{C^2 L\left[\frac{L}{C} - R_C^2\right]}$$

$$\omega_r^2 = \frac{R_L^2 - L/C}{LC(R_C^2 - L/C)}$$

$$\omega_r = \sqrt{\frac{R_L^2 - L/C}{LC(R_C^2 - L/C)}}$$

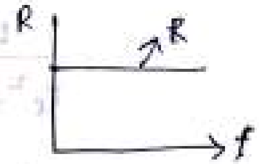
$$2\pi f_r = \sqrt{\frac{R_L^2 - L/C}{LC(R_C^2 - L/C)}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}} \quad \#3} \rightarrow \text{Resonant frequency.}$$

Graphical representation of various elements in parallel Resonance

(1) Resistance (R):-

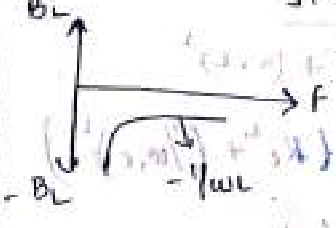
R is independent of frequency.



(2) Inductive Reactance :- (B_L)

$$B_L = \frac{1}{jX_L} = \frac{-j}{X_L} = j\left(\frac{-1}{X_L}\right) = j\left(\frac{-1}{\omega L}\right)$$

$$= j\left(\frac{-1}{2\pi f L}\right)$$



$$\boxed{B_L \propto \frac{1}{f}}$$

(3) Capacitive reactance :-

$$B_C = \frac{1}{-jX_C} \cdot \frac{j}{j} = \frac{j}{X_C}$$

$$B_C = j\omega C = j2\pi f C$$

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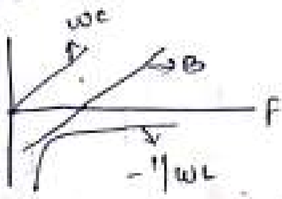
$\boxed{B_c \propto F}$

(4) Susceptance (B)



$Z = R + j(X_L - X_C)$

$Y = G + j(B_C - B_L)$

$= G + j\left(\omega C - \frac{1}{\omega L}\right)$.

(5) Impedence & admitance:

we know,

$Y = G + jB$

At resonance, $B = 0$

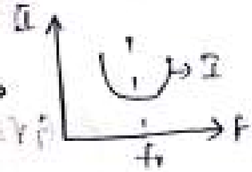
$\therefore Y = G$, i.e. Y is minimum

$Z = 1/Y \Rightarrow$

(6) Voltage & Current :-

we know, $I = VY$

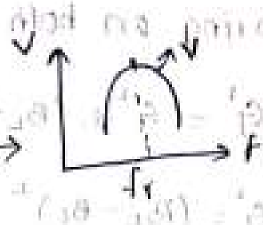
$\rightarrow$ $I \propto Y$



$I = VY$

$V = I/Y$ $\rightarrow$

$V \propto 1/Y$



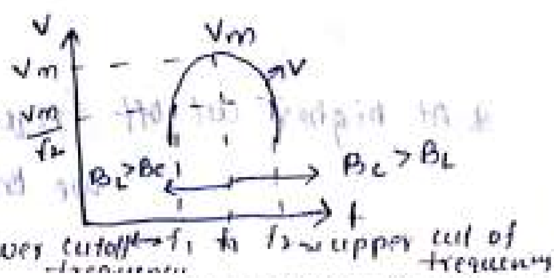
Note :- In parallel resonance, $V = V_m$

$I = I_R$

Bandwidth (BW) :-

Like Series resonance, bandwidth of parallel resonance also defined as the difference in frequency at which voltage is equal to 70.7 % is of maximum value.

Bandwidth $= f_2 - f_1$ Hz.



Relation b/w Bandwidth & RLC:

* At f_r:-

We know, $i = VY$

At resonance,

$$I = V_m \cdot G$$ [$\because Y = G + j\cancel{B}^{0}$]

$Y = G.$

$$\boxed{V_m = \frac{I}{G}} \rightarrow (1)$$

* At lower cut off frequency (f_1):

we know,

$$I = VY$$

$$I = \frac{V_m}{\sqrt{2}} (G + j(B_L - B_C))$$

$$I = \frac{I/G}{\sqrt{2}} (G + j(B_L - B_C))$$

$$1 = \frac{1}{\sqrt{2}G} (G + j(B_L - B_C))$$

$$G\sqrt{2} = G + j(B_L - B_C) \Rightarrow G\sqrt{2} = \sqrt{G^2 + (B_L - B_C)^2}$$

Squaring on both sides.

$$2G^2 = G^2 + (B_L - B_C)^2$$

$$G^2 = (B_L - B_C)^2$$

$$G = B_L - B_C$$

$$1/R = \frac{1}{X_L} - \frac{1}{X_C}$$

$$\frac{1}{R} = \frac{1}{\omega_1 L} - \frac{1}{\omega_1 C}$$

$$1/R = \frac{1}{\omega_1 L} - \omega_1 C \rightarrow (2)$$

* At higher cut off frequency:-(f_2)

we know;

$$I = VY$$

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$$I = \frac{V_m}{\sqrt{2}} \cdot (G + j(B_C - B_L))$$

$$Z = \frac{V/G}{\sqrt{2}} (G + j(B_C - B_L))$$

$$G\sqrt{2} = G + j(B_C - B_L)$$

$$G\sqrt{2} = \sqrt{G^2 + (B_C - B_L)^2}$$

squarring on both sides

$$2G^2 = G^2 + (B_C - B_L)^2$$

$$G^2 = (B_C - B_L)^2$$

$$G = (B_C - B_L)$$

$$\frac{1}{R} = \frac{1}{X_C} - \frac{1}{X_L}$$

$$\frac{1}{R} = \omega_2 C - \frac{1}{\omega_2 L} \rightarrow (3)$$

Equating (2) & (3)

$$\frac{1}{\omega_1 L} - \omega_1 C = \omega_2 C - \frac{1}{\omega_2 L}$$

$$\frac{1}{\omega_1 L} + \frac{1}{\omega_2 L} = \omega_1 C + \omega_2 C$$

$$\frac{1}{L}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = (\omega_1 + \omega_2) C$$

$$\frac{1}{L}\left(\frac{\omega_2 + \omega_1}{\omega_1 \omega_2}\right) = (\omega_1 + \omega_2) C$$

$$\frac{1}{L(\omega_1 \omega_2)} = C$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

we know from RLC parallel ckt $\omega_r^2 = \frac{1}{LC}$

$$\omega_1 \omega_2 = \omega_r^2$$

$$\omega_r = \sqrt{\omega_1 \omega_2} \Rightarrow \boxed{f_r = \sqrt{f_1 f_2}} \rightarrow (5)$$

$\therefore$ Resonant frequency (f_r) is the geometrical mean of f_1 & f_2.

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Adding eq ② & ③

$$\frac{1}{R} + \frac{1}{R} = \frac{1}{\omega_1 L} - \omega_1 C + \omega_2 C - \frac{1}{\omega_2 L}$$

$$\frac{2}{R} = \frac{1}{L}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] + C(\omega_2 - \omega_1)$$

$$\frac{2}{R} = \frac{1}{L}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) + C(\omega_2 - \omega_1)$$

$$= (\omega_2 - \omega_1)\left[\frac{1}{L(\omega_1 \omega_2)} + C\right]$$

$$= (\omega_2 - \omega_1)\left(\frac{1}{L \times \frac{1}{LC}} + C\right)$$

$$= (\omega_2 - \omega_1)(C + C)$$

$$\frac{2}{R} = (\omega_2 - \omega_1)(2C)$$

$$\omega_2 - \omega_1 = 1/RC$$

$$f_2 - f_1 = \frac{1}{2\pi RC}$$

$$\boxed{\text{Band width} = \frac{1}{2\pi RC}}$$

Half power frequencies interms of resonant frequency (f_r):-

for Symmetrical wave,

$$f_r = \frac{f_1 + f_2}{2}$$

let $f_1 = 2f_r - f_2 \rightarrow$ ①

we know,

$$B.\omega = f_2 - f_1 = \frac{1}{2\pi RC} \longrightarrow ②$$

⑤ in ①

$$f_2 - 2f_r + f_2 = \frac{1}{2\pi RC}$$

$$2f_2 = \frac{1}{2\pi RC} + 2f_r$$

$$\boxed{f_2 = \frac{1}{4\pi RC} + f_r}$$

Similarly,

$$f_r = \frac{f_1 + f_2}{2}$$

$$f_2 = 2f_r - f_1 \rightarrow (3)$$

(3) in eqn (2)

$$2f_r - f_1 - f_1 = \frac{1}{2\pi RC}$$

$$2f_r - 2f_1 = \frac{1}{2\pi RC}$$

$$f_r - f_1 = \frac{1}{4\pi RC}$$

$$-f_1 = \frac{1}{4\pi RC} - f_r$$

$$\boxed{f_1 = f_r - \frac{1}{4\pi RC}\ \text{Hz}}$$

Quality factor:-

It is defined as "The ratio of current through the inductor (or) capacitor to the Supply current.

$$Q = \frac{I_L}{I}$$

At resonance, $I = I_R$

$$Q = \frac{I_L}{I_R}$$

$$= \frac{V/X_L}{V/R}$$

$$\boxed{Q = \frac{R}{X_L}}$$

$$\boxed{Q = \frac{R}{\omega L}}$$

$$Q = \frac{I_C}{I}$$

At resonance, $I = I_R$

$$Q = \frac{I_C}{I_R}$$

$$= \frac{V/X_C}{V/R}$$

$$\boxed{R = \frac{R}{X_C}}$$

$$R = \frac{R}{1/\omega C}$$

$$\boxed{Q = R\omega C}$$

Relation b/w Bandwidth, Quality factor & Resonant frequency:-

we know, $\text{Bandwidth} = \frac{1}{2\pi RC}$

Multiply numerator & dinaminator by f_r

$$B.\omega = \frac{f_r}{2\pi f_r RC}$$

$$\boxed{B.\omega = \frac{f_r}{\omega_r RC}}$$

$$\boxed{B\omega = \frac{f_r}{Q}}$$ were $\boxed{\text{where} = Q = R\omega C}$

Also,
$$B.\omega = \frac{f_r}{\omega_r RC}$$
$$= f_r \cdot \frac{x_c}{R}$$

Selectivity :- (S) =

Selectivity is the reciprocal of Quality factor.

$$S = 1/Q = \frac{1}{R/\omega L}$$
$$S = \frac{\omega L}{R}$$

$$S = 1/Q$$
$$S = \frac{1}{\omega RC}$$

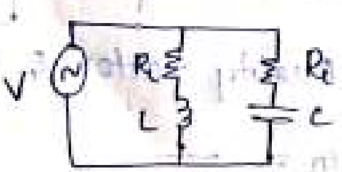
1) What is dynamic impedence?

A) Dynamic imdepence :- Dynamic impedence ~~are~~ or simply dynamic resistance is a pure resistance which is defined only at resonant frequency.

Dynamic impedence :- Z_{dy} (or) $R_{dy} = \frac{L}{CR}$

2) Write the Equation for R_L & R_C which causes the ckt to resonant to at all frequencies.

A)



$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

if R_L & R_C are very small,

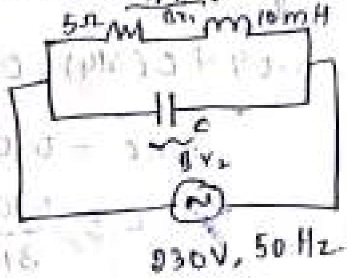
$R_L^2 - L/C = 0$ | $R_C^2 - L/C = 0$

$R_L^2 = \frac{L}{C}$ | $R_C^2 = L/C$

$R_L = \sqrt{\frac{L}{C}}$ | $R_C = \sqrt{L/C}$

problems on Parallel Resonance:-

1) A parallel ckt has 2nd branches, 1st branche has a resistances of 5 Ω connected in series with an inductance of 10 mH. A capacitor is connected in 2nd branch. The parallel ckt is connected across a 230V, 50 Hz supply. If the ckt to be in resonance. find the value of capacitance & also current drawn from the supply & also find currents in branchs 1 & 2.



let $Y_1 = \frac{1}{R+jX_L} = \frac{1}{R+j\omega L}$

$= \frac{1}{R+j2\pi fL}$

$= \frac{1}{5+j(2\times3.14\times50\times10\times10^{-3})}$

$= \frac{1}{5+j(3.14)}$

$Y_2 = \frac{1}{-jX_C} = \frac{j}{X_C} = j\omega C$

$= j(2\pi f)C$

$= j(2\times3.14\times50)(C)$

$= j(314)C$

$Y = Y_1 + Y_2$

$$Y_1 = \frac{1}{5+j(3.14)}$$

$$= \frac{5-j(3.14)}{5^2+(3.14)^2}$$

$$= \frac{5-j(3.14)}{25+9.85}$$

$$= 0.14 - j(0.09).$$

$$Y = Y_1 + Y_2$$

$$= 0.14 - j(0.09) + j(314)C$$

$$= 0.14 + j(-0.09 + C\,314)$$

At resonance, imaginary part = 0

$$-0.09 + C(314) = 0$$

$$314C = 0.09$$

$$C = \frac{0.09}{314}$$

$$= 2.86 \times 10^{-4}$$

$$= 286 \times 10^{-6}$$

$$= 286\ \mu F$$

Current in branch 1,

$$I_1 = \frac{V}{Z_1} = \frac{230\angle 0}{R+j\omega L}$$

$$= \frac{230\angle 0}{5+j(314)}$$

$$= \frac{230\angle 0}{5.9\angle 32.12}$$

$$= 38.98\angle -32.12.$$

Current in branch 2, $I_2 = \frac{V}{Z_2} = \frac{V}{-jX_C}$

$$= \frac{230\angle 0}{-j11.10}$$

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$= \frac{220 \angle 0}{11.10 \angle -90^\circ}$

$= 22.52 \angle 90^\circ$

Supply current $(I) = I_1 + I_2$

$= 38.98 \angle -32.12 + 22.52 \angle 90^\circ$

$= 38.98(\cos(-32.12) + j\sin(-32.12)) + 22.52(\cos 90 + j\sin 90)$

$= 38.98[0.84 + j(-0.53)] + 22.52(0+j)$

$= 32.74 - j(20.65) + j(22.52)$

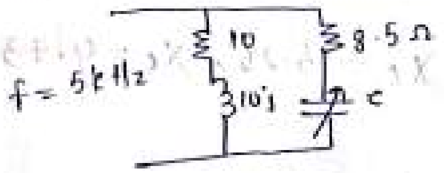
$= 32.74 + j(22.52 - 20.65)$

$= 32.74 + j(1.87)$

$= 32.79 \angle 3.26$

2) An impedence of $Z_1 = 10 + 10j\ \Omega$ is connected with ||el to another impedence of resistances $8.5\ \Omega$ & variable capacitance connected in series. find C such that the ckt is in resonance at 5 kHz.

A)



$Y_1 = \frac{1}{10 + 10j}$, $Y_2 = \frac{1}{8.5 - jX_C}$

$Y_1 = \frac{10 - j10}{10^2 + 10^2}$, $Y_2 = \frac{1}{8.5 - jX_C}$

$Y = Y_1 + Y_2$

$= \frac{10 - j10}{200} + \frac{8.5 + jX_C}{(8.5)^2 + (X_C)^2}$

$= \frac{10}{200} - j\frac{10}{200} + \frac{8.5 + jX_C}{72.25 + X_C^2}$

$$= \frac{10}{200} + \frac{8.5}{72.25+X_c^2} + j\left(\frac{-10}{200} + \frac{X_c}{72.25+X_c^2}\right)$$

$$= \frac{1}{20} + \frac{8.5}{72.25} + j\left(\frac{-1}{20} + \frac{X_c}{72.25+X_c^2}\right)$$

$$= 0.16 + j\left(-0.05 + \frac{X_c}{72.25+X_c^2}\right)$$

At resonance, imaginary part = 0

$$-0.05 + \frac{X_c}{72.25+X_c^2} = 0$$

$$\frac{X_c}{72.25+X_c^2} = 0.05$$

$$X_c = (0.05)(72.25+X_c^2)$$

$$X_c = 3.61 + 0.05X_c^2$$

$$3.61 + 0.05X_c^2 - X_c = 0$$

$$3.61 + X_c(0.05X_c - 1) = 0$$

$$3.61 = -X_c(0.05X_c - 1)$$

$$3.61 = X_c(1 - 0.05X_c)$$

$$X_c = 15.26,\ X_c = 4.73.$$

$$C_1 = X_c = \frac{1}{\omega C_1}$$

$$15.26 = \frac{1}{\omega C_1}$$

$$15.26 = \frac{1}{2\pi f C_1}$$

$$15.26 = \frac{1}{2\times 3.14\times 5\times 10^3 C_1} \Rightarrow 15.26 = \frac{3.18\times 10^{-5}}{C_1}$$

$$\cancel{15.26 = \frac{3.18\times10^{-5}}{C_1}} \Rightarrow C_1 = \frac{3.18\times 10^{-5}}{15.26}$$

$$\cancel{C = \frac{1}{3.18\times10^{-5}\times 15.26}} = 2.08\times 10^{-6}$$

$$= 2.08\ \mu F.$$

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$$C_2 = X_c = \frac{1}{\omega C}$$

$$4.73 = \frac{1}{2\pi f C}$$

$$4.73 = \frac{1}{2 \times 3.14 \times 5 \times 10^{3} \times C}$$

$$C_2 = \frac{1}{2 \times 3.14 \times 5000 \times 4.73}$$

$$= 6.73 \times 10^{-6}$$

$$\boxed{C_2 = 6.73\ \mu F}$$

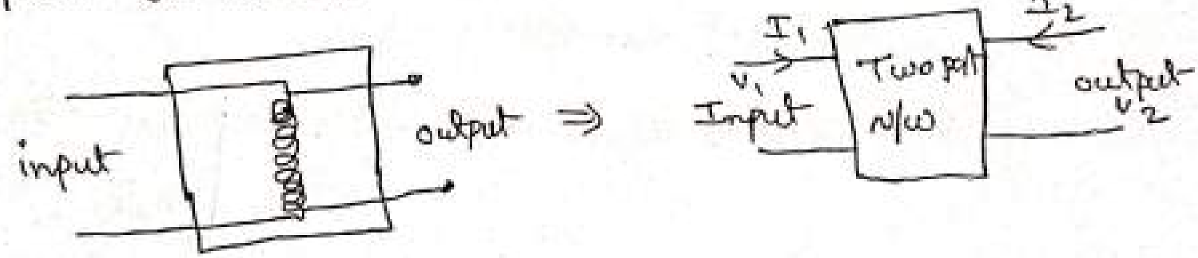
Unit-2

Two port Networks

Two port Parameters - Impedance - admittance - Transmission & hybrid parameters and their relation - concept of Transformed N/w - Two port n/w parameters using transformed variables.

Two port n/w

It is the rectangular box represents a network, consists of two pairs of network [Four terminals (V_1, I_1, V_2, I_2)] where one pair of terminals can be designated as input, the other pair being output. It is called two port n/w.



Types of two port parameters

- In Two port n/w, Four variables are V_1, I_1, V_2, I_2
- out of Four variables, Two variables are dependent and other two variables are independent variables.
- These dependent & Independent variables are depends on the type of parameter.

There are Five types of parameters

1. Z-parameters (or) open ckt parameters
2. Y-parameters (or) short ckt parameters

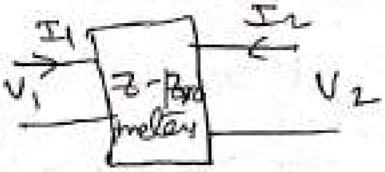
3. Hybrid or h-Parameters
4. Transmission or ABCD Parameters
5. Inverse hybrid or Inverse-h parameters (or) g-Parameters

1. Z-Parameters (or) open ckt parameters:

Z-Parameters are represented as

$V_1 = Z_{11} I_1 + Z_{12} I_2$ —(1)

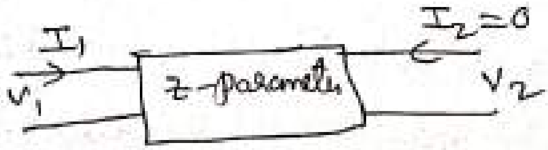
$V_2 = Z_{21} I_1 + Z_{22} I_2$ —(2)



From the above equations
Dependent variables : V_1 & V_2
Independent variables : I_1 & I_2

From eq (1) & (2) Z-Parameters are $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

Case(i): To find Z_{11} & Z_{21} by open ckting the output port i.e $I_2 = 0$.



put $I_2 = 0$ in eq (1) & (2)

From eq(1) $V_1 = Z_{11} I_1$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \rightarrow \text{Driving point Input Impedance}$$

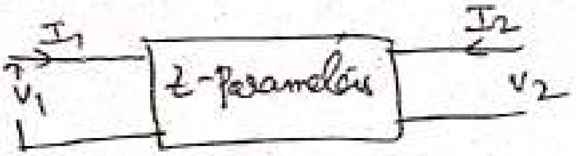
From eq (2)

$$V_2 = Z_{21} I_1$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} \rightarrow \text{Forward transfer Impedance}$$

case(2): To find Z_{12} & Z_{22} by open circuiting port 1

i.e $I_1 = 0$.



Put $I_1 = 0$ in eq (1) & (2)

From eq(1), $V_1 = Z_{12} I_2$

$$\boxed{Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}}$$ → Reverse transfer Impedance.

From eq (2)

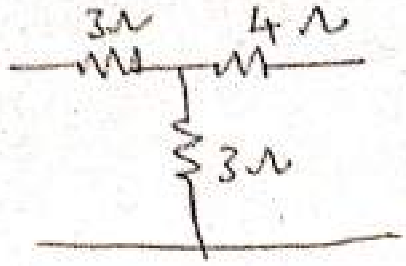
$V_2 = Z_{22} I_2$

$$\boxed{Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}}$$ – Driving point output Impedance.

Problems

Z-Parameters (Problem)

① Find Z-Parameters for the following ckt



Sol

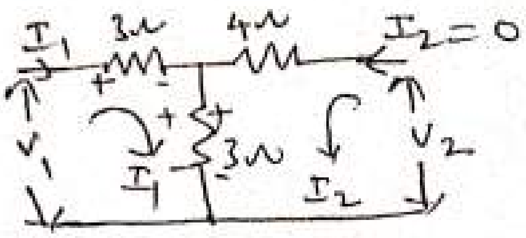
Z-Parameters

$V_1 = Z_{11}I_1 + Z_{12}I_2$ — ①

$V_2 = Z_{21}I_1 + Z_{22}I_2$ — ②

case (1) To find Z_{11} & Z_{21}, put $I_2 = 0$ $\left[Z_{11} = \frac{V_1}{I_1},\ Z_{21} = \frac{V_2}{I_1}\right]$

Apply KVL for loop 1



$V_1 - 3I_1 - 3I_1 - 3I_2 = 0$

put $I_2 = 0$

$V_1 - 6I_1 = 0$

$V_1 = 6I_1$

$\boxed{\frac{V_1}{I_1} = 6\,\Omega = Z_{11}}$

Apply KVL for loop 2

$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$

put $I_2 = 0$

$V_2 - 3I_1 = 0$

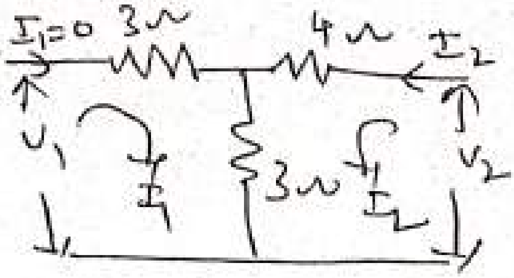
$V_2 = 3I_1$

$\boxed{\frac{V_2}{I_1} = Z_{21} = 3}\ \Omega$

Case (2) : To obtain Z_{12} & Z_{22}, put $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2}, \quad Z_{22} = \frac{V_2}{I_2}$$

apply KVL for loop 1



$$V_1 - 3I_1 - 3I_1 - 3I_2 = 0$$

put $I_1 = 0$

$$V_1 - 3I_2 = 0$$

$$V_1 = 3I_2$$

$$\boxed{\frac{V_1}{I_2} = Z_{12} = 3\,\Omega}$$

apply KVL for loop 2

$$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$$

put $I_1 = 0$

$$V_2 - 7I_2 = 0$$

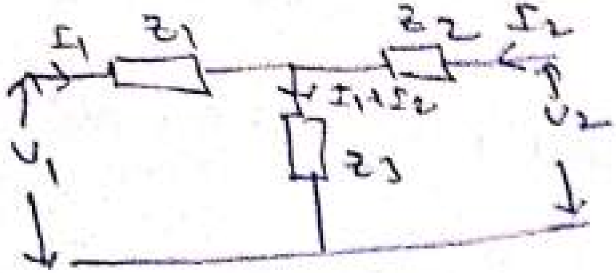
$$V_2 = 7I_2$$

$$\boxed{\frac{V_2}{I_2} = Z_{22} = 7\,\Omega}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 3 & 7 \end{bmatrix}$$

Prob 1

1. Find the z-parameters for the n/w



Sol using KVL

$$V_1 = I_1 Z_1 + (I_1 + I_2) Z_3$$

$$V_1 = I_1(Z_1 + Z_3) + I_2 Z_3 \quad -(1)$$

$$V_2 = I_2 Z_2 + (I_1 + I_2) Z_3$$

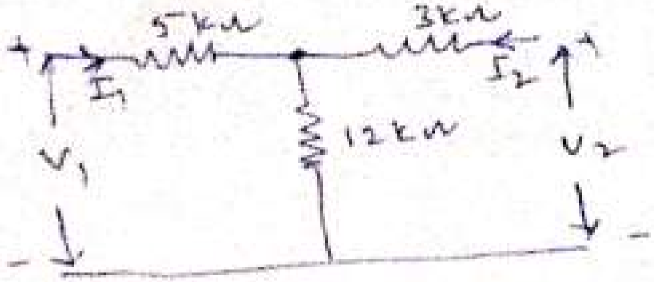
$$V_2 = I_1 Z_3 + I_2 (Z_2 + Z_3) \quad -(2)$$

Z - parameters

$$Z_{11} = Z_1 + Z_3, \; Z_{12} = Z_3$$

$$Z_{21} = Z_3 \;, \; Z_{22} = Z_2 + Z_3$$

(2) Find the z-Parameter for the n/w shown in fig (3)



Sol

Using KVL ⇒

$$V_1 = 5\times10^3 I_1 + (I_1+I_2)\,12\times10^3$$

$$= 17\times10^3 I_1 + 12\times10^3 I_2 \quad -(1)$$

$$V_2 = 3\times10^3 I_2 + 12\times10^3 (I_2+I_1)$$

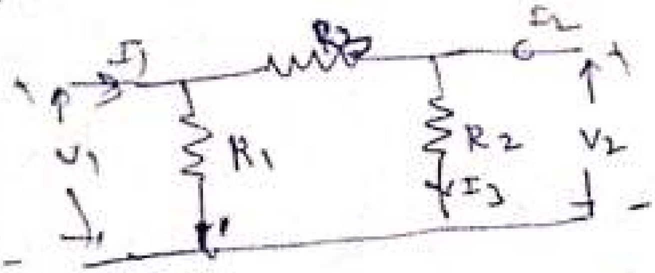
$$= 12\times10^3 I_1 + 15\times10^3 I_2 \quad -(2)$$

comparing two equations

$z_{11} = 17\times10^3\,\Omega$, $z_{12} = 12\times10^3$

$z_{21} = 12\times10^3$, $z_{22} = 15\times10^3$

(3) Determine the z-parameters of the π-type attenuator section.



$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

Sol Assume that output port is open ckted

i.e $I_2 = 0$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$= R_1 \parallel (R_2+R_3)$$

$$= \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \quad -(1)$$

However

$$I_3 = I_1 \times \frac{R_1}{R_1+R_2+R_3} \quad -(2)$$

$$V_2 = I_3 R_2$$

$$= I_1 \frac{R_1 R_2}{R_1+R_2+R_3} \quad -③$$

Again $z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$

$$= \frac{R_1 R_2}{R_1+R_2+R_3}$$

Now assume input open ckted; i.e $I_1 = 0$

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$= R_2 \parallel (R_1+R_3)$$

$$= \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$$

assume I_3' current through R_1 resistor

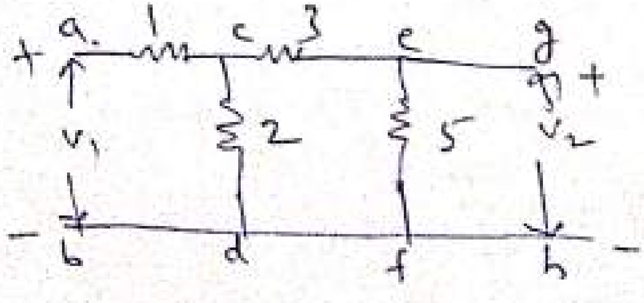
$$V_1 = I_3' R_1$$

where $I_3' = I_2 \times \frac{R_2}{R_1+R_2+R_3}$

$$V_1 = I_2 \frac{R_2 R_1}{R_1+R_2+R_3}$$

$$z_{12} = \frac{V_1}{I_2} \quad \frac{R_1 R_2}{R_1+R_2+R_3}$$

(6) Find the z-parameters for the ckt shown in fig



Sol

Assuming o/p terminals open circuited i.e $I_2 = 0$

$$R_{eq1} = (3+5) \parallel 2 + 1$$

$$= \frac{8 \times 2}{10} + 1$$

$$= 1.6 + 1$$

$$= 2.6\,\Omega$$

Let us assume input current I_1

$$V_1 = I_1 R_{eq1}$$

$$Z_{11} = \frac{V_1}{I_1} = R_{eq1}$$

$$= 2.6\,\Omega$$

Let us assume that current through $5\,\Omega$ resistor is I_x

$$\therefore I_x = I_1 \times \frac{2}{2+3+5} = \frac{I_1}{5}\ A.$$

$$V_2 = I_x \cdot 5$$

$$= \frac{I_1}{5} \times 5 = I_1 \text{ volts.}$$

$$\frac{V_2}{I_1} = Z_{21} = 1\,\Omega.$$

Now assume I/P open circuited i.e $I_1 = 0$, then

$$R_{eq2} = (3+2) \parallel 5$$

$$= 2.5\,\Omega.$$

$$V_2 = I_2 R_{eq2}$$

$$Z_{22} = \frac{V_2}{I_2} = R_{eq2} = 2.5\,\Omega.$$

Let I_y be the current through $2\,\Omega$ resistor

$$V_1 = I_y\, 2$$

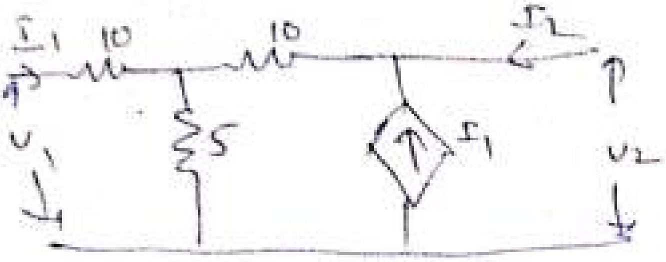
$$I_y = I_2 \times \frac{5}{2+3+5} = \frac{I_2}{2}\ A.$$

$\therefore \quad V_1 = I_y \times 2$

$= \frac{I_2}{2} \times 2$

$\frac{V_1}{I_2} = Z_{12} = 1\,\Omega$

(5) Determine the Z-parameter



Sol

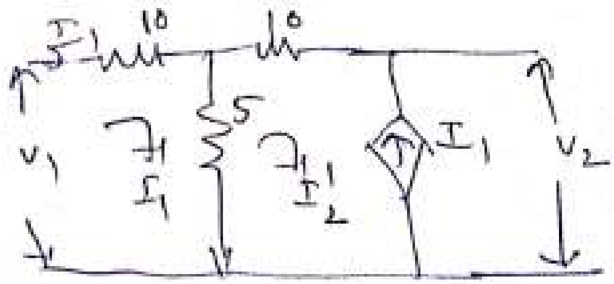
First o/p open ckted $I_2 = 0$, and applying vol source at V_1 at input port, the loop equations are given by

~~Req~~ $=$

$V_1 = 10 I_1 + 5(I_1 - I_2')$

$V_1 = (10+5) I_1 - 5 I_2'$ —

$V_1 = 15 I_1 - 5 I_2'$ — (1)



but from I_2' loop

$I_1 = -I_2'$ — (2)

sub eq (2) in eq (1)

$V_1 = 15 I_1 + 5 I_1$

$V_1 = 20 I_1$ — (3)

$Z_{11} = \frac{V_1}{I_1} = 20\,\Omega$

$\therefore I_2' = -I_1 = -\frac{V_1}{20}$

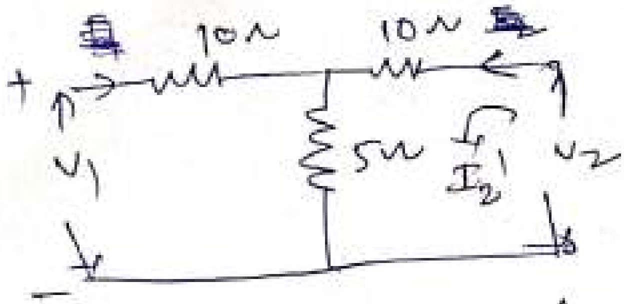
Again $V_2 = (I_1 - I_2') 5 - 10 I_2'$

$= (I_1 + I_1) 5 + 10 I_1$ $\quad (I_2' = -I)$

$V_2 = 20 I_1$

$Z_{21} = V_2 / I_1 = 20\,\Omega$

next Input is opened $I_1 = 0$



here
The current source becomes useless as with input open.

here $I_2 = I_2'$

$$V_2 = I_2(10+5) = 15 I_2$$

$$Z_{22} = \frac{V_2}{I_2} = 15\,\Omega$$

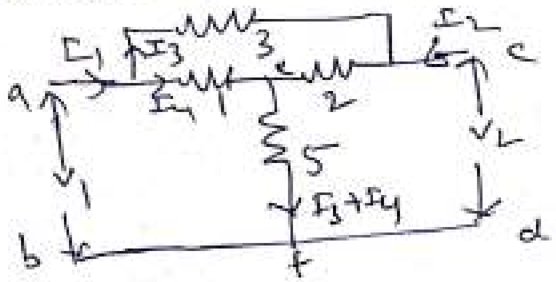
also $V_1 = I_2' \times 5 = 5 I_2$

$$Z_{12} = \frac{V_1}{I_2} = 5\,\Omega.$$

$\therefore$ $Z_{11} = 20\,\Omega$, $Z_{12} = 5\,\Omega$

$Z_{21} = 20\,\Omega$, $Z_{22} = 15\,\Omega$.

(6) obtain the open ckt parameters and o/p equations of the network



Sol Let o/p open circuited i.e $I_2 = 0$. Then

$$V_1 = R_{eq} I_1$$
$$= \left((3+2)\,||\,1 + 5\right) I_1$$
$$= \left(\frac{5 \times 1}{6} + 5\right) I_1 \Rightarrow \frac{35}{6}\,\Omega\,(I_1)$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{35}{6}\,\Omega.$$

$\cancel{V_1 = (I_3 + I_1)5}$ $V_2 = I_3 \times 2 + I_1 \times 5$

$$I_3 = I_1 \times \frac{1}{1+5}$$

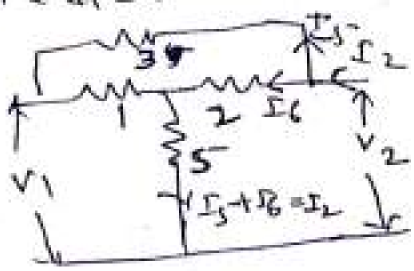
$$I_3 = I_1 \times \frac{1}{6}$$

$$\therefore V_2 = \frac{I_1}{6} \times 2 + I_1 \times 5$$

$$= \frac{16}{3} I_1$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{16}{3}\,\Omega.$$

Similarly Input open circuited i.e $I_1 = 0$



$$V_2 = I_2 \cdot R_{eq}$$

$$= I_2((3+1) \parallel 2 + 5)$$

$$= I_2\left(\frac{4 \times 2}{6} + 5\right)$$

$$= I_2\left(\frac{19}{3}\right)$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{19}{3}\,\Omega$$

$$V_1 = I_5 \times 1 + I_2 \times 5$$

$$= I_2 \times \frac{2}{2+4} \times 1 + I_2 \times 5$$

$$= \frac{I_2}{3} + I_2 \times 5$$

$$= I_2 \frac{16}{3}$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{16}{3}\,\Omega$$

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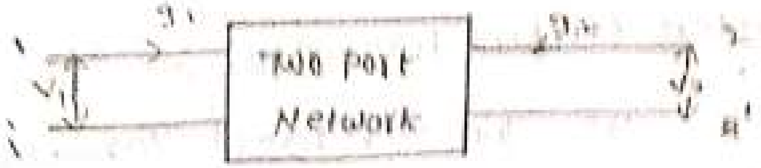
Y-Parameter:

In this parameter the dependent variable are I_1 & I_2 and independent variables are V_1 & V_2 then Y-parameter are usually expressed as the following equation.



I_1 & I_2 → D
V_1 & V_2 → I

$I_1 = Y_{11}V_1 + Y_{12}V_2$ → (1)
$I_2 = Y_{21}V_1 + Y_{22}V_2$ → (2)

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow (3)$$

Case(i):- To find Y_{11} & Y_{21}

Let $V_2 = 0$ → (4)

(1) → $I_1 = Y_{11}V_1 + Y_{12}(0)$

$Y_{11} = I_1/V_1 \mid V_2 = 0$

→ short circuited driving point admittance.

(2) → $I_2 = Y_{21}V_1 + 0$

$Y_{21} = \frac{I_2}{V_1} \mid V_2 = 0$

short circuited forward transfer admittance

case(ii):- To find Y_{12} & Y_{22}

Let $V_1 = 0$ → (5)

(1) → $I_1 = Y_{12}V_2$

$Y_{12} = I_1/V_2 \mid V_1 = 0$

short circuited reverse transform admittance.

② ⇒ $I_2 = Y_{21}(0) + Y_{22}V_2$

$$Y_{22} = \frac{I_2}{V_2}$$

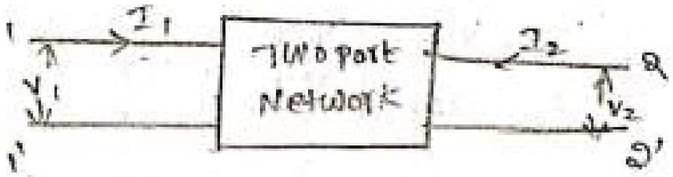
short circuted driving output admittance.

H-parameters (Hybrid parameters) :-

In this case the dependent variables are V_1 & I_2 and independent variables are I_1 & V_2 then the h- parameters are usually expressed by following two equations.

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad -①$$

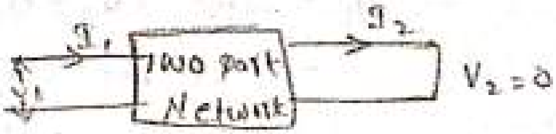
$$I_2 = h_{21}I_1 + h_{22}V_2 \quad -②$$



In matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Case(i) :- $V_2 = 0$

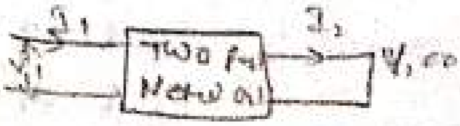


① ⇒ $V_1 = h_{11}I_1 + 0$

$h_{11} = V_1/I_1 \,/ V_2 = 0$ (Ω) short circuited driving input impedence.

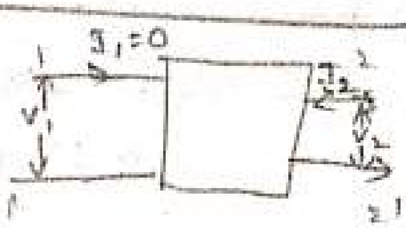
② ⇒ $I_2 = h_{21}I_1$

$h_{21} = I_2/I_1 \mid V_2 = 0$



short circuited forward transfer current gain.

Case (ii):- $I_1 = 0$

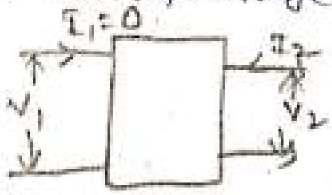


① ⇒ $V_1 = 0 + h_{12}V_2$

$h_{12} = V_1/V_2 \; |_{I_1=0}$

open circuited transfer voltage gain.

② ⇒ $I_2 = 0 + h_{22}V_2$

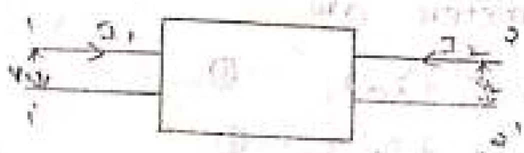


$h_{22} = \frac{I_2}{V_2} \; |_{I_1=0}$ ℧ (mho)

open circuited driving o/p admitance.

TRANSMISSION PARAMETERS (ABCD PARAMETERS):-

In this the dependent variables are V_1 & I_1 and independent variables are V_2 & I_2. Then ABCD parameters are usually expressed by the following figure equations.



$V_1 = AV_2 - BI_2$ → ①

$I_1 = CV_2 - DI_2$ → ②

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Case (i):- $I_2 = 0$

① ⇒ $V_1 = AV_2$

$A = V_1/V_2$ open circuited reverse transfer voltage gain.

② ⇒ $I_1 = CV_2$

$C = I_1/V_2$ (℧) mho

open circuited reverse transfer admittance.

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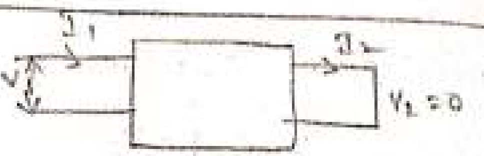
Case (ii):- $V_2 = 0$



$$V_1 = -BI_2$$

$$B = -\frac{V_1}{I_2} \, (\Omega)$$ short circuited reverse transfer impedance.

$$I_1 = -DI_2$$

$$D = -\frac{I_1}{I_2}$$ short circuited reverse transfer current gain.

Relation b/w Parameters

Z-Parameter in terms of various Parameters:-

(A) Z-Parameters in terms of Y-Parameters:-

we know the

Z-Parameters are

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad -①$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad -②$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

and the Y-parameters are

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad -③$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad -④$$

$$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}^{-1}$$

$$= \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

so $z_{11} = \frac{Y_{22}}{\Delta Y}$, $z_{22} = \frac{Y_{11}}{\Delta Y}$

$z_{12} = \frac{-Y_{12}}{\Delta Y}$

$z_{21} = \frac{-Y_{21}}{\Delta Y}$

(b) Z-parameters in terms of h-parameters :-

We know the

Z-parameters are

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad -(1)$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \quad -(2)$$

h-parameters are

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad -(3)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad -(4)$$

From eq (4)

$$h_{22}V_2 = I_2 - h_{21}I_1$$

$$V_2 = \frac{1}{h_{22}}I_2 - \frac{h_{21}}{h_{22}}I_1$$

$$V_2 = -\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2 \quad -(5)$$

comparing (5) & (2)

$z_{21} = \frac{-h_{21}}{h_{22}}$, $z_{22} = \frac{1}{h_{22}}$

sub eq (5) in eq (3)

$$V_1 = h_{11}I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2\right]$$

$$V_1 = I_1\left[h_{11} - \frac{h_{21}h_{12}}{h_{22}}\right] + \frac{h_{12}}{h_{22}}I_2$$

$$= I_1\left[\frac{h_{11}h_{22} - h_{21}h_{12}}{h_{22}}\right] + \frac{h_{12}}{h_{22}}I_2$$

$$V_1 = I_1\left(\frac{\Delta h}{h_{22}}\right) + \frac{h_{12}}{h_{22}}I_2 \quad -(6)$$

compare (6) & (9)

$$Z_{11} = \frac{\Delta h}{h_{22}} \quad , \quad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

(c) Z Parameters interms of ABCD Parameter.

we know the

Z-Parameters are

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad -①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad -②$$

and the ABCD parameters are

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_2$$

$$V_1 = AV_2 - BI_2 \quad -(3)$$

$$I_1 = CV_2 - DI_2 \quad -(4)$$

from (4)

$$CV_2 = I_1 + DI_2$$

$$V_2 = \frac{1}{C} I_1 + D/C\, I_2 \quad -⑤$$

compare ② & (5)

$$Z_{21} = 1/C \quad , \quad Z_{22} = D/C$$

sub eq⑤ in eq (3)

$$V_1 = A\left[\frac{1}{C} I_1 + D/C\, I_2\right] - BI_2$$

$$V_1 = A/C\, I_1 + \left[AD/C - B\right] I_2$$

$$V_1 = A/C\, I_1 + \left[\frac{AD - BC}{C}\right] I_2 \quad -(6)$$

compare ① & (6)

$$Z_{11} = A/C \quad , \quad Z_{12} = \frac{AD - BC}{C} = \frac{\Delta T}{C}$$

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$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{V_c}{} & \frac{\Delta T}{} \\ V_c & \frac{1}{h_{22}} \end{bmatrix}$$

Y-parameters in terms of various parameters:

Y-parameters are

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{--(1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{--(2)}$$

Z-parameters are

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{--(3)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{--(4)}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1}$$

$$= \frac{1}{z_{11}z_{22} - z_{12}z_{21}} \begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$$

Y-Parameters Interms of h-parameters:-

We know that Y-parameters are

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{--(1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{--(2)}$$

h-parameters are

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{--(3)}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{--(4)}$$

(3) $\Rightarrow$ $$h_{11}I_1 = V_1 - h_{12}V_2$$

$$I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2 \quad \text{--(5)}$$

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compare eq(1) & (5).

$$Y_{11} = \frac{1}{h_{11}} \quad , Y_{12} = \frac{-h_{12}}{h_{11}}$$

substitute eq(5) in eq(4)

$$I_2 = h_{21}\left[\frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 - \frac{h_{21}h_{12}}{h_{11}}V_2 + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 + V_2\left(\frac{h_{22}h_{11} - h_{21}h_{12}}{h_{11}}\right)$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 + V_2\frac{\Delta h}{h_{11}} \quad -(6)$$

compare (2) & (6)

$$Y_{21} = \frac{h_{21}}{h_{11}} \quad , Y_{22} = \frac{\Delta h}{h_{11}}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$$

Y-parameters interms of ABCD parameters:-

we know that Y parameters are

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad -(1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad -(2)$$

ABCD Parameters are

$$V_1 = AV_2 - BI_2 \quad -(3)$$

$$I_1 = CV_2 - DI_2 \quad -(4)$$

(3) $\Rightarrow$ $BI_2 = AV_2 - V_1$

$$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2 \quad -(5)$$

compare ③ & ⑤

$$Y_{21} = \frac{-1}{B}, \quad Y_{22} = A/B$$

sub ⑤ in eq ④

$$I_1 = CV_2 - D\left[-1/B\, V_1 + A/B\, V_2\right]$$

$$I_1 = CV_2 + D/B\, V_1 - AD/B\, V_2$$

$$I_1 = D/B\, V_1 + \left[\frac{-AD}{B} + C\right] V_2$$

$$I_1 = D/B\, V_1 + \left[\frac{-AD + BC}{B}\right] V_2$$

$$I_1 = D/B\, V_1 - \left[\frac{AD - BC}{B}\right] V_2$$

$$I_1 = \frac{D}{B} V_1 - \frac{\Delta T}{B} V_2 \quad \text{—⑥}$$

compare ⑦ & ⑧

$$Y_{11} = D/B, \quad Y_{12} = \frac{-\Delta T}{B}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} D/B & \frac{-\Delta T}{B} \\ -1/B & A/B \end{bmatrix}$$

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h-parameters to various parameters.

(A) h-parameters to z-parameters:-

We know h-parameters are

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad -(1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad -(2)$$

The z-parameters are

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad -(3)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad -(4)$$

from (4)

$$Z_{22}I_2 = V_2 - Z_{21}I_1$$

$$I_2 = \frac{1}{Z_{22}}V_2 - \frac{Z_{21}}{Z_{22}}I_1 \quad -(5)$$

compare (2) & (5)

$$h_{21} = \frac{-Z_{21}}{Z_{22}}, \quad h_{22} = \frac{1}{Z_{22}}$$

sub eq (5) in (3)

$$V_1 = Z_{11}I_1 + Z_{12}\left[\frac{1}{Z_{22}}V_2 - \frac{Z_{21}}{Z_{22}}I_1\right]$$

$$V_1 = I_1\left[Z_{11} - \frac{Z_{21}}{Z_{22}}.Z_{12}\right] + \frac{Z_{12}}{Z_{22}}V_2$$

$$V_1 = I_1\left[\frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{22}}\right] + \frac{Z_{12}}{Z_{22}}V_2$$

$$V_1 = I_1\frac{\Delta Z}{Z_{22}} + \frac{Z_{12}}{Z_{22}}V_2 \quad -(6) \qquad (\Delta Z = Z_{11}Z_{22} - Z_{21}Z_{12})$$

compare (6) & (1)

$$h_{11} = \frac{\Delta Z}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

h-parameters interms of Y-parameters:-

We know that h-parameters are

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad -(1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad -(2)$$

The Y-parameters are

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad -(3)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad -(4)$$

$$(3) \Rightarrow \quad V_1 Y_{11} = I_1 - Y_{12}V_2$$

$$V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \quad -(5)$$

compare eq-(5) & eq(1)

$$h_{11} = 1/Y_{11} \quad ; \quad h_{12} = \frac{-Y_{12}}{Y_{11}}$$

eq(5) in eq (4)

$$I_2 = Y_{21}\left(\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right) + Y_{22}V_2$$

$$= \frac{Y_{21}}{Y_{11}} I_1 + V_2\left[Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}}\right]$$

$$= \frac{Y_{21}}{Y_{11}} I_1 + V_2\left[\frac{Y_{22}Y_{11} - Y_{12}Y_{21}}{Y_{11}}\right]$$

$$I_2 = \frac{Y_{21}}{Y_{11}} I_1 + V_2\left[\frac{\Delta Y}{Y_{11}}\right] \quad -(6)$$

compare (2) & (6)

$$h_{21} = \frac{Y_{21}}{Y_{11}} \quad ; \quad h_{22} = \frac{\Delta Y}{Y_{11}}$$

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$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{bmatrix}$$

h-parameters interms of ABCD parameters :-

we know that

h-parameters are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -① $$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -② $$

ABCD parameters are

$$V_1 = A V_2 - B I_2 \quad -③ $$
$$I_1 = C V_2 - D I_2 \quad -④ $$

④ $\Rightarrow$ $D I_2 = C V_2 - I_1$

$$I_2 = \frac{C}{D} V_2 - \frac{1}{D} I_1 \quad -⑤ $$

compare ⑤ & ②

$$h_{21} = -\frac{1}{D}, \quad h_{22} = \frac{C}{D}$$

sub. eq ⑤ in eq ③

$$V_1 = A V_2 - \left[\frac{C}{D} V_2 - \frac{1}{D} I_1\right] B$$

$$V_1 = A V_2 - \left[\frac{BC}{D} V_2 - \frac{B}{D} I_1\right]$$

$$V_1 = \left[A - \frac{BC}{D}\right] V_2 + \frac{B}{D} I_1$$

$$V_1 = \frac{B}{D} I_1 + \left(\frac{AD - BC}{D}\right) V_2$$

$$V_1 = \frac{B}{D} I_1 + \frac{\Delta}{D} V_2 \quad -⑥ $$

compare ① & ⑥

$$h_{11} = \frac{B}{D}, \quad h_{12} = \frac{\Delta T}{D}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} B/D & \Delta T/D \\ -1/D & C/D \end{bmatrix}$$

we know that ABCD parameters are

$$V_1 = AV_2 - BI_2 \quad -①$$
$$I_1 = CV_2 - DI_2 \quad -②$$

Z-parameters are

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad -③$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad -④$$

$$④ \Rightarrow \quad Z_{21}I_1 = V_2 - Z_{22}I_2$$
$$I_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2 \quad -⑤$$

compare ⑤ & ②

$$C = \frac{1}{Z_{21}}, \quad D = \frac{Z_{22}}{Z_{21}}$$

eq⑤ in eq③

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2\right] + Z_{12}I_2$$

$$V_1 = \frac{Z_{11}V_2}{Z_{21}} - I_2\left[-Z_{12} + \frac{Z_{22} \cdot Z_{11}}{Z_{21}}\right]$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - I_2\left[\frac{-Z_{12}Z_{21} + Z_{22} \cdot Z_{11}}{Z_{21}}\right]$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - I_2 \cdot \frac{\Delta Z}{Z_{21}} \quad -⑥$$

① & ⑥ $\quad A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{\Delta Z}{Z_{21}}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$$

we know that ABCD parameters are

$$V_1 = AV_2 - BI_2 \quad -①$$

$$I_1 = CV_2 - DI_2 \quad -②$$

Y-parameters are

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad -③$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad -④$$

$$④ \Rightarrow \quad Y_{21}V_1 = I_2 - Y_{22}V_2$$

$$V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2 \quad -⑤$$

compare ① & ⑤

$$A = \frac{-Y_{22}}{Y_{21}}, \quad B = -\frac{1}{Y_{21}}$$

sub. eq ⑤ in ③

$$I_1 = Y_{11}\left[\frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2\right] + Y_{12}V_2$$

$$I_1 = \left[Y_{12} - \frac{Y_{22} \cdot Y_{11}}{Y_{21}}\right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$I_1 = \left[\frac{Y_{21}Y_{12} - Y_{22}Y_{11}}{Y_{21}}\right] V_2 + \frac{Y_{11}}{Y_{21}} I_2$$

$$I_1 = \frac{-\Delta}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 \quad -⑥$$

② & ⑥ $\quad C = \frac{-\Delta}{Y_{21}}, \quad D = \frac{-Y_{11}}{Y_{21}}$

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$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$$

We know that ABCD parameters are

$$V_1 = AV_2 - BI_2 \quad -①$$

$$I_1 = CV_2 - DI_2 \quad -②$$

h-parameters are

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad -③$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -④$$

④ $\Rightarrow$ $h_{21} I_1 = I_2 - h_{22} V_2$

$$I_1 = \frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \quad -⑤$$

compare ⑤ & ②

$$C = \frac{-h_{22}}{h_{21}}, \quad D = \frac{-1}{h_{21}}$$

sub. eq ⑤ in eq ③

$$V_1 = h_{11}\left[\frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2\right] + h_{12} V_2$$

$$V_1 = V_2\left[\frac{-h_{11} h_{22}}{h_{21}} + h_{12}\right] + \frac{h_{11}}{h_{21}} I_2$$

$$V_1 = -V_2\left[\frac{h_{22} h_{11} - h_{12} h_{21}}{h_{21}}\right] + \frac{h_{11}}{h_{21}} I_2 \quad -⑥$$

compare ① & ⑥

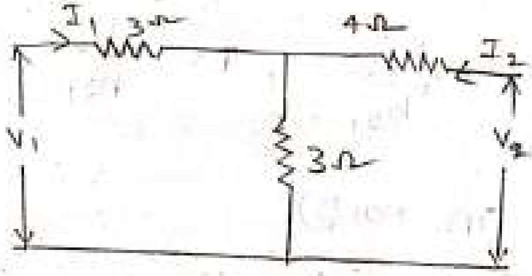
$$A = \frac{-\Delta h}{h_{21}}, \quad B = \frac{h_{11}}{h_{12}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$$

Parameters	condition for symmety	condition for reciprocal.
Z	$z_{11} = z_{22}$	$z_{12} = z_{21}$
Y	$y_{11} = y_{22}$	$y_{12} = y_{21}$
h	$\Delta h = 1$	$h_{12} = -h_{21}$
ABCD	$A = D$	$AD - BC = 1$

PROBLEMS ON parameters:-

① Find Z & Y parameters in the following circuit.



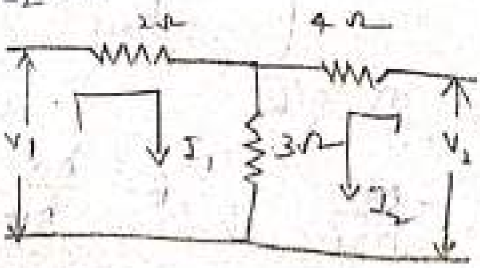
Z parameters are

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

case(i):- To find z_{11} & z_{21} by putting $I_2 = 0$

Let $I_2 = 0$



then $V_1 = z_{11} I_1$

$$z_{11} = \frac{V_1}{I_1}$$

$$V_2 = z_{21} I_1$$

$$z_{21} = \frac{V_2}{I_1}$$

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$$V_1 - 2I_1 - 3I_1 = 0$$

$$V_1 = 5I_1$$

$$\frac{V_1}{I_1} = 5$$

$$Z_{11} = 5$$

Apply KVL for loop 2,

$$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$$

$$V_2 = 3I_1$$

$$\frac{V_2}{I_1} = 3$$

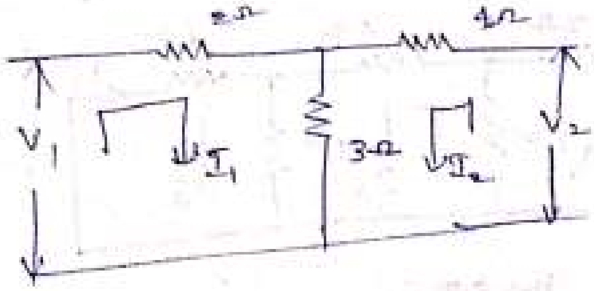
$$Z_{21} = 3$$

Case 2:- to find Z_{12} & Z_{22}, by putting $I_1 = 0$

$$I_1 = 0$$

$$V_1 = Z_{12} I_2 \quad , \quad V_2 = Z_{22} I_2$$

$$Z_{12} = \frac{V_1}{I_2} \quad , \quad Z_{22} = \frac{V_2}{I_2}$$



apply KVL for loop 1,

$$V_1 - 2I_1 - 3I_1 - 3I_2 = 0$$

$$V_1 = 3I_2$$

$$\frac{V_1}{I_2} = Z_{12} = 3\Omega$$

apply KVL for loop 2,

$$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$$

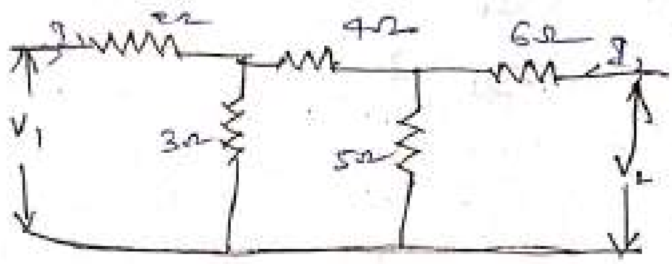
$$V_2 = 7I_2 \Rightarrow \frac{V_2}{I_2} = Z_{22} = 7\Omega$$

Y - Parameters

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$= \begin{bmatrix} 7/26 & -3/26 \\ -3/26 & 5/26 \end{bmatrix}$$

(7) Find Y parameters for the following crt.

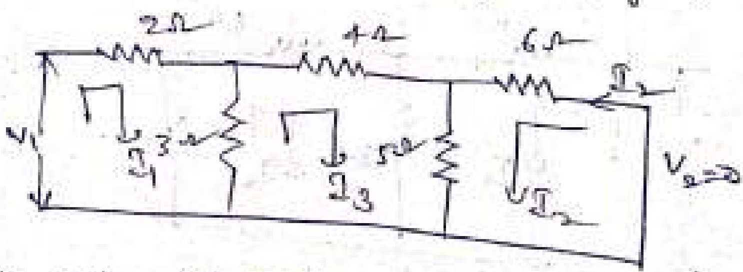


Case(i):- ≠0

$I_1 = Y_{11}V_1 + Y_{12}V_2$ — ①

$I_2 = Y_{21}V_1 + Y_{22}V_2$ — ②

Case(ii):- To find Y_{11} & Y_{21} by putting $V_2 = 0$



Apply KVL for loop 1,

$$V_1 - 2I_1 - 3I_1 + 3I_3 = 0$$

$$V_1 = 5I_1 - 3I_3 \quad —③$$

Apply KVL for loop 3

$$-4I_3 - 5I_3 - 5I_2 - 3I_3 + 3I_1 = 0$$

$$3I_1 - 5I_2 - 12I_3 = 0 \quad —④$$

Apply KVL for loop 2

$-6I_2 - 5I_2 - 5I_3 = 0$

$11I_2 + 5I_3 = 0$

$11I_2 = -5I_3$ — (5)

$V_2 = 0 \Rightarrow$

$I_1 = Y_{11}V_1$ | $I_2 = Y_{21}V_1$

$Y_{11} = I_1/V_1$ | $Y_{21} = I_2/V_1$

from (5) $\Rightarrow I_2 = -5/11\, I_3$ — (A)

(4) $\Rightarrow$ $3I_1 - 5(-5/11)I_3 - 12I_3 = 0$

$3I_1 + 25/11\, I_3 - 12I_3 = 0$

$3I_1 + 2.27I_3 - 12I_3 = 0$

$3I_1 - 9.73I_3 = 0$ — (6)

$3I_1 = 9.73I_3$

$I_3 = \frac{3I_1}{9.73}$

$I_3 = 0.308I_1$ — (7)

eq (7) in eq (3)

$V_1 = 5I_1 - 3(0.308)I_1$

$V_1 = 5I_1 - 0.924I_1$

$V_1 = 4.076I_1 \Rightarrow \frac{I_1}{V_1} = 0.24\ \mho = Y_{11}$

V_1 in eq (3)

$\cancel{4.076I_1} = 5I_1 - 3I_3$

$0.924I_1 - 3I_3 = 0$ — (8)

(7) $\Rightarrow$ $I_1 = \frac{1}{0.308}I_3$

$I_1 = 3.24I_3$ — (8)

eq-⑥ sub. in eq③

$V_1 = 5(3.26 I_3) - 3I_3$

$V_1 = 13.3 I_3$

from ⓐ $I_2 = -0.45 I_3$

$I_3 = -\frac{1}{0.45} I_2$

$I_3 = -2.22 I_2$ — ⑦

sub eq ⑦ in $V_1 = 13.3 I_3$

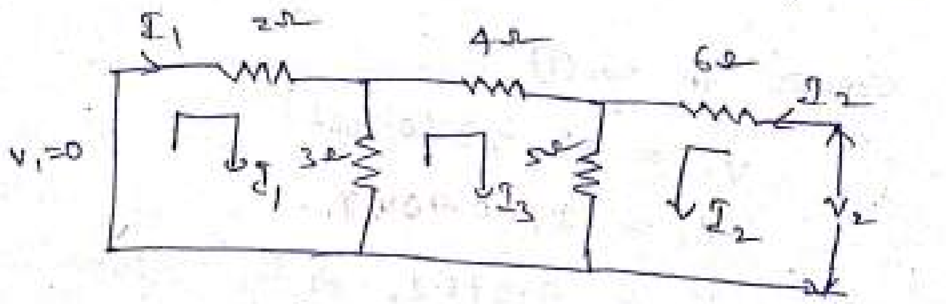
$V_1 = 13.3(-2.22) I_2$

$V_1 = -29.04 I_2$

$Y_{21} = \frac{V_1}{I_2} =$

$Y_{21} = \frac{I_2}{V_1} = \frac{-1}{29.04} = -0.03.$

Case (ii): To find Y_{12} & Y_{22} by putting $V_1 = 0$



apply KVL for loop 1,

$-2I_1 - 3I_1 + 3I_3 = 0$

$3I_3 - 5I_1 = 0$ — ①

apply KVL for loop 2,

$V_2 - 6I_2 - 5I_2 - 5I_3 = 0$

$V_2 - 11I_2 - 5I_3 = 0$ — ②

$V_2 = 11I_2 + 5I_3$

if $V_1 = 0$

$I_1 = Y_{12} V_2$

$\boxed{Y_{12} = \frac{I_1}{V_2}}$

$I_2 = Y_{22} V_2$

$\boxed{Y_{22} = \frac{I_2}{V_2}}$

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Apply KVL for loop ③,

$$-4I_3 - 5I_3 - 5I_2 - 3I_3 + 3I_1 = 0$$

$$3I_1 - 5I_2 - 12I_3 = 0 \quad -③$$

① $\Rightarrow$ $3I_3 = 5I_1$

$$I_3 = 5/3\, I_1$$

$$I_3 = 5/3\, I_1 \quad -④$$

③ $\Rightarrow$ $3I_1 - 5I_2 - 12(5/3)I_1 = 0$

$$3I_1 - 5I_2 - \frac{20}{3}I_1 = 0$$

$$3I_1 - 5I_2 - 20I_1 = 0$$

$$-17I_1 = 5I_2$$

$$I_1 = -5/17\, I_2 \quad -⑧$$

$$I_2 = -17/5\, I_1 \quad -⑤$$

④ & ⑤ write in ②

$$V_2 = 11(-17/5)I_1 + 5(5/3)I_1$$

$$V_2 = -37.4I_1 + 8.33I_1$$

$$V_2 = -29.07I_1$$

$$\frac{I_1}{V_2} = \frac{1}{29.07}$$

$$\frac{I_1}{V_2} = 0.034 = Y_{12}$$

$$Y_{12} = 0.034$$

⑤ $\Rightarrow$ $I_1 = -5/17\, I_2 \quad -⑥$

eq ⑥ write in eq ④

$$I_3 = 5/3(-5/17)I_2$$

$$I_3 = -0.49I_2$$

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eq (2) ⇒ $V_2 = 11I_2 + 5(-0.49)I_2$

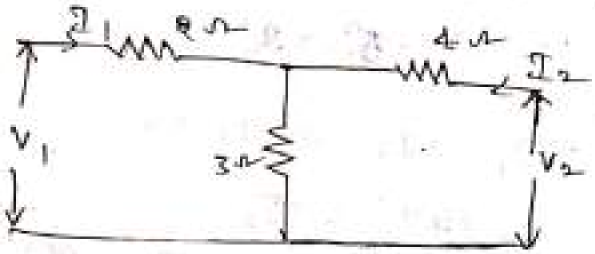
$$V_2 = 11I_2 - 2.45I_2$$

$$V_2 = 8.55\,I_2$$

$$\frac{I_2}{V_2} = \frac{1}{8.55}$$

$$y_{22} = \frac{I_2}{V_2} = 0.11$$

Find the Z-parameters for the following circuit

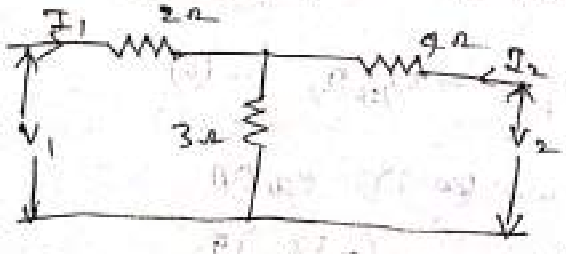


$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2+3 & 3 \\ 3 & 4+3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ 3 & 7 \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 7 \end{bmatrix}^{-1}$$

Find the ABCD parameters for the following circuit.



ABCD parameters are

$$V_1 = AV_2 - BI_2 \quad -(1)$$

$$I_1 = CV_2 - DI_2 \quad -(2)$$

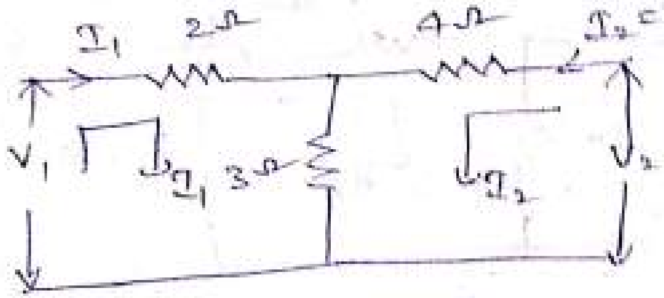
case(i):- To find A & C, put $I_2 = 0$

then eq ① ⇒ $V_1 = AV_2$

$A = V_1/V_2$

eq ② ⇒ $I_1 = CV_2$

$C = \frac{I_1}{V_2}$



Apply KVL for loop ①

$V_1 - 2I_1 - 3I_1 - 3I_2 = 0$

$V_1 - 5I_1 - 0 = 0$

$V_1 = 5I_1$ — ③

Apply KVL for loop ②

$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$

$V_2 + 0 - 3I_1 = 0$

$V_2 = 3I_1$ — ④

$\frac{③}{④} \Rightarrow \frac{V_1}{V_2} = \frac{5I_1}{3I_1}$

$A = \frac{V_1}{V_2} = 1.67$

from ④ ⇒ $\frac{I_1}{V_2} = 1/3 = 0.33$

$C = 0.33$

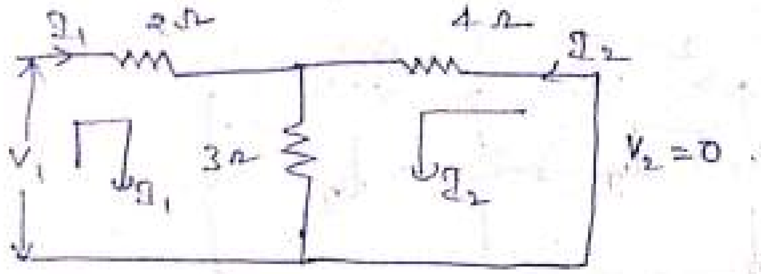
Case(iii):- To find B & D, by putting $V_2 = 0$.

$① \rightarrow \quad V_1 = -BI_2$

$$B = -\frac{V_1}{I_2}$$

$② \Rightarrow \quad I_1 = -DI_2$

$$D = -\frac{I_1}{I_2}$$



Apply KVL for loop 1,

$$V_1 - 2I_1 - 3I_1 - 3I_2 = 0$$

$$V_1 - 5I_1 - 3I_2 = 0$$

$$V_1 = 5I_1 + 3I_2 \quad — ③$$

Apply KVL for loop 2,

$$-4I_2 - 3I_2 - 3I_1 = 0$$

$$-7I_2 - 3I_1 = 0$$

$$3I_1 + 7I_2 = 0 \quad — ④$$

$$3I_1 = -7I_2$$

$$\frac{-I_1}{I_2} = 7/3$$

$$\frac{-I_1}{I_2} = 2.33$$

$$D = 2.33$$

$$I_1 = -7/3\, I_2$$

$④ \Rightarrow \quad I_2 = -3/7\, I_1$

$③ \Rightarrow \quad V_1 = 5I_1 + 3(-7/3) I_2$

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$$V_1 = 5I_1 - 9/_7 I_2$$

$$= 5I_1 - 1.28 I_2$$

$$V_1 = 3.72 I_1$$

$$V_1 = 2I_2$$

$$-\frac{V_1}{I_2} = -2$$

$$B = -2$$

$$③ \Rightarrow V_1 = 5(-7/_3)I_2 + 3I_2$$

$$= -\frac{35}{3} I_2 + 3I_2$$

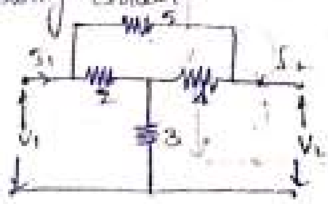
$$= -11.66 I_2 + 3I_2$$

$$V_1 = -8.6 I_2$$

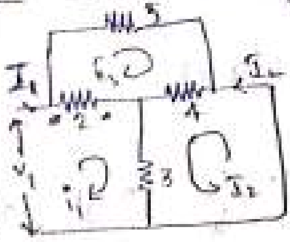
$$-\frac{V_1}{I_2} = +8.6$$

$$B = 8.6$$

1. Calculate y-parameter for the following circuit



Case 1:- Short circuit port ② i.e $V_2 = 0$



Loop①

$V_1 = 2(i_1 - i_3) + 3(i_1 + i_2)$

$5i_1 + 3i_2 - 2i_3 = V_1 \longrightarrow ①$

Loop②

$4(i_2 + i_3) + 3(i_2 + i_1) = 0$

$3i_1 + 7i_2 + 4i_3 = 0 \longrightarrow ②$

Loop③:

$2(i_3 - i_1) + 5i_3 + 4(i_3 + i_2) = 0$

$2i_1 - 4i_2 - 11i_3 = 0 \longrightarrow ③$

from eq③ $i_2 = \frac{2}{4} i_1 - \frac{11}{4} i_3$

$i_2 = 0.5i_1 - 2.75i_3 \longrightarrow ④$

from eq② $i_3 = -\frac{3}{4} i_1 - \frac{7}{4} i_2$

$i_3 = -0.75i_1 - 1.75i_2 \longrightarrow ⑤$

put eq ⑤ in ④

$i_2 = 0.5i_1 - 2.75(-0.75i_1 - 1.75i_2)$

$i_2 = 0.5i_1 + 2.0625i_1 + 4.8125i_2$

$-3.8125 i_2 = 2.5625 i_1$

$i_2 = -0.672 i_1 \longrightarrow ⑥$

put eq ⑥ & ⑤ in eq① we get

$5i_1 + 3(-0.672 i_1) - 2(-0.75i_1 - 1.75i_2) = V_1$

$5i_1 - 2.016i_1 + 1.5i_1 + 3.5i_2 = V_1$

$4.484i_1 + 3.5(-0.672)i_1 = V_1$

$2.132 i_1 = V_1$

$\frac{I_1}{V_1} = \frac{1}{2.132} = \boxed{0.469\mho = y_{11}}$

from eq② $i_1 = -\frac{7}{3} i_2 - \frac{4}{3} i_3$

$i_1 = -2.33i_2 - 1.33i_3 \longrightarrow ⑦$

put eq ⑤ in eq ① we get

$5i_1 + 3i_2 - 2[-0.75i_1 - 1.75i_2] = V_1$

$5i_1 + 3i_2 + 1.5i_1 + 3.5i_2 = V_1$

$6.5i_1 + 7.5i_2 = V_1 \longrightarrow ⑨$

from eq③ $i_3 = +\frac{2}{11} i_1 - \frac{4}{11} i_2$

$i_3 = 0.181i_1 - 0.363i_2 \longrightarrow ⑧$

Put eq ⑧ in ⑦ we get

$i_1 = -2.33i_2 - 1.33[0.181i_1 - 0.363i_2]$

$= -2.33i_2 - 0.241i_1 + 0.483i_2$

$i_1 = -0.241i_1 - 1.847i_2$

$1.241i_1 = -1.847i_2 \Rightarrow i_1 = -1.488 i_2$

eq ⑨ $\Rightarrow$

$6.5(-1.488 i_2) + 7.5 i_2 = V_1$

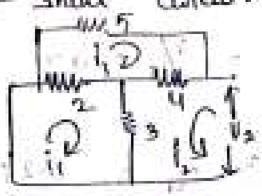
$-9.672 i_2 + 7.5 i_2 = V_1$

$-2.172 i_2 = V_1$

$\frac{I_2}{V_1} = \frac{1}{-2.172} = -0.46$

$\boxed{y_{21} = -0.313\mho}$

Case:2 Short circuit loop① i.e $V_1 = 0$



Apply KVL to loop①

$2(i_1 - i_3) + 3(i_1 + i_2) = 0$

$5i_1 + 3i_2 - 2i_3 = 0 \longrightarrow ①$

$V_2 = 4(i_2 + i_3) + 3(i_2 + i_1)$

$3i_1 + 7i_2 + 4i_3 = V_2 \longrightarrow ②$

$2(i_3 - i_1) + 5i_3 + 4(i_3 + i_2) = 0$

$-2i_1 + 4i_2 + 11i_3 = 0 \longrightarrow ③$

③ $i_1 = 2i_2 + 5.5i_3 \longrightarrow ④$

① $i_3 = 2.5i_1 + 1.5i_2 \longrightarrow ⑤$

$i_1 = 2i_2 + 5.5(2.5i_1 + 1.5i_2)$

$i_1 = 13.75i_1 + 10.25i_2 \longrightarrow \boxed{i_1 = -0.804 i_2}$

put eq ① in eq ②

$3i_1 + 7i_2 + 4[3.5i_1 + 1.5i_2] = V_2$

$13i_1 + 13i_2 = V_2$

$13(0.804 i_2) + 13i_2 = V_2$

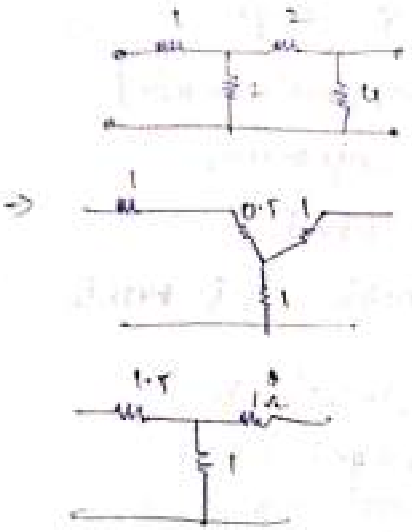
$(10.452 + 13)i_2 = V_2$

$i_2 / V_2 = \frac{1}{10.452}$

$\boxed{Y_{22} = 0.0426\ \mho} = -0.425$

$Y_{21} = -0.315\ \Omega^{-1} = -0.315\ \mho$

③ find y-parameters



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2.5 & 1 \\ 1 & 2 \end{bmatrix}$$

$Y_{11} = \frac{Z_{22}}{4} = 2/4 = 0.5$

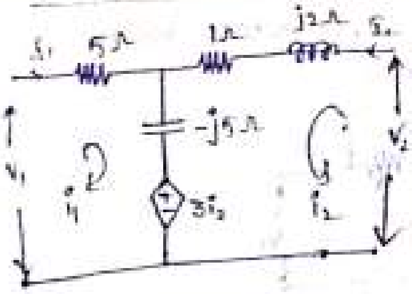
$Y_{12} = \frac{-Z_{12}}{4} = \frac{-1}{4} = -0.25$

$Y_{21} = -0.25$

$Y_{22} = \frac{2.5}{4} = 0.625$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.625 \end{bmatrix}$$

⑤ obtain z-parameters for following circuit.



Case 1:

Open circuit port ② (i.e) ($i_2 = 0$)

loop ①

$V_1 = 5i_1 - j5i_1 - j5i_2 + 3i_2$

$V_1 = (5 - j5)i_1 \Rightarrow \frac{V_1}{i_1} = \boxed{5 - j5 = Z_{11}}$

$\frac{i_1}{V_1} = \frac{1}{5 - j5}$

Loop ②

$V_2 = (1 + j2)i_2 - j5i_2 - j5i_1 + 3i_2$

$V_2 = -j5 i_1$

$\frac{V_2}{i_1} = \boxed{Z_{21} = -j5}$

Case 2: open circuit Loop ① (i.e) $i_1 = 0$

Loop ①

$V_1 = 5i_1 - j5i_1 - j5i_2 + 3i_2$

$V_1 = (3 - j5)i_2 \Rightarrow \frac{V_1}{i_2} = \boxed{3 - j5 = Z_{12}}$

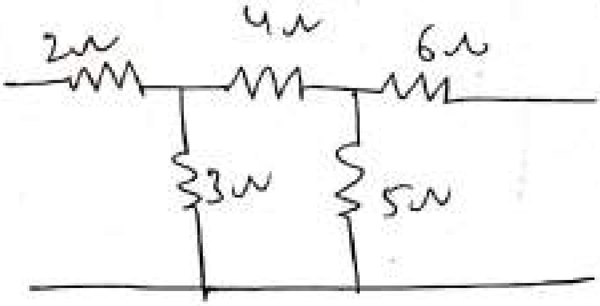
Loop 2

$V_2 = (1 + j2)i_2 - j5i_2 - j5i_1 + 3i_2$

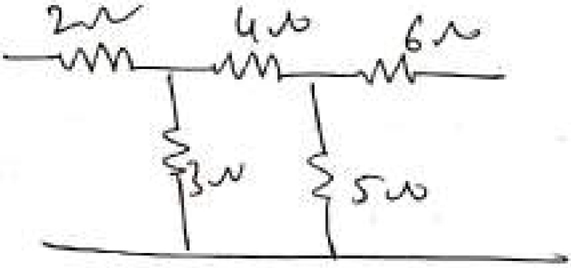
$V_2 = (4 - j3)i_2$

$\Rightarrow \frac{V_2}{I_2} = \boxed{Z_{22} = 4 - j3}$

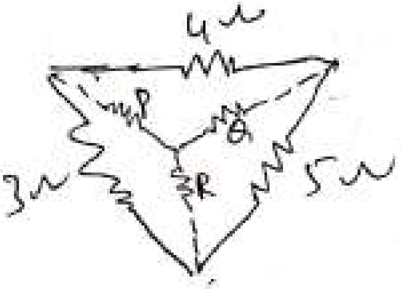
(2) Find Y-parameters for the following ckt

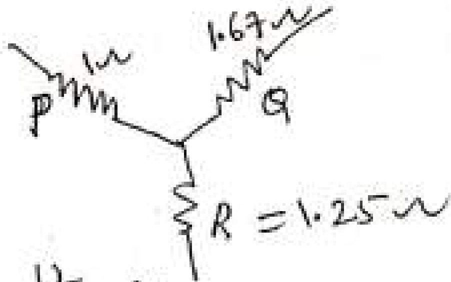


Sol



First convert Δ to Y





$$P = \frac{3 \times 4}{3+4+5} = \frac{12}{12} = 1\Omega$$

$$Q = \frac{4 \times 5}{3+4+5} = \frac{\cancel{20}^{5}}{\cancel{12}_{3}} = \frac{5}{3} = 1.67\ \Omega$$

$$R = \frac{3 \times 5}{3+4+5} = \frac{\cancel{15}^{5}}{\cancel{12}_{4}} = \frac{5}{4} = 1.25\ \Omega$$

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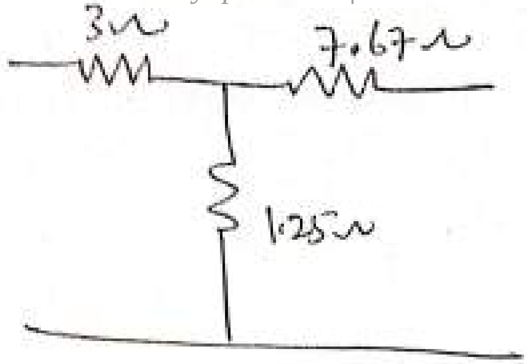


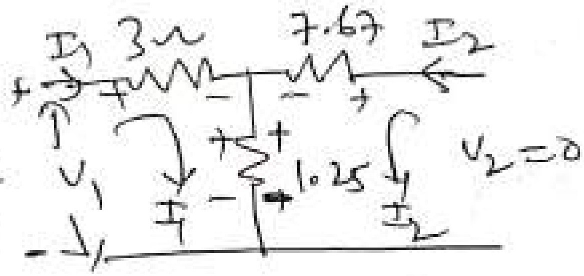
Y - parameters

$I_1 = Y_{11}V_1 + Y_{12}V_2$ — (1)

$I_2 = Y_{21}V_1 + Y_{22}V_2$ — (2)

case (1) To find Y_{11} & Y_{21} put $V_2 = 0$

apply KVL for loop 1



$-V_1 + 3I_1 + 1.25I_1 + 1.25I_2 = 0$

$V_1 - 4.25I_1 - 1.25I_2 = 0$ — (1)

apply KVL for loop 2

$-V_2 + 7.67I_2 + 1.25I_2 + 1.25I_1 = 0$

put $V_2 = 0$

$8.92I_2 + 1.25I_1 = 0$

$I_2 = \frac{-1.25 I_1}{8.92} = -0.14I_1$ — (2)

put eq (2) in eq (1)

$V_1 - 4.25I_1 - (-0.14 \times 1.25)I_1 = 0$

$V_1 - 4.25I_1 + 0.175I_1 = 0$

$V_1 = 4.075I_1 \Rightarrow \frac{I_1}{V_1} = \frac{1}{4.075} = \boxed{0.246 = Y_{11}}$

From eq (2)

$$I_1 = -7.14 I_2 \quad -(3)$$

Sub eq (3) in eq (1)

$$V_1 - 4.25[-7.14 I_2] - 1.25 I_2 = 0$$

$$V_1 + 30.34 I_2 - 1.25 I_2 = 0$$

$$V_1 + 29.09 I_2 = 0$$

$$29.09 I_2 = -V_1$$

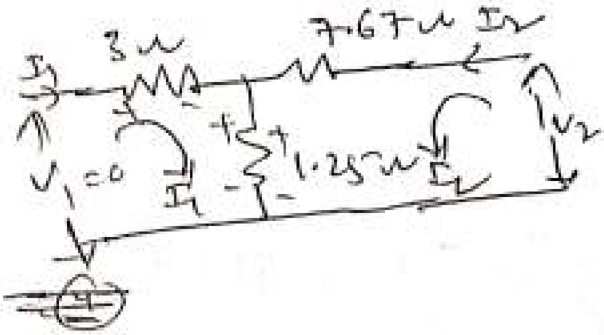
$$\frac{I_2}{V_1} = -0.034$$

$$\boxed{Y_{21} = -0.034}$$

Case(2): To find Y_{12} & Y_{22}, put $V_1 = 0$

$$Y_{12} = \frac{I_1}{V_2}, \quad Y_{22} = \frac{I_2}{V_2}$$

apply KVL for loop I



$$-V_1 + 3I_1 + 1.25 I_1 + 1.25 I_2 = 0$$

$$V_1 - 4.25 I_1 - 1.25 I_2 = 0$$

put $V_1 = 0$

$$4.25 I_1 = -1.25 I_2 \quad -(4)$$

apply KVL for loop II

$$-V_2 + 7.67 I_2 + 1.25 I_2 + 1.25 I_1 = 0$$

$$V_2 - 8.92 I_2 - 1.25 I_1 = 0 \quad -(5)$$

From eq(3) $\Rightarrow$ $I_2 = \frac{-4.25}{1.25} I_1$

$I_2 = -3.4 I_1$ — (5.1)

put eq (5.1) in eq(5)

~~$V_2 - 8.42$~~

$V_2 - 8.42(-3.4 I_1] - 1.25 I_1 = 0$

$V_2 + 28.62 I_1 - 1.25 I_1 = 0$

$V_2 + 27.37 I_1 = 0$ — (6)

$\frac{I_1}{V_2} = \frac{-1}{27.37}$ ℧

$\boxed{Y_{12} = -0.0365}$ ℧ — (7)

also from eq (5.1)

$I_1 = \frac{-1.25}{4.25} I_2$

$I_1 = -0.29 I_2$ — (8)

sub eq (8) in eq (5)

$V_2 - 8.92 I_2 + 1.25 \times 0.29 I_2 = 0$

$V_2 - 8.92 I_2 + 0.36 I_2$

$V_2 = 8.92 I_2 - 0.36 I_2$

$V_2 = 8.56 I_2$

$\frac{I_2}{V_2} = 0.117 \Rightarrow \boxed{Y_{22} = 0.117}$

$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{bmatrix} 0.246 & -0.036 \\ -0.034 & 0.117 \end{bmatrix}$

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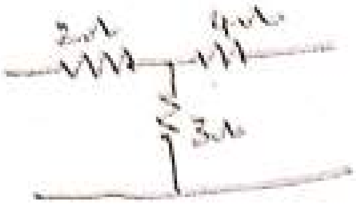
3. Find h-parameters for the following ckt



Sol

h-parameters

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad —(1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad —(2)$$

case(1) To find h_{11} & h_{21}, put $V_2 = 0$

$$\boxed{h_{11} = \frac{V_1}{I_1}}, \quad \boxed{h_{21} = \frac{I_2}{I_1}}$$

apply KVL for loop 1

$$-V_1 + 2I_1 + 3I_1 + 3I_2 = 0$$

$$V_1 - 5I_1 - 3I_2 = 0 \quad —(3)$$

apply KVL for loop 2

$$-V_2 + 4I_2 + 3I_2 + 3I_1 = 0$$

put $V_2 = 0$

$$7I_2 + 3I_1 = 0$$

$$7I_2 = -3I_1 \quad —(4)$$

$$\boxed{h_{21} = \frac{I_2}{I_1} = \frac{-3}{7} = -0.428}$$

From eq (4)

$$I_2 = -0.428 I_1 \quad —(5)$$

put eq (5) in eq (1)

$$V_1 - 5I_1 - 3(-0.428) I_1 = 0$$

$$V_1 - 5I_1 + 1.285 I_1 = 0$$

$$\boxed{h_{11} = \frac{V_1}{I_1} = 3.715\ \Omega}$$

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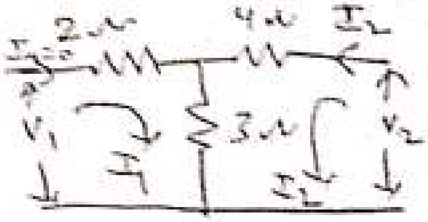
case(2) : To find h_{12} & h_{22} . put $I_1 = 0$

$$\boxed{h_{12} = \frac{V_1}{V_2}} \quad , \quad \boxed{h_{22} = \frac{I_2}{V_2}}$$

Apply KVL for loop1



$$-V_1 + 2I_1 + 3I_1 + 3I_2 = 0$$

$$V_1 - 5I_1 - 3I_2 = 0$$

put $I_1 = 0$

$$V_1 = 3I_2 \quad \text{--- (6)}$$

apply KVL for loop2

$$-V_2 + 4I_2 + 3I_2 + 3I_1 = 0$$

$$V_2 - 7I_2 - 3I_1 = 0$$

put $I_1 = 0$

$$V_2 = 7I_2 \quad \text{--- (7)}$$

$$\boxed{h_{22} = \frac{I_2}{V_2} = \frac{1}{7}}$$

put $I_2 = \frac{1}{7} V_2$ --- (8)

sub eq (8) in eq (6)

$$V_1 = 3 \times \frac{1}{7} V_2$$

$$\boxed{h_{12} = \frac{V_1}{V_2} = \frac{3}{7}}$$

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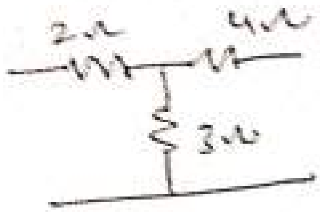
(4) Find ABCD parameters for the following ckt



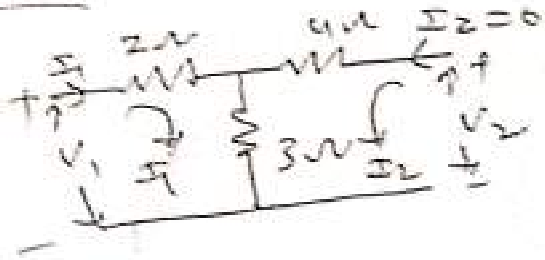
Sol

ABCD parameters

$V_1 = AV_2 - BI_2$ —(1)

$I_1 = CV_2 - DI_2$ —(2)

case(i) To find A & C, put $I_2 = 0$



apply KVL for loop1

$-V_1 + 2I_1 + 3I_1 + 3I_2 = 0$

& put $I_2 = 0$

$V_1 - 5I_1 = 0$

$V_1 = 5I_1$ —(3)

apply KVL for loop2

$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$

put $I_2 = 0$

$V_2 = 3I_1$ —(4)

$C = \boxed{\frac{I_1}{V_2} = \frac{1}{3}} \Rightarrow \boxed{C = \frac{1}{3}}$

From eq (4) $I_1 = \frac{1}{3}V_2$ —(5)

put eq (5) in eq(3)

$V_1 = 5 \times \frac{1}{3} V_2$

$\boxed{A = \frac{V_1}{V_2} = \frac{5}{3}}$

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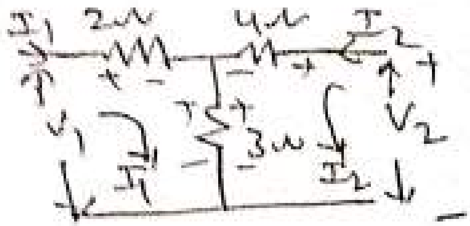
Case(2): To find B & D by putting $V_2 = 0$ $\left[B = \frac{-V_1}{I_2}, D = \frac{-I_1}{I_2}\right]$



apply KVL for loop 1

$$-V_1 + 2I_1 + 3I_1 + 3I_2 = 0$$

$$V_1 = 5I_1 + 3I_2 \quad -(6)$$

apply KVL for loop 2

$$V_2 - 4I_2 - 3I_2 - 3I_1 = 0$$

put $V_2 = 0$

$$7I_2 + 3I_1 = 0$$

$$7I_2 = -3I_1 \quad -(7)$$

$$\boxed{D = \frac{-I_1}{I_2} = \frac{7}{3}}$$

From eq (7) $I_1 = \frac{-7}{3} I_2$ — (8)

sub eq(8) in eq(6)

$$V_1 = 5\left(\frac{-7}{3} I_2 + 3I_2\right)$$

$$V_1 = -8.66 I_2$$

$$\boxed{B = \frac{-V_1}{I_2} = 8.66}$$

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Two-Port Parameter Conversion Table

Desired Parameters	Given Parameters [z]	[y]	[h]	[t]
[z]	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_t}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
[y]	$\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_t}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_t}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$
[t]	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_t = AD - BC$

Problems on interrelation between two port parameters

① Determine Y, h and ABCD Parameters if Z parameters are

$Z_{11} = 10,\ Z_{12} = 5,\ Z_{21} = 5,\ Z_{22} = 9.$

Sol Given

$Z_{11} = 10,\ Z_{12} = 5,\ Z_{21} = 5,\ Z_{22} = 9.$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & 9 \end{bmatrix}$$

(i) Y - Parameters

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$$
$$= 10 \times 9 - 5 \times 5$$
$$= 90 - 25 = 65$$

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{9}{65}$$

$$Y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-5}{65}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-5}{65}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{10}{65}.$$

(2) h - Parameters

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{pmatrix}$$

$$= \begin{bmatrix} \frac{65}{9} & \frac{5}{9} \\ -\frac{5}{9} & \frac{1}{9} \end{bmatrix}$$

(3) ABCD Parameters

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{5} & \frac{65}{5} \\ \frac{1}{5} & \frac{9}{5} \end{bmatrix}$$

II Find Z, Y, ABCD Parameters if h-parameters are

$h_{11} = 8$, $h_{12} = 5$, $h_{21} = 3$ $h_{22} = 12$

Sol (1) Z - parameters

$$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

$\Delta h = h_{11}h_{22} - h_{12}h_{21}$
$= 8\times12 - 5\times3$
$= 96 - 15$
$= 81$

$$= \begin{bmatrix} \frac{81}{12} & \frac{5}{12} \\ \frac{-3}{12} & \frac{1}{12} \end{bmatrix}$$

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(2) Y - Parameter

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{-h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{8} & -\frac{5}{8} \\ -\frac{3}{8} & \frac{81}{8} \end{pmatrix}$$

(3) ABCD Parameters

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-81}{3} & \frac{-8}{3} \\ \frac{-12}{3} & \frac{-1}{3} \end{pmatrix}$$

III Find Z, h, ABCD parameters if Y-Parameters are

$Y_{11} = 9$, $Y_{12} = 6$, $Y_{21} = 6$, $Y_{22} = 15$

$\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21} = 15 \times 9 - 36$
$= 135 - 36$
$= 99$

Sol

① Z parameters

$$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{pmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{pmatrix} = \begin{pmatrix} \frac{15}{99} & -\frac{6}{99} \\ \frac{-6}{99} & \frac{9}{99} \end{pmatrix}$$

② h - parameters

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{pmatrix} = \begin{pmatrix} \frac{1}{9} & \frac{-6}{9} \\ \frac{6}{9} & \frac{99}{9} \end{pmatrix}$$

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(3) ABCD Parameters

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{-\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{pmatrix} = \begin{pmatrix} \frac{-15}{6} & \frac{-1}{6} \\ \frac{-99}{6} & \frac{-9}{6} \end{pmatrix}$$

IV Find Z, Y, h parameters if $A = 6$, $B = 2$, $C = 3$, $D = 8$.

$$\Delta t = AD - BC = 6 \times 8 - 2 \times 3 = 48 - 6 = 42$$

Sol (1) Z parameters

$$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{pmatrix} \frac{A}{C} & \frac{\Delta t}{C} \\ \frac{1}{C} & \frac{D}{C} \end{pmatrix} = \begin{pmatrix} \frac{6}{3} & \frac{42}{3} \\ \frac{1}{3} & \frac{8}{3} \end{pmatrix}$$

(2) Y - Parameters

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} \frac{D}{B} & \frac{-\Delta t}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{pmatrix} = \begin{pmatrix} \frac{8}{2} & \frac{-42}{2} \\ \frac{-1}{2} & \frac{6}{2} \end{pmatrix} = \begin{pmatrix} 4 & -21 \\ -0.5 & 3 \end{pmatrix}$$

(3) h - Parameters

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} \frac{B}{D} & \frac{\Delta t}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{pmatrix} = \begin{pmatrix} \frac{2}{8} & \frac{42}{8} \\ \frac{-1}{8} & \frac{3}{8} \end{pmatrix}$$

Unit - III: Transient Analysis

D.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for D.C Excitation - Initial Conditions in network - Initial Conditions in elements - Solution Method Using Differential Equation and Laplace Transforms - Response of R-L & R-C Networks to Pulse Excitation.

A.C Transient Analysis: Transient Response of R-L, R-C, R-L-C Series Circuits for Sinusoidal Excitations - Solution Method Using Differential Equations and Laplace Transforms.

Unit Outcomes:

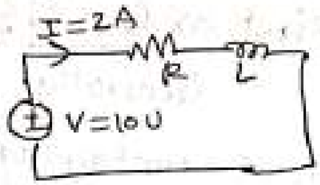
The student will be able to

- Distinguish between classical method and Laplace transform approach in analysing transient phenomenon in DC excitations
- Distinguish between classical method and Laplace transform approach in analysing transient phenomenon in sinusoidal excitations

TRANSIENT RESPONSE ANALYSIS

1. Steady state :

A circuit having voltage source or current source is said to be steady state if voltage and currents doesn't change with respect to time.



Ex : t = 2 sec , V = 10V , I = 2A

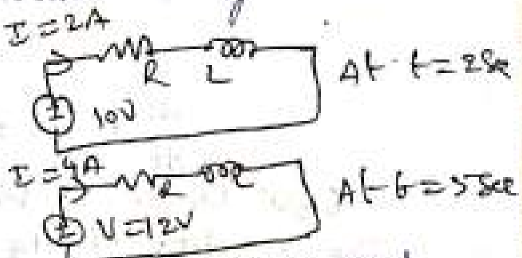
t = 5 sec , V = 10V , I = 2A

2. Transient state :

A circuit having voltage source, current source, circuit parameters like R, L & C. Now either change in voltage or current source or any one of the circuit parameters, then voltage and currents are changed from one state to another state is called transient state.

(OR)

The behaviour of voltage and currents when it is changed from one steady state to another steady state is called transient state.



Ex : t = 2 sec , V = 10 V , I = 2A

t = 5 sec , V = 12V , I = 4A

3. Transient time :

The time required for changing the voltage and currents from one state to another state is called Transient time.

4. Natural Response :

The circuit having circuit parameters like R, L, C with independent current and voltage sources, the response of output depends on the nature of the circuit, then the responce is called natural response.

5. Transient Response :

The circuit having circuit parameters like R, L, C with independent voltage and current sources, then the energy storing elements (L&C) which delivers energy to the resistors, the response or o/p change with time and get saturated.

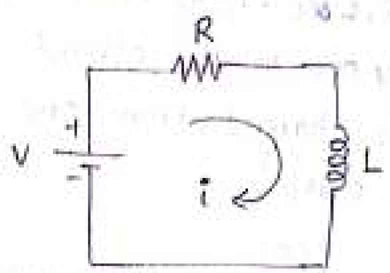
6 Forced Response :

The circuits having circuit parameters like R, L, with dependent voltage and current sources, the response or output depends on the sources, then it is called forced response.

First order differential equations

Generally kirchoff's voltage law is applied to circuit containing R-L, R-C & R-L-C which gives differential equation.

for example, take R-L circuit



$$V = IR + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \longrightarrow (1)$$

Eq(1) is called First order differential equation

The solution of the differential equation is obtained by two cases

Case 1 : Differential eqn's method

Case 2 : Laplace transforms method

Differential Equation Methods :-

In the case of DE method, the eqns are of two types.

1. Non-homogeneous DE
2. Homogeneous DE

Non-Homogenous Differential Equation:-

$$\frac{di}{dt} + Pi = Q \longrightarrow (2)$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

where $P = \frac{R}{L}$, $Q = \frac{V}{L}$

The solution is

i(t) = Complimentory func. + partial integer
= Transient response + steady state
= Natural response + steady state.

$$i(t) = i_{CF} + i_{ss}$$

$$= Ke^{-Pt} + e^{-Pt} \int Q e^{Pt} dt$$

$$= Ke^{-Pt} + e^{-Pt} Q \frac{e^{Pt}}{P}$$

$$\boxed{i(t) = Ke^{-Pt} + \frac{Q}{P}}$$

Homogeneous Differential Equation:-

$$\frac{di}{dt} + Pi = 0$$

The solution is

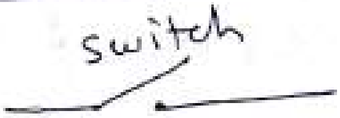
i(t) = complementary func. + Partial integral
= transient response + S.S
=

$$i(t) = Ke^{-Pt} + e^{-Pt} \int 0 \cdot e^{Pt} dt$$

$$\boxed{i(t) = Ke^{-Pt}}$$

Initial conditions in elements

Assume That



$t=0$, The switch is ready to operate

$t=0^-$, the time just before the switch is closed.

$t=0^+$, The time just immediately after closing the switch.

Now let us study the switching operation on passive elements such as R, L, C.

Resistor (R)

* It consumes energy.
* The property of resistor is it opposes the flow of current.

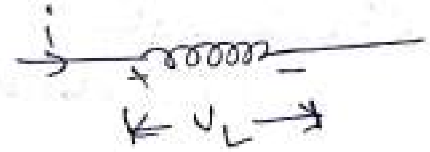
voltage across resister $(V) = iR$

current through resistor $(i) = \frac{V}{R}$

* 

* if switch is closed the current through the resistor changes instantaneously when voltages changes.

Inductor (L)

* Inductor stores energy in magnetic field.
* The property of Inductor is it opposes the flow of change in current (or) does not allow sudden change in current.
* 

Voltage across inductor (L) is $v_L = L\frac{di}{dt}$

current through inductor is $i_L = \frac{1}{L}\int_0^t v\,dt$

* 

switch open   switch closed

* If switch is closed

At $t=0^+$, Inductor does not allow sudden change of current. so it acts as open circuit

open

At $t=\alpha$ (steady state), Inductor continuously takes full current. so it acts as short circuit

short circuit.

capacitor (c)

* It stores energy in electric field.
* The property of capacitor is it does not allow sudden change in voltage.

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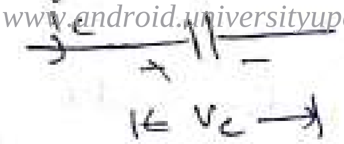
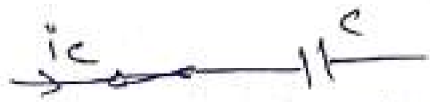
* 

current through the capacitor is $i_c = C\frac{dV_c}{dt}$

voltage across capacitor is $V_c = \frac{1}{C}\int_0^t i\,dt$.

* 

switch is opened



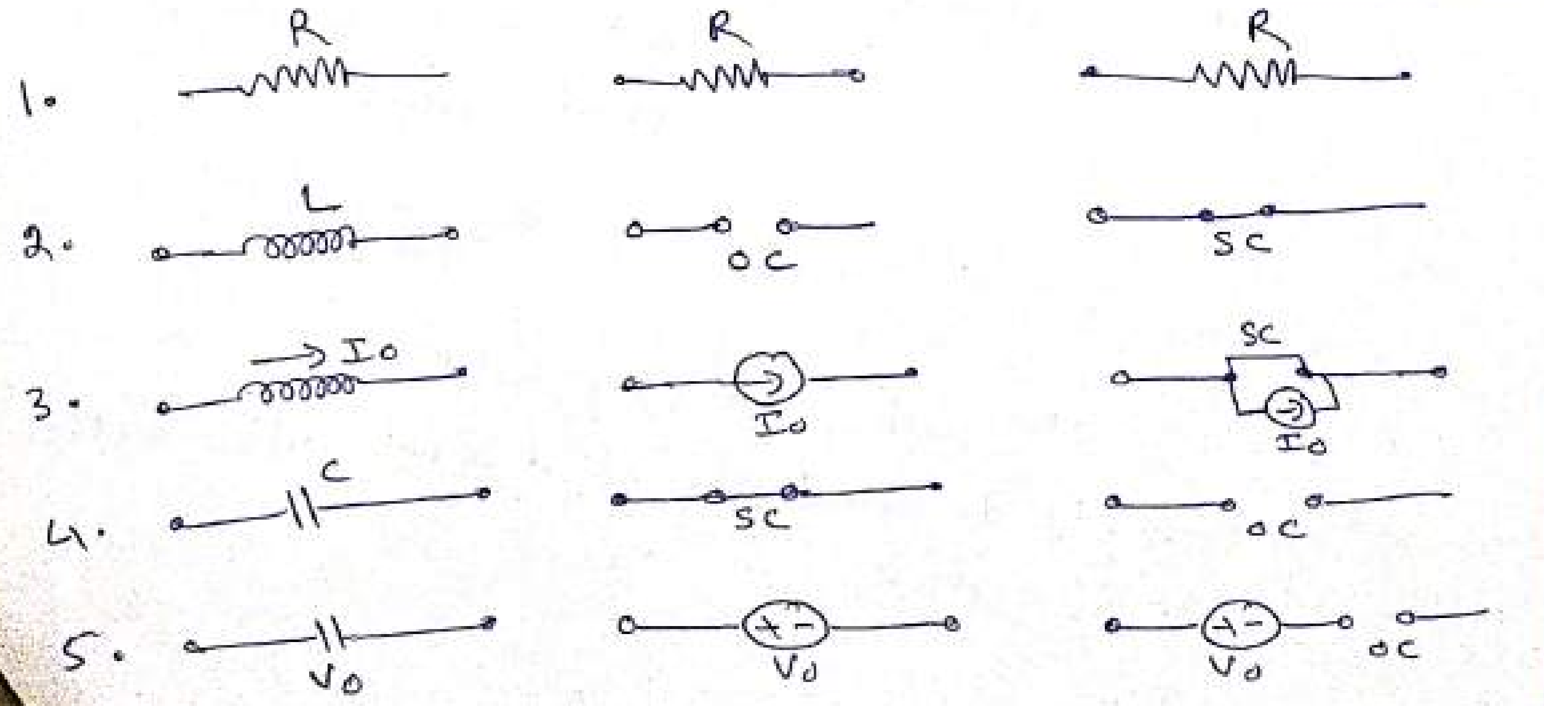
switch is closed.

* <u>If the switch is closed</u>

At $t=0^+$, capacitor does not allow sudden change in voltage. so it acts as short circuit

At $t=\alpha$, capacitor builds up full voltage & no current so it acts as open circuit

S.No	Parameter	At $t=0^+$ (Transient state)	At $t=\alpha$ (~~steady~~ (steady state) ~~state~~)



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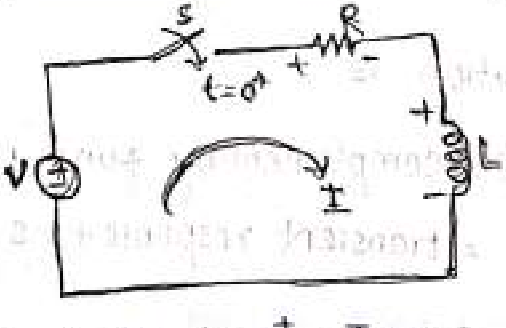
* Initial Conditions :

Resistor (R)	Inductor (L)	Capacitor (C)
At $t = 0^+$ (transient state) $V = IR$ $I = \frac{V}{R}$	At $t = 0^+$, $I = 0$ full voltage. o/c open circuit	At $t = 0^+$, $V = 0$ full load current s/c
At $t = \infty$ (steady state) $V = IR$ $I = \frac{V}{R}$	At $t = \infty$ (steady state) s/c Full load current $V = 0$	At $t = \infty$ (S.S) o/c full voltage $I = 0$

Note: The main purpose of initial conditions is to find out the unknown value of K.

* Transient Analysis of Series R-L Circuit:

Series R-L circuit is excited by DC input which is connected through switch as shown in the figure.



At $t = 0^-$, $I = 0$; At $t = 0^+$, $I = 0$

Initial Conditions :

$t = 0$, $I = 0$ ⟶ (1)

Now, apply KVL for the closed circuit

$$-V + IR + L\,di/dt = 0$$

$$V - IR - L\frac{di}{dt} = 0$$

$$V = IR + L\frac{di}{dt}$$

divide both sides with 'L'

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \longrightarrow (2)$$

It is a non-homogeneous equation

$$\frac{di}{dt} + Pi = Q \text{ , where } P = \frac{R}{L}, \; Q = \frac{V}{L}$$

The solution for eqn (2) is

$$i(t) = Ke^{-pt} + \frac{Q}{P}$$

$$i(t) = Ke^{-\frac{R}{L}t} + \frac{V}{R} \longrightarrow (3)$$

Subst. eqn (1) in eqn (3)

$$0 = Ke^{0} + \frac{V}{R}$$

$$\boxed{K = \frac{-V}{R}} \longrightarrow (4)$$

Subst. eqn (4) in eqn (3)

$$i(t) = -\frac{V}{R} e^{-\frac{R}{L}t} + \frac{V}{R}$$

$$\boxed{i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)} \longrightarrow (5)$$

In case of series R-L circuit, the time constant.

$$\tau = \frac{L}{R}$$

subst. τ in eqn (5)

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \longrightarrow (6)$$

Now, $t = \tau$, then

$$i(t) = \frac{V}{R}(1 - e^{-1})$$

$$i(t) = \frac{V}{R}(1 - 0.367)$$

$$i(t) = 0.633 \frac{V}{R}$$

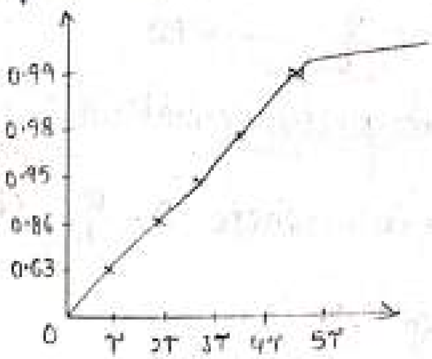
at $t = 2\tau$, $i(t) = \frac{V}{R}(1 - e^{-2}) = 0.865 \frac{V}{R}$

$t = 3\tau$, $i(t) = \frac{V}{R}(1 - e^{-3}) = 0.951 \frac{V}{R}$

$t = 4\tau$, $i(t) = \frac{V}{R}(1 - e^{-4}) = 0.98 \frac{V}{R}$

$t = 5\tau$, $i(t) = \frac{V}{R}(1 - e^{-5}) = 0.99 \frac{V}{R}$

Now, the graph b/w i(t) vs t



Voltage across resistor:

$$V_R = iR$$

$$= \frac{V}{R}(1-e^{-\frac{t}{\tau}})R$$

$$\boxed{V_R = V(1-e^{-t/\tau})}$$

Voltage across inductor:

$$V = V_R + V_L$$

$$V_L = V - V_R$$

$$= V - V(1-e^{-\frac{t}{\tau}})$$

$$\boxed{V_L = Ve^{-t/\tau}}$$

Q: Find the current in series R-L series circuit having $R=2\Omega$ and $L=10H$, while a DC voltage of 100 V is applied. What is the value of current after 5 sec of switch ON. what is the time taken by the current to reach half of its final value.

Sol: Given that

$R=2\Omega$
$L=10\ H$
$V=100\ V$
$t=5$ sec

WKT $i(t) = \frac{V}{R}(1-e^{-\frac{R}{L}t})$

$$= \frac{100}{2}(1-e^{-\frac{2}{10}t})$$

$$= 50(1-e^{-1/5\, t})$$

at $t=5$, $i = 50(1-e^{-5/5})$

$$\boxed{i = 31.606\ A}$$

final value $i = \frac{V}{R} = \frac{100}{2} = 50\ A$

half of its final value $= 25\ A$

$$25 = 50\left(1-e^{-t/5}\right)$$

$$-25 = -50\,e^{-t/5}$$

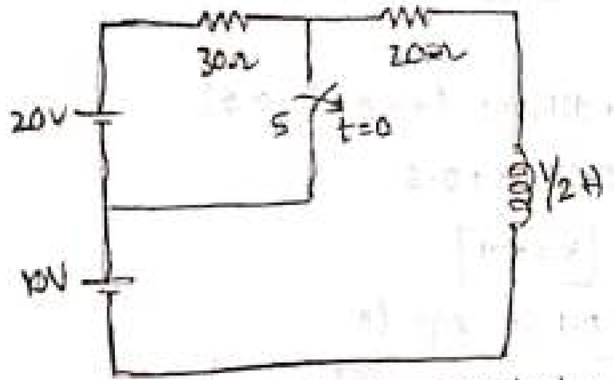
$$\frac{1}{2} = e^{-t/5}$$

$$\log\left(\frac{1}{2}\right) = \frac{-t}{5}$$

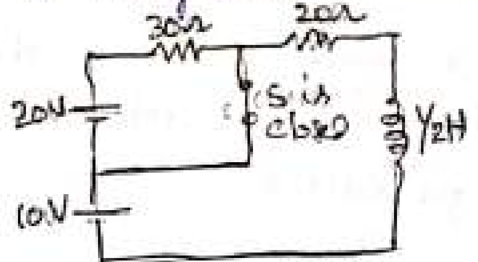
$$t = -5\ln\left(\tfrac{1}{2}\right)$$

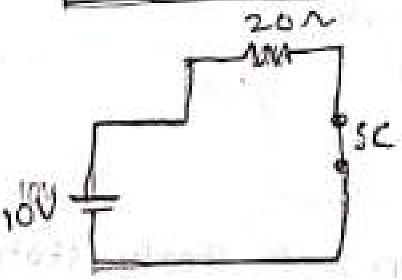
$$\boxed{t = 3.46\ \text{sec}}$$

2. In the figure switch 's' is closed and steady state is reached. At $t=0$, the switch 's' is opened, find $i_L(t)$ for $t>0$



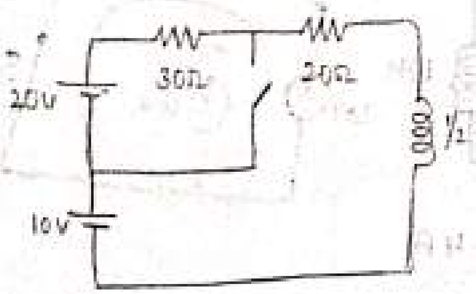
Ans: S is closed & steady state is reached:

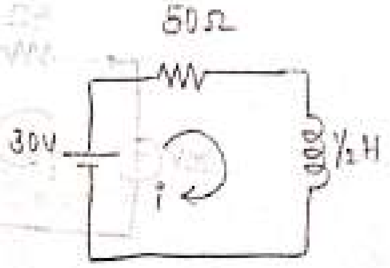




$$i = \frac{V}{R} = \frac{10}{20} = 0.5\,A$$

At $t=0$ switch is opened.





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Initial conditions are $t=0$, $i=0.5$ A

Applying KVL to the closed circuit

$$30-50\,i_L(t)-\frac{1}{2}\frac{d\,i_L(t)}{dt}=0$$

$$50\,i_L(t)+\frac{1}{2}\frac{d\,i_L(t)}{dt}=30$$

$$\frac{d\,i_L(t)}{dt}+100\,i_L(t)=60 \longrightarrow (2)$$

It is like a non-homogeneous eqn, the solution is

$$i_L(t)=ke^{-Pt}+\frac{Q}{P} \quad [P=100, Q=60]$$

$$=ke^{-100t}+0.6 \longrightarrow (3)$$

To find k:

Use initial conditions $(t=0, i=0.5)$

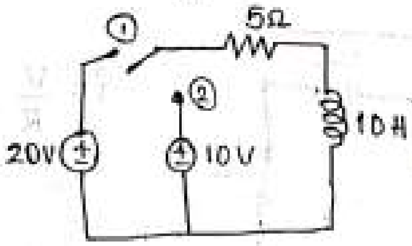
$$0.5=k+0.6$$

$$\boxed{k=-0.1}$$

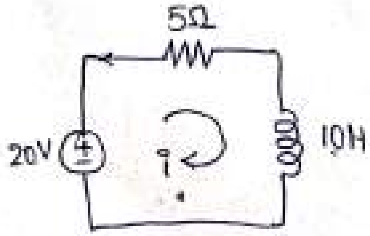
Substitute $k=-0.1$ in eqn (3)

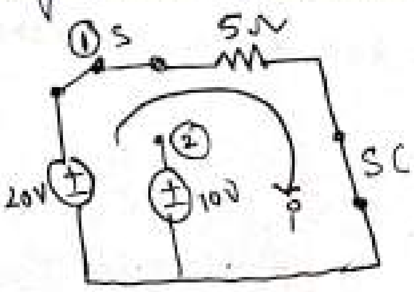
$$\boxed{i_L(t)=-0.1\,e^{-100t}+0.6}$$

3. In the ckt shown in figure switch is at position 1 and steady state is reached. At $t=0$, switch is moved to position 2. Find the expression for current.



Ans: Switch is at position 1 & steady state is reached.





$$i=\frac{V}{R}=\frac{20}{5}=4\text{A}$$

4

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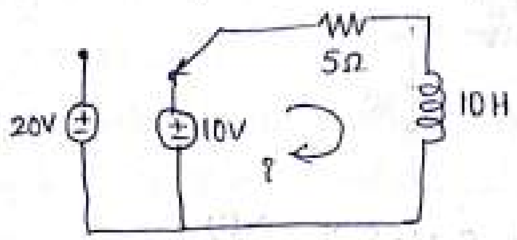
At $t=0$, switch is moved to position 2:



Initial conditions:

$$t=0\ ,\ i=4A$$

Now, apply KVL to the circuit

$$10-5i-10\frac{di}{dt}=0$$

$$10\frac{di}{dt}+5i=10$$

$$\frac{di}{dt}+\frac{1}{2}i=1 \longrightarrow (1)$$

It is like a non-homogeneous eqn, the solution is

$$i(t)=Ke^{-Pt}+\frac{Q}{P} \quad [P=1/2\ ;\ Q=1]$$

$$=Ke^{-t/2}+2 \longrightarrow (2)$$

To find K:

Use initial conditions; $(t=0,\ i=4A)$

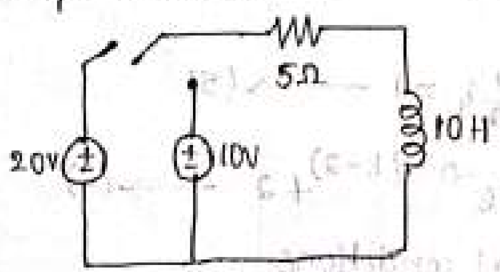
$$4=K+2$$

$$\boxed{K=2}$$

substitute $K=2$ in eqn (2)

$$\boxed{i(t)=2e^{-t/2}+2.}$$

4. In the following circuit at $t=0$, switch is moved to position 1. At $t=2$ sec, switch is moved to position 2. Find the current through inductor.



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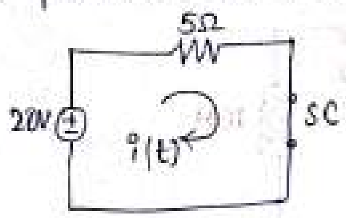
Ans: Switch is at position 1 at t=0



Initial conditions, $t=0, i=0 \longrightarrow (1)$

applying KVL to the circuit

$$t=0, \quad 20 - 5i = 10\frac{di}{dt}$$

$$\cancel{5i=20}$$

$$\boxed{i=4A}$$

Switch is at ~~position 1 at t~~:

$$10\frac{di}{dt} + 5i = 20$$

$$\frac{di}{dt} + \frac{1}{2}i = 2 \longrightarrow (2)$$

It is in non-homogeneous eqn., the solution is

$$i(t) = Ke^{-Pt} + \frac{Q}{P} \quad [P = ½, Q = 2]$$

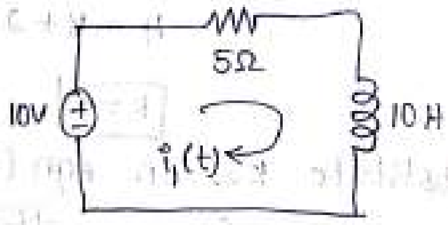
$$i(t) = -4e^{-0.5t} + 4 \longrightarrow (3)$$

Now switch is moved to position 2 at t=2:

Initial condition:

$$t=2, \; i = 4-4e^{-0.5t}$$

$$= 4-4e^{-0.5(2)}$$



$$t=2, \; i = 2.5A \longrightarrow (4)$$

Now, apply KVL to the closed circuit

$$10 - 5i_1 - 10\frac{di_1}{dt} = 0$$

$$10\frac{di_1}{dt} + 5i_1 = 10$$

$$\frac{di_1}{dt} + \frac{1}{2}i_1 = 1 \longrightarrow (5)$$

Solution is $i(t) = Ke^{-0.5(t-2)} + 2 \longrightarrow (6)$

to find K, use initial conditions,

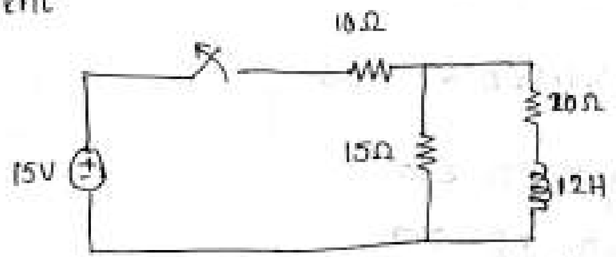
$$t=2, \; i = 2.5A$$

$$\boxed{K = 0.5}$$

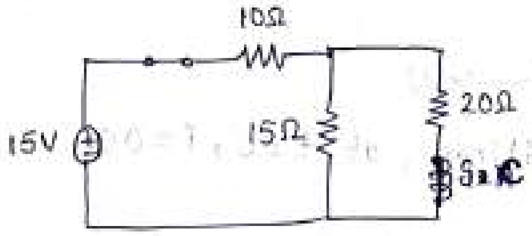
Substitute $K = 0.5$ in eqn (6)

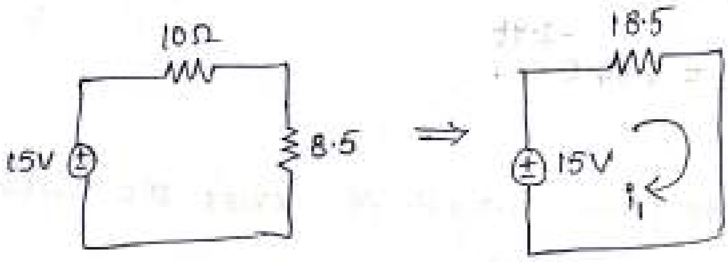
$$i_1(t) = 0.5e^{-0.5(t-2)} + 2$$

5. In the following circuit switch is closed and steady state is reached. At $t=0$, switch is suddenly opened, find the expression for current.



Sol: Switch is closed and & s.s is reached.





$$i_1 = \frac{15}{18.5} = 0.81\text{ A}$$

According to current division rule

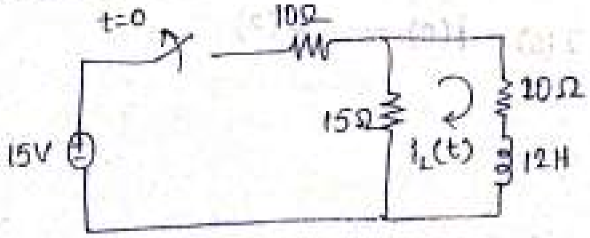
$$i_3 = i_1 \times \frac{15}{15+20}$$

$$= 0.81 \times \frac{15}{35}$$

$$i_3 = 0.34\text{ A}$$

Initial current $i_0 = i_3 = 0.34\text{ A} \longrightarrow (1)$ at $t=0$

Switch is opened suddenly at $t=0$:



apply KVL to the closed ckt

$$-15\, i_L(t) - 20\, i_L(t) - 12\frac{di_L(t)}{dt} = 0$$

$$-35\, i_L(t) - 12\frac{di_L(t)}{dt} = 0$$

$$\frac{di_L(t)}{dt} + \frac{35}{12}\, i_L(t) = 0$$

$$\frac{di_L(t)}{dt} + 2.9\, i_L(t) = 0 \longrightarrow (2)$$

eqn (2) is a homogeneous eqn.

$$\frac{di}{dt} + Pi = 0 \quad , P = 2.9$$

Solution is $i_L(t) = ke^{-pt}$

$$i_L(t) = Ke^{-2.9t} \longrightarrow (3)$$

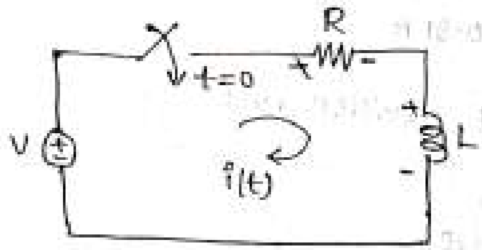
to find k, use initial conditions, at $t=0$, $i = 0.34$

$$\boxed{0.34 = K}$$

$$i_L(t) = 0.34\, e^{-2.9t}$$

Method 2:

DC transient response analysis of series R-L circuit by Laplace transform method:



KVL equation, $-V + Ri(t) + L\frac{di(t)}{dt} = 0$

$$V = Ri(t) + L\frac{di(t)}{dt} \longrightarrow (1)$$

In laplace transform

$$L(V) = V/s$$

$$L[i(t)] = F(s)$$

$$L\left[\frac{di(t)}{dt}\right] = s\,I(s) - i(0) \longrightarrow (2)$$

initial condition

Initial conditions:

$t=0, i=0$

apply initial current $i=0$ in eqn (2)

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$$L\left[\frac{di(t)}{dt}\right] = SI(S)$$

by apply laplace transform

$$\frac{V}{S} = RI(S) + L\left[S\,I(S)\right]$$

$$I(S)\left[R + SL\right] = \frac{V}{S}$$

$$I(S) = \frac{V/S}{R+SL}$$

$$I(S) = \frac{V/S}{L\left(S+\frac{R}{L}\right)}$$

$$I(S) = \frac{(V/L)}{S\left(S+\frac{R}{L}\right)} \longrightarrow (3)$$

Using partial fractions,

$$\frac{(V/L)}{S(S+R/L)} = \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$$

$$\frac{A}{S} = \left.\frac{V/L}{S\left(S+\frac{R}{L}\right)}\right|_{S=0} = \frac{V/L}{R/L}$$

$$\boxed{A = \frac{V}{R}}$$

$$\frac{B}{S+R/L} = \left.\frac{V/L}{S(S+R/L)}\right|_{S=-\frac{R}{L}}$$

$$B = \frac{(V/L)}{-(R/L)}$$

$$\boxed{B = -\frac{V}{R}}$$

from eqn (3) $I(S) = \frac{V/R}{S} - \frac{V/R}{S+R/L}$

$$= \frac{V}{R}\left[\frac{1}{S} - \frac{1}{S+R/L}\right] \longrightarrow (4)$$

apply inverse Laplace

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

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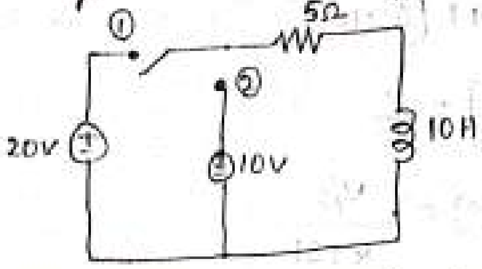
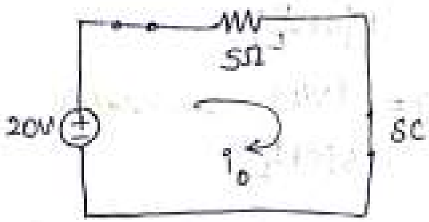
1. In the circuit shown in fig. switch is at position 1 & steady state is reached. At $t=0$ switch is moved to position 2. Find current through inductor using Laplace transform method.

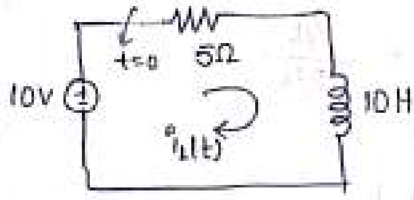


Ans: Switch is at position 1 & steady state is reached.



$$i_0 = \frac{20}{5} = 4A$$

Switch is moved to position 2 at $t=0$



Initial condition,

$$t=0,\ i_0 = 4A \longrightarrow (1)$$

Apply KVL,

$$10 - 5i_L(t) - 10\frac{di_L(t)}{dt} = 0$$

$$10\frac{di_L(t)}{dt} + 5i_L(t) = 10$$

$$\frac{di_L(t)}{dt} + \frac{1}{2}i_L(t) = 1 \longrightarrow (2)$$

apply laplace transform,

$$L\left[\frac{di_L(t)}{dt} + 0.5\, i_L(t)\right] = L[1]$$

$$[sI(s) - i(0)] + 0.5\, I(s) = \frac{1}{s}$$

$$I(s)\,[s+0.5] = \frac{1}{s} + 4$$

$$I(s) = \frac{1+4s}{s(s+0.5)} \longrightarrow (3)$$

2

Ans

$$\frac{1+4s}{s(s+0.5)} = \frac{A}{s} + \frac{B}{s+0.5}$$

$$\frac{A}{s} = \left.\frac{1+4s}{s(s+0.5)}\right|_{s=0}$$

$$A = \frac{1+4(0)}{0+0.5}$$

$$\boxed{A = 2}$$

$$\frac{B}{s+0.5} = \left.\frac{1+4s}{s(s+0.5)}\right|_{s=-0.5}$$

$$B = \frac{1+4(-0.5)}{-0.5}$$

$$= \frac{1-2}{-0.5}$$

$$\boxed{B = 2}$$

$$\therefore I(s) = \frac{2}{s} + \frac{2}{s+0.5}$$

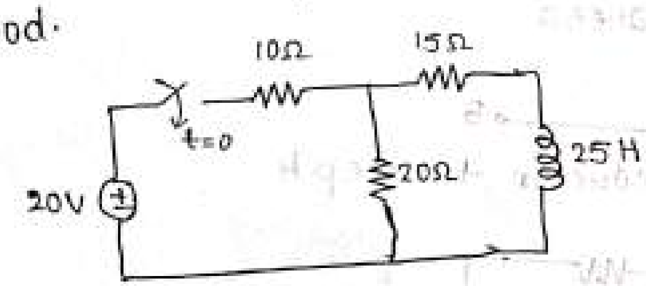
By using inverse Laplace

$$L^{-1} I(s) = L^{-1}\left\{\frac{2}{s}\right\} + L^{-1}\left\{\frac{2}{s+0.5}\right\}$$

$$i(t) = 2 + 2e^{-0.5t}$$

$$i(t) = 2(1+e^{-0.5t}).$$

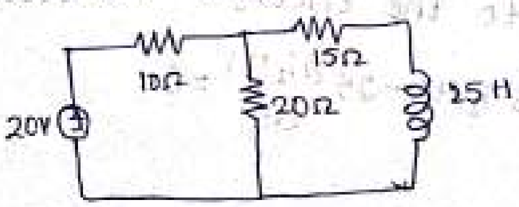
2 In the following figure, the switch is closed at $t=0$, find current through inductor using Laplace transform method.



Ans. Initial condition,

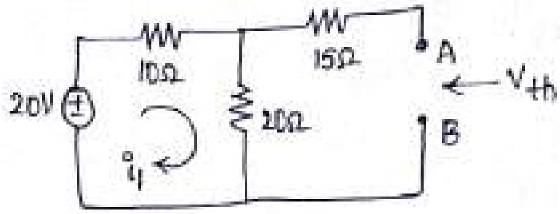
at $t=0$, $i=0 \longrightarrow (1)$

Switch is closed at $t=0$



Apply Thevenin theorem to the above circuit

Step 1: remove 25 H inductor from the circuit



Step 2: find V_{th}

apply KVL to the closed circuit

$$20 - 10i_1 - 20i_1 = 0$$

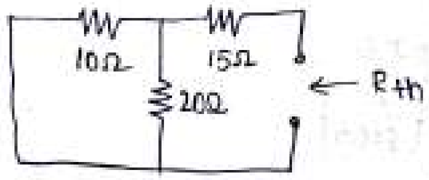
$$30i_1 = 20$$

$$i_1 = 0.66$$

$$V_{th} - 20 \times 0.66 = 0$$

$$\boxed{V_{th} = 13.2\,V}$$

Step 3: To find R_{th}, short circuit the voltage source

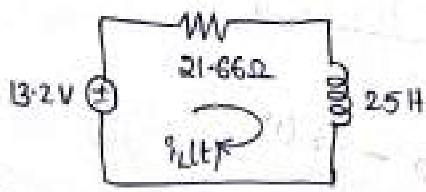


$$R_{th} = 15 + \frac{10 \times 20}{30}$$

$$\boxed{R_{th} = 21.66\,\Omega}$$

Step 4: Draw thevenin's equivalent circuit

Step 5: Add 25 H inductor to step 4.



Now apply KVL to the circuit

$$13.2 - 21.66\, i_L(t) - 25\frac{di_L(t)}{dt} = 0$$

$$21.66 i_L(t) + 25 \frac{d i_L(t)}{dt} = 13.2 \longrightarrow (2)$$

Apply laplace transform,

$$21.66 i_L(s) + 25 [s i_L(s) - I_L(0)] = \frac{13.2}{s}$$

$$21.66 i_L(s) + 25 s i_L(s) = \frac{13.2}{s}$$

$$i_L(s) [21.66 + 25s] = \frac{13.2}{s}$$

$$i_L(s) = \frac{13.2}{s(21.66 + 25s)}$$

$$i_L(s) = \frac{\frac{13.2}{25}}{s(s + \frac{21.66}{5})}$$

$$i_L(s) = \frac{13.2/25}{s(s + 0.8664)}$$

$$i_L(s) = \frac{0.528}{s(s + 0.8664)} \longrightarrow (3)$$

Let $\frac{0.528}{s(s+0.86)} = \frac{A}{s} + \frac{B}{s+0.86}$

$$0.528 = A(s+0.86) + Bs$$

$$0.528 = (A+B)s + 0.86A$$

$$A+B = 0, \quad A = \frac{0.528}{0.86}$$

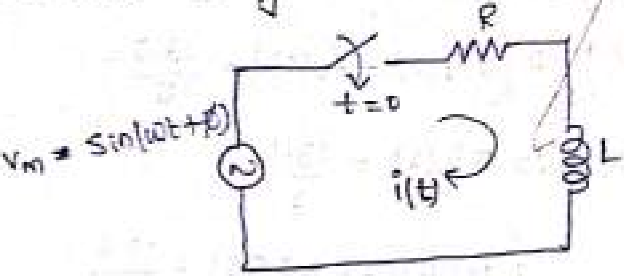
$$B = -0.613 \quad A = 0.613$$

$$I_L(s) = \frac{0.613}{s} - \frac{0.613}{s+0.86}$$

apply inverse laplace transform,

$$i_L(t) = 0.613 \left[1 - e^{-0.86}\right]$$

Method 3: AC transient response analysis series R-L circuit using DE method.



Initial condition $t=0, i=0 \longrightarrow (1)$

Apply KVL to the circuit

$$\frac{iR}{L} + \frac{di}{dt} = \frac{V_m}{L}\sin(\omega t+\phi) \longrightarrow (2)$$

It is non-homogeneous eqn.

$$\frac{di}{dt} + Pi = Q$$

The solution is $i = i_{CF} + i_{PI}$

$$i_{CF} = ke^{-Pt} = ke^{-\frac{R}{L}t} \longrightarrow (3)$$

Consider,

$$i_{PI} = A\cos(\omega t+\phi) + B\sin(\omega t+\phi) \longrightarrow (4)$$

$$\frac{di}{dt} = -A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi) \longrightarrow (5)$$

Substitute eqn (4) & eqn (5) in eqn (2)

$$-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi) + \frac{R}{L}[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)]$$

$$= \frac{V_m}{L}\sin(\omega t+\phi) \longrightarrow (6)$$

Equating like terms, we get

$$\left(-A\omega + B\frac{R}{L}\right)\sin(\omega t+\phi) = \frac{V_m}{L}\sin(\omega t+\phi)$$

$$-A\omega + B\frac{R}{L} = \frac{V_m}{L} \longrightarrow (7)$$

lly, $B\omega + A\frac{R}{L} = 0 \longrightarrow (8)$

from (8) $B = -\frac{AR}{\omega L} \longrightarrow (9)$

substitute eqn (9) in eqn (7)

$$-A\omega + \left(-\frac{AR}{\omega L}\cdot\frac{R}{L}\right) = \frac{V_m}{L}$$

$$-A\left[\omega + \frac{R^2}{\omega L^2}\right] = \frac{V_m}{L}$$

$$-A\left[\frac{(\omega L)^2+R^2}{\omega L}\right]=V_m$$

$$A=\frac{V_m\omega L}{(\omega L)^2+R^2}\longrightarrow(10)$$

substitute eqn (10) in eqn (9)

$$B=\frac{-(-V_m\omega L)}{R^2+(\omega L)^2}\times\frac{R}{\omega L}$$

$$B=\frac{V_m R}{R^2+(\omega L)^2}\longrightarrow(11)$$

from eqn (4)

$$i_{PI}=\frac{V_m\omega L}{(\omega L)^2+R^2}\cos(\omega t+\phi)+\frac{V_m R}{R^2+(\omega L)^2}\sin(\omega t+\phi)\longrightarrow(12)$$

take, $\frac{V_m R}{R^2+(\omega L)^2}=K_1\cos\theta\longrightarrow(13)$ $\tan\theta=\frac{-\omega L}{R}$

$\frac{-V_m\omega L}{R^2+(\omega L)^2}=K_1\sin\theta\longrightarrow(14)$ $\theta=\tan^{-1}\left(\frac{-\omega L}{R}\right)$

solve eqn (13) & 14 & add

$$K_1^2(\cos^2\theta+\sin^2\theta)=\frac{V_m^2(R^2+(\omega L)^2)}{(R^2+(\omega L)^2)^2}$$

$$K_1=\frac{V_m}{\sqrt{R^2+(\omega L)^2}}\longrightarrow(15)$$

eqn (12) becomes

$$i_{PI}=K_1\sin\theta\cos(\omega t+\phi)+K_1\cos\theta\sin(\omega t+\phi)$$

$$i_{PI}=K_1[\sin(\omega t+\phi+\theta)]\longrightarrow(16)$$

substitute K_1 in eqn (16)

$$i_{PI}=\frac{V_m}{\sqrt{R^2+(\omega L)^2}}\left[\sin\left(\omega t+\phi-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)\right]\longrightarrow(17)$$

The complete solution is

$$i(t)=i_{CF}+i_{PI}$$

$$i(t)=ke^{-\frac{R}{L}t}+\frac{V_m}{\sqrt{R^2+(\omega L)^2}}\left[\sin\left(\omega t+\theta-\tan^{-1}\frac{\omega L}{R}\right)\right]$$

To find k, use initial condition, i.e., $t=0$; $i=0$

$$k=\frac{V_m}{\sqrt{R^2+(\omega L)^2}}\left[\sin\left(\omega t+\phi-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)\right].$$

1. Frequency 50 Hz, 400 V (peak value) sinusoidal voltage is applied at $t=0$ to a series R-L circuit having resistance 5Ω and inductance 0.2 H. Obtain an expression of current at any instant 'K'. Calculate the value of the transient current after 0.01 sec switching ON!

Sol: Given data,

$$\text{frequency} = 50\ Hz$$

$$\text{voltage peak value} = 400V$$

$$R = 5\Omega$$

$$L = 0.2\ H$$

$$\text{Sinusoidal voltage } (V) = V\sin\omega t$$

$$= 400\sin 2\pi f t$$

$$= 400\sin(2\times 3.14\times 50)t$$

$$(V) = 400\sin 314t$$

for series RL circuit in AC

$$Ri + L\frac{di}{dt} = 400\sin 314t$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{400}{0.2}\sin 314t$$

$$\frac{di}{dt} + \frac{5}{0.2}i = 2000\sin 314t$$

$$\frac{di}{dt} + 25i = 2000\sin 314t \longrightarrow (1)$$

The solution is

$$i(t) = i_{CF} + i_{PF}$$

$$i(t) = Ke^{-25t} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}}\sin\left(314t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$i(t) = Ke^{-25t} + \frac{400}{\sqrt{5^2+(2\pi\times 0.2\times 50)^2}}\sin(314t - \tan^{-1} 12.56)$$

$$i(t) = Ke^{-25t} + 6.34\sin(314t - 85.4) \longrightarrow (2)$$

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To find K, use initial conditions

i.e., $t=0, i=0$

$$0 = Ke^{0} + 6.34 \sin(314(0) - 85.4)$$

$$= K + 6.34 \sin(-85.4)$$

$$= K + 6.34(-0.997)$$

$$= K - 6.320$$

$$\boxed{K = 6.320}$$

Substitute in eqn (2)

$$i(t) = 6.32\, e^{-25t} + 6.34 \sin(314t - 85.4)$$

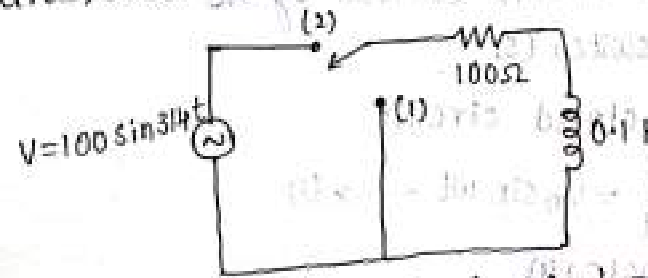
Transient current:

$$i(t) = 6.32\, e^{-25 \times 0.01}$$

$$i(t) = 4.92 \text{ amp}$$

$$\boxed{i_t = 4.92\text{ A}}$$

2. Obtain the current and $t>0$, if AC voltage be is applied when the switch 'S' is moved to position 2 from position 1, at $t=0$. Assume steady state current of 1A in the RL circuit, when the switch was at position 1.



Sol: Given initial conditions $i = 1A$, $t=0$

sinusoidal voltage $(V) = 100 \sin 314t$

for R-L series circuit

$$Ri + L\frac{di}{dt} = 100 \sin 314t \longrightarrow (1)$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{100}{L} \sin 314t$$

$$\frac{di}{dt} + 1000i = 1000 \sin 314t$$

The solution is $i(t) = i_{CF} + i_{PI}$

$$i(t) = Ke^{-Pt} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin(314t - \tan^{-1}\frac{\omega L}{R})$$

$$i(t) = Ke^{-1000t} + 0.9541 \sin(314t - 17.432) \longrightarrow (2)$$

To find K :

Use initial conditions $t=0$, $i=1A$

Substitute these values in eqn (2)

$$1 = Ke^{-1000(0)} + 0.9541 \sin(-17.432)$$

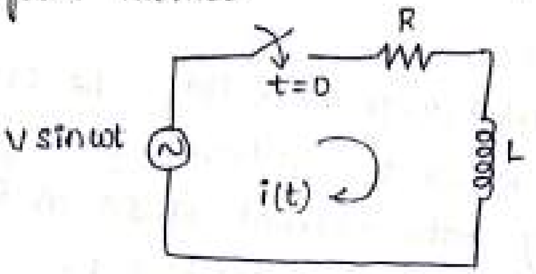
$$1 = K - 0.286$$

$$K = 1.286$$

$$\therefore i(t) = 1.3e^{-1000t} + 0.9541 \sin(314t - 17.432).$$

Method 4 :

AC transient analysis of sound RL circuit using Laplace transform method.



Consider series RL circuit excited by AC source $V_m \sin \omega t$ through switch (S)

Apply KVL to the closed circuit

$$iR + L\frac{di}{dt} = V_m \sin \omega t \longrightarrow (1)$$

Apply Laplace transform

$$I(s)R + L[sI(s) - I(0)] = V_m \frac{\omega}{s^2+\omega^2} \longrightarrow (2)$$

$$I(s)[R+sL] = \frac{V_m\omega}{s^2+\omega^2}$$

$$I(s) = \frac{V_m\omega}{L(s+\frac{R}{L})(s^2+\omega^2)}$$

$$I(s) = \frac{(V_m\omega/L)}{(s+\frac{R}{L})(s^2+\omega^2)} \longrightarrow (3)$$

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Using Partial fractions

$$\frac{V_m\omega/L}{(s+\frac{R}{L})(s^2+\omega^2)} = \frac{A}{s+\frac{R}{L}} + \frac{B}{s^2+\omega^2} \longrightarrow (4)$$

$$A = \left.\frac{\frac{V_m\omega}{L}}{s^2+\omega^2}\right|_{s=-\frac{R}{L}}$$

$$= \frac{\left(\frac{V_m\omega}{L}\right)}{\left(\frac{R}{L}\right)^2+\omega^2}$$

$$= \frac{V_m\omega L}{R^2+\omega^2L^2}$$

$$\boxed{A = \frac{V_m\omega L}{R^2+(\omega L)^2}}$$

Cross multiplying eqn (4)

$$\frac{V_m\omega}{L} = A(s^2+\omega^2)+(Bs+C)(s+\frac{R}{L})$$

$$\frac{V_m\omega}{L} = As^2+A\omega^2+Bs^2+Bs\frac{R}{L}+Cs+\frac{CR}{L}$$

s^2 terms

$$A+B=0 \longrightarrow (6) \qquad \boxed{B=-A=\frac{-V_m\omega L}{R^2+(\omega L)^2}}$$

Comparing 's' terms

$$\frac{BR}{L}+C=0 \longrightarrow (7)$$

$$C = \frac{-BR}{L}$$

$$= \frac{V_m\omega RL}{L[R^2+(\omega L)^2]}$$

$$\boxed{C = \frac{V_m\omega R}{R^2+(\omega L)^2}}$$

Comparing constants

$$\frac{CR}{L}+A\omega^2 = \frac{V_m\omega}{L}$$

$$\frac{CR}{L}+\frac{V_m\omega L}{R^2+(\omega L)^2} = \frac{V_m\omega}{L}$$

$$\frac{CR}{L} = \frac{V_m\omega\,[R^2+(\omega L)^2-L]}{L\,(R^2+(\omega L)^2)}$$

Substitute A, B & C values in eqn (4)

$$I(s) = \frac{\left(\frac{V_m\omega L}{R^2+(\omega L)^2}\right)}{s+\frac{R}{L}} + \frac{\frac{-V_m\omega L}{R^2+(\omega L)^2}s + \frac{V_m\omega R}{R^2+(\omega L)^2}}{s^2+\omega^2}$$

$$= \frac{\left(\frac{V_m\omega L}{R^2+(\omega L)^2}\right)}{s+\frac{R}{L}} - \frac{V_m\omega L}{R^2+(\omega L)^2}\times\frac{s}{s^2+\omega^2} + \frac{V_m R}{R^2+(\omega L)^2}\times\frac{\omega}{s^2+\omega^2} \longrightarrow (5)$$

Apply inverse laplace

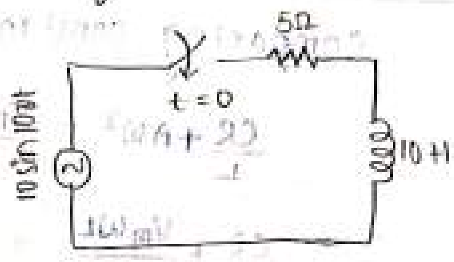
$$i(t) = \frac{V_m\omega L}{R^2+(\omega L)^2}e^{-\frac{R}{L}t} - \frac{V_m\omega L}{R^2+(\omega L)^2}\cos\omega t + \frac{V_m R}{R^2+(\omega L)^2}\sin\omega t \longrightarrow (6)$$

$$= \frac{V_m\omega L}{R^2+(\omega L)^2}e^{-\frac{R}{L}t} - \frac{V_m}{R^2+(\omega L)^2}\left[\omega L\cos\omega t - R\sin\omega t\right] \longrightarrow (7)$$

$$= \frac{V_m\omega L}{R^2+(\omega L)^2}e^{-\frac{R}{L}t} - \frac{V_m}{R^2+(\omega L)^2}\left[\sqrt{R^2+(\omega L)^2}\sin\phi\cos\omega t - \sqrt{R^2+(\omega L)^2}\cos\phi\sin\omega t\right]$$

$$= \frac{V_m\omega L}{R^2+(\omega L)^2}e^{-\frac{R}{L}t} - \frac{V_m}{R^2+(\omega L)^2}\left[\sin(\phi-\omega t)\right]$$

1. A sinusoidal voltage 10 sin 100t is applied to a series R-L circuit having R = 5Ω & L = 10 H. Then find current flowing through the conductor after switch is closed at t = 0. Using laplace transform method.



Initial condition, at t=0, i=0 ⟶ (1)

Apply KVL to the closed circuit

$$5i + 10\frac{di}{dt} = 10\sin 100t \longrightarrow (2)$$

Apply laplace transform,

$$5I(s) + 10[sI(s) - I(0)] = 10\,\frac{100}{s^2+100^2} \longrightarrow (3)$$

$$5\,I(s) + 10\,[s\,I(s) - 0] = 10\,\frac{\omega}{s^2+\omega^2}$$

$$I(s)\,[5 + 10s] = 10\,\frac{\omega}{s^2+\omega^2}$$

$$I(s) = \frac{10\omega}{(10s+5)(s^2+\omega^2)}$$

$$= \frac{10\,\omega}{L(s+\frac{R}{L})(s^2+\omega^2)}$$

$$= \frac{10\,\omega/L}{(s+R/L)(s^2+\omega^2)}$$

Using partial fractions

$$\frac{10\,\omega/L}{(s+\frac{R}{L})(s^2+\omega^2)} = \frac{A}{s+\frac{R}{L}} + \frac{B}{s^2+\omega^2}$$

$$A = \left.\frac{10\omega/L}{s^2+\omega^2}\right|_{s=\frac{-R}{L}}$$

$$= \frac{\frac{10\omega}{10}}{(\frac{-R}{L})^2+\omega^2}$$

$$= \frac{\omega}{\frac{5^2}{10^2}+\omega^2}$$

$$= \frac{\omega\times 100}{25+\omega^2 100}$$

$$= \frac{100\times 100}{25+(100)^2 100}$$

$$\boxed{A = 0.009}$$

$$\text{Now, } \frac{Bs+C}{s^2+\omega^2} = \frac{10\omega/L}{(s+R/L)(s^2+\omega^2)}$$

$$\frac{(10\omega/L)}{(s+R/L)(s^2+\omega^2)} = \frac{A(s^2+\omega^2)}{(s+R/L)} + \frac{(Bs+C)(s+R/L)}{(s^2+\omega^2)}$$

$$\frac{\left(\frac{10\times 100}{10}\right)}{(s+\frac{R}{L})(s^2+\omega^2)} = \frac{A(s^2+\omega^2)+(Bs+C)(s+\frac{R}{L})}{(s+R/L)\,(s^2+\omega^2)}$$

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$$100 = As^2 + A\omega^2 + Bs^2 + Bs\frac{R}{L} + Cs + \frac{CR}{L}$$

comparing "s^2" terms

$A+B=0 \longrightarrow (5)$

$B = -A$

$B = -0.009$

Comparing 's' terms

$\frac{BR}{L} + C = 0$

$C = \frac{-BR}{L}$

$C = -\frac{(0.009)5}{10}$

$\boxed{C = 0.0045}$

Substitute A, B & C values in eqn (4)

$$I(s) = \frac{0.009}{s+1/2} + \frac{-0.009s + 0.0045}{s^2+10000}$$

$$I(s) = \frac{0.009}{s+2.5} - \frac{0.009s}{s^2+100^2} + \frac{0.0045}{s^2+100^2}$$

Apply inverse laplace

$$i(t) = 0.009e^{-1/2t} - 0.009\cos 100t + \frac{0.0045}{s^2+100^2} \times \frac{100}{100}$$

$$= 0.009e^{-1/2t} - 0.009\cos\omega t + \frac{0.0045}{100} \times \frac{100}{s^2+100^2}$$

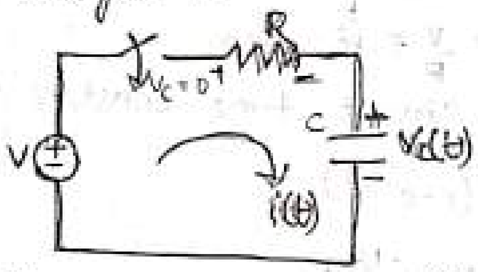
$$= 0.009e^{-1/2t} - 0.009\cos\omega t + 0.0045\sin 100t$$

$$i(t) = 0.009e^{-1/2t} - 0.009\cos\omega t + 0.0045\sin\omega t.$$

* Series R-C circuit

Method 1:

DC transient analysis of series R-C circuit using DE method:



Initial conditions, $t=0, V_c=0 \longrightarrow (1)$

Now apply KVL,

$$-V + i(t)R + V_c(t) = 0$$

$$V - RI(t) - V_c(t) = 0$$

$$RI(t) + V_c(t) = V \longrightarrow (2)$$

$$I(t) = \frac{V - V_c(t)}{R}$$

$$\text{Now, } i(t) = C\frac{dV_c(t)}{dt}$$

$$RC\frac{dV_c(t)}{dt} + V_c(t) = V \longrightarrow (3)$$

$$\frac{dV_c(t)}{dt} + \frac{1}{RC}V_c(t) = \frac{V}{RC} \longrightarrow (4)$$

It is non-homogeneous eqn

$$\frac{dV_c(t)}{dt} + PV_c(t) = Q$$

The solution is

$$V_c(t) = ke^{-\frac{1}{RC}t} + \frac{V/RC}{1/RC}$$

$$V_c(t) = ke^{\frac{-t}{RC}} + V \longrightarrow (5)$$

To find k, use initial conditions

i.e., $t=0, V_c=0$

$$\boxed{k=-V}$$

Substitute $k=-V$ in eqn (5)

$$V_c(t) = -Ve^{\frac{-t}{RC}} + V$$

$$V_c(t) = -V\left[1 - e^{\frac{-t}{RC}}\right] \text{ Volts.}$$

$$i(t) = C\frac{dv}{dt}$$

$$= C(-Ve^{-\frac{t}{RC}}) \times \frac{-1}{RC}$$

$$i(t) = \frac{V}{R}e^{\frac{-t}{RC}} \text{ amp.}$$

In case of series RC circuit time constant $\tau = RC$

Now,
$$V_c(t) = V(1-e^{-t/\tau})$$

$$i(t) = \frac{V}{R}(e^{-t/\tau})$$

The graph for $V_c(t)$ is

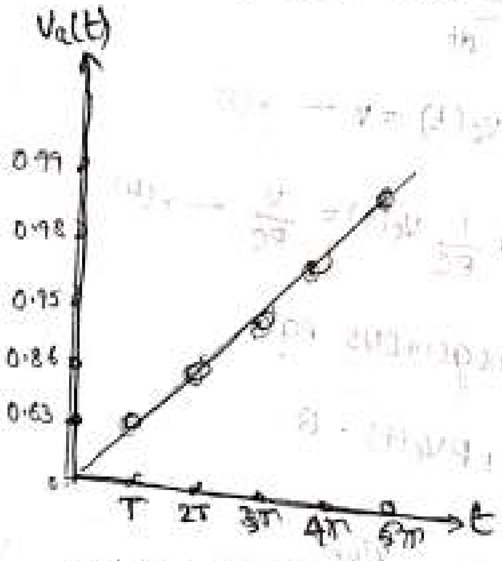
$t=\tau,\ V_c(t) = 0.63V$

$t=2\tau,\ V_c(t) = 0.86V$

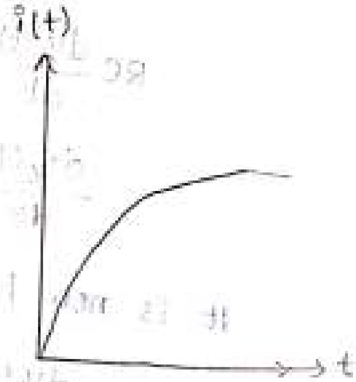
$t=3\tau,\ V_c(t) = 0.95V$

$t=4\tau,\ V_c(t) = 0.98V$

$t=5\tau,\ V_c(t) = 0.99V$

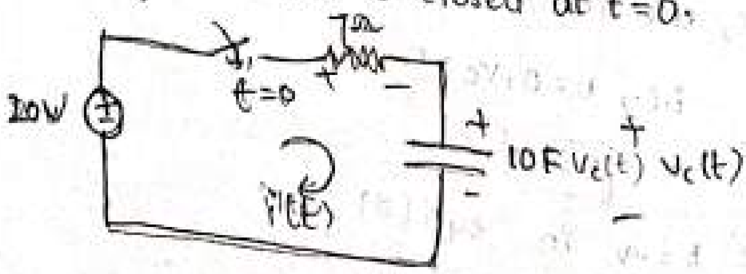


voltage wave form



current wave form

1. A 20 V dc supply is applied to a series RC circuit with $R=7\Omega$ and $C=10F$. Find voltage and current through the capacitor after switch is closed at $t=0$.



Initial conditions $t=0, i=0 \longrightarrow (1)$

Now apply KVL to the circuit is, ~~$-20+7i(t)+\,=0$~~

$$-20V + 7i(t) + V_c(t) = 0$$

$$7i(t) + V_c(t) = 20 \longrightarrow (2)$$

$$7C\frac{dV_c(t)}{dt} + V_c(t) = 20 \longrightarrow (3)$$

$$[C = 10F]$$

$$70\frac{dV_c(t)}{dt} + V_c(t) = 20$$

$$\frac{dV_c(t)}{dt} + \frac{1}{70}V_c(t) = \frac{20}{70} \longrightarrow (4)$$

It is a non-homogeneous eqn

$$P = \frac{1}{70},\ Q = \frac{20}{70}$$

solution is $V_c(t) = ke^{-Pt} + \frac{Q}{P}$

$$= ke^{-\frac{1}{70}t} + \frac{20/70}{1/70}$$

$$V_c(t) = k.e^{-\frac{1}{70}t} + 20 \longrightarrow (5)$$

To find k :-

use initial conditions $t=0, V_c=0$

$$ke^0 + 20 = 0$$

$$k = -20$$

Substitute 'k' in eqn (5)

$$V_c(t) = -20e^{-1/70t} + 20$$

$$= 20\left[1 - e^{-\frac{1}{70}t}\right]$$

differentiate the eqn

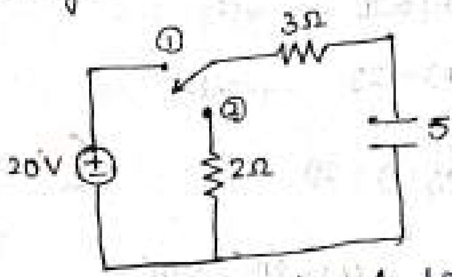
$$i(t) = C \cdot \frac{dV_c}{dt}$$

$$= C\left[-Ve^{-t/RC}\right] \times \frac{-1}{RC}$$

$$= \frac{V}{R} \cdot e^{-t/RC}$$

$$i(t) = \frac{20}{7}e^{-\frac{1}{70}t}\ A$$

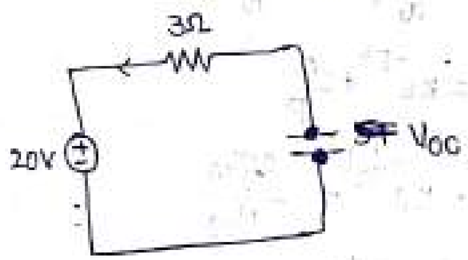
The circuit shown in fig: switch is at position 1 for long time. At $t=0$, switch is moved to position 2, then find voltage and current through the capacitor.



Initially switch at position 1 for long time:

long time = steady state

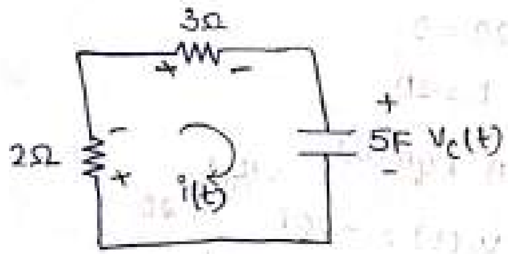
At steady state, capacitor acts as open circuit



$$V_{oc} - 20 = 0$$

$$V_{oc} = 20$$

Switch is moved to position 2 at $t=0$



Initial condition $t=0, V_0 = 20V \rightarrow (1)$

Apply KVL to the circuit

$$(+2+3)\, i(t) + V_c(t) = 0$$

$$+5\left[c\frac{dV_c(t)}{dt}\right] + V_c(t) = 0$$

$$\frac{dV_c(t)}{dt} + \frac{1}{25} V_c(t) = 0 \rightarrow (2)$$

$$p = \frac{-1}{25}$$

Solution is $V_c(t) = ke^{-\frac{1}{25}t} \rightarrow (3)$

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To find k, use initial condition

$$20 = k e^{0}$$

$$\boxed{k = 20}$$

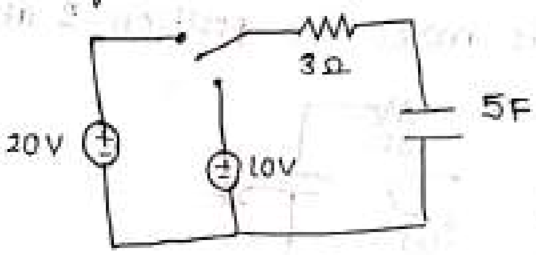
$$V_c(t) = 20\, e^{-\frac{t}{25}} \text{ volts}$$

current(i) $\Rightarrow$ $i(t) = C \dfrac{dv}{dt}$

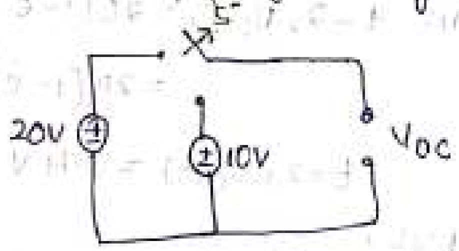
$$= 5 \times 20 \times \frac{-1}{25} e^{-t/25}$$

$$= -4\, e^{-t/25} \text{ amps}$$

In the circuit shown in fig. switch is moved to position 1 at t = 0 switch is moved to position 2. Then find the voltage and current through the capacitor.

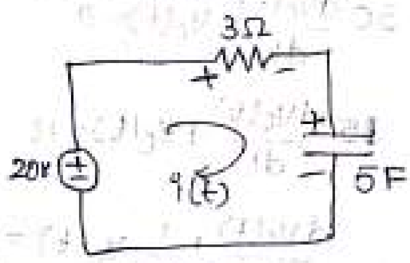


Initially switch is open for long time



$V_{oc} = 0$

Now, switch is moved to position 1 at t=0



Initial condition,

$t = 0,\ V_c = 0 \longrightarrow (1)$

Apply KVL for the circuit.

$$-20 + 3i(t) + V_c(t) = 0$$

$$20 - 3i(t) - V_c(t) = 0$$

$$20 - 3C \frac{dV_c(t)}{dt} - V_c(t) = 0$$

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$$15\frac{dV_c(t)}{dt} + V_c(t) = 20$$

$$\frac{dV_c(t)}{dt} + \frac{1}{15}V_c(t) = \frac{20}{15} \longrightarrow (2)$$

Non-homogeneous eqn

$$P = \frac{1}{15}, Q = \frac{20}{15}$$

Solution is $V_c(t) = Ke^{-pt} + \frac{Q}{P}$

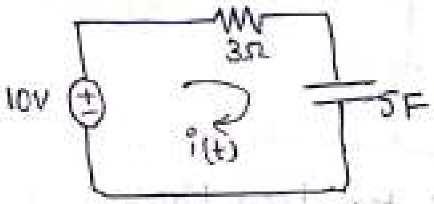
$$V_c(t) = Ke^{\frac{-t}{15}} + 20 \longrightarrow (3)$$

To find k use initial condition

$$K = -20$$

$$\boxed{V_c(t) = -20e^{\frac{-t}{15}} + 20} \longrightarrow (4)$$

Now, switch is moved to position 2 at t=2 sec



Initial conditions $t=2, V_c(t) = 20(1-e^{\frac{-t}{15}})$

$$= 20(1-e^{\frac{-2}{15}})$$

$$t=2, V_c(t) = 2.4\,V \longrightarrow (5)$$

Apply KVL,

$$10 - 3i_c(t) - V_c(t) = 0$$

$$10 - 3C\frac{dV_c(t)}{dt} - V_c(t) = 0$$

$$15\frac{dV_c(t)}{dt} + V_c(t) = 10$$

$$\frac{dV_c(t)}{dt} + \frac{1}{15}V_c(t) = \frac{10}{15}$$

Non-homogeneous eqn, $P = \frac{1}{15}, Q = \frac{10}{15}$

Solution is $P = \frac{1}{15}, Q = \frac{10}{15}$

$$V_c(t) = Ke^{-\frac{1}{15}(t-2)} + 10$$

To find K, use initial conditions

$$2.4 = Ke^{-\frac{1}{15}(0)} + 10$$

$$2.4 = Ke^{0} + 10$$

$$2.4 = Ke^{\cancel{2/15}}$$

$$\boxed{K = -7.6}$$

$$V_c(t) = -7.6\, e^{-\frac{1}{15}(t-2)} + 10.$$

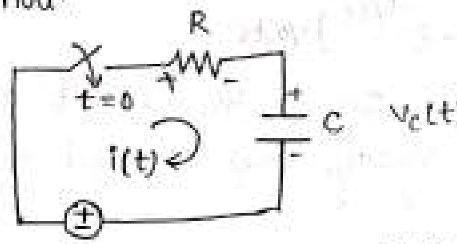
$$i_c(t) = C\frac{dV_c(t)}{dt}$$

$$= 5(-7.6)e^{-\frac{1}{15}(t-2)} \times \left(\frac{-1}{15}\right)$$

$$= 2.53\, e^{-\frac{1}{15}(t-2)} \text{ amp.}$$

Method 2:

DC transient analysis of series RC circuit using Laplace transform method.



Apply KVL to the circuit, $-V + i(t)R + v_c(t) = 0$

$$V = I(t)R + V_c(t)$$

$$I(t) = C\frac{dV_c(t)}{dt}.$$

$$C\frac{dV_c(t)}{dt}R + V_c(t) = V \longrightarrow (1)$$

Apply Laplace transform.

$$CR[sV_c(s) - V_c(0)] + V_c(s) = \frac{V}{s} \qquad V_c(0) = 0$$

$$CRsV_c(s) + V_c(s) = \frac{V}{s}$$

$$V_c(s) = \frac{V/s}{CR(s + \frac{1}{CR})}$$

$$V_c(s) = \frac{V/CR}{s(s+\frac{1}{CR})} \longrightarrow (2)$$

To find k, use initial conditions

$$2.4 = k e^{\frac{-1}{15}(2)} + 10$$

$$2.4 = k e^{0} + 10$$

$$7.4 = k e^{\frac{2}{15}}$$

$$\boxed{K = -7.6}$$

$$V_c(t) = -7.6\, e^{\frac{-1}{15}(t-2)} + 10.$$

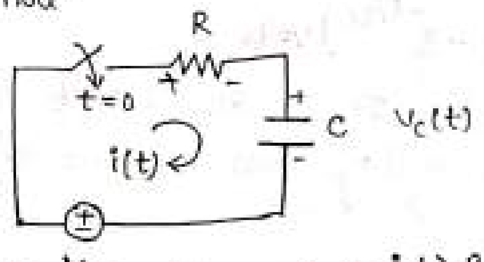
$$i_c(t) = C\frac{dV_c(t)}{dt}$$

$$= 5(-7.6)e^{\frac{-1}{15}(t-2)} \times \left(\frac{-1}{15}\right)$$

$$= 2.53\, e^{\frac{-1}{15}(t-2)} \text{ amp.}$$

Method 2 :

DC transient analysis of series RC circuit using Laplace transform method.



Apply KVL to the circuit, $-V + i(t)R + V_c(t) = 0$

$$V = I(t)R + V_c(t)$$

$$I(t) = C\frac{dV_c(t)}{dt}.$$

$$C\frac{dV_c(t)}{dt}R + V_c(t) = V \longrightarrow (1)$$

Apply Laplace transform.

$$CR[S\,V_c(S) - V_c(0)] + V_c(S) = \frac{V}{S} \qquad V_c(0) = 0$$

$$CRSV_c(S) + V_c(S) = \frac{V}{S}$$

$$V_c(S) = \frac{V/S}{CR(S+\frac{1}{CR})}$$

$$V_c(S) = \frac{V/CR}{S(S+\frac{1}{CR})} \longrightarrow (2)$$

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Apply laplace transform by partial fraction

$$\frac{V/CR}{s\left(s+\frac{1}{CR}\right)} = \frac{A}{s} + \frac{B}{s+1/CR}$$

at $s=0$ $\quad A = \frac{V/\cancel{CR}}{\frac{1}{\cancel{CR}}}$, $\quad B = \frac{\frac{V}{CR}}{s\left(s+\frac{1}{CR}\right)}\Bigg|_{s=\frac{-1}{CR}}$

$A = V$

$$= \frac{\frac{V}{CR}}{s}\Bigg|_{s=\frac{-1}{CR}}$$

$||^{ly}$, $B = -V$ $\quad \boxed{B = -V}$

$$V_c(s) = \frac{V}{s} - \frac{V}{s+\frac{1}{RC}}$$

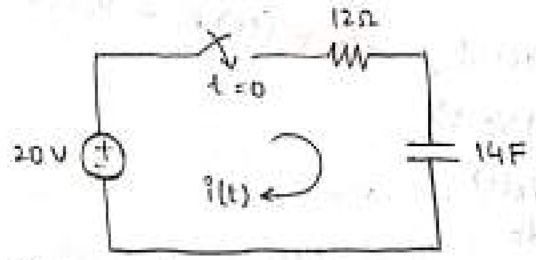
$$V_c(s) = V\left(\frac{1}{s} - \frac{1}{s+\frac{1}{RC}}\right) \longrightarrow (3)$$

Apply inverse laplace transform

$$V_c(t) = V - Ve^{-t/RC}$$

$$V_c(t) = V\left[1-e^{-t/RC}\right] \text{Volts}$$

1. A 20V is applied to series RC circuit having $R=12\Omega$ and $C=14F$. Find the voltage and current after switch is closed at $t=0$.



Initial conditions $t=0, V_c=0$

Apply KVL to the circuit

$$20 - i_c(t)\,12 - V_c(t) = 0$$

$$12\, i_c(t) + V_c(t) = 20$$

$$i_c(t) = C\frac{dV_c(t)}{dt}$$

$$12C\frac{dV_c(t)}{dt} + V_c(t) = 20$$

$$168\frac{dV_c(t)}{dt} + V_c(t) = 20 \longrightarrow (1)$$

Apply Laplace transform

$$168\left[sV_c(s) - V_c(0)\right] + V_c(s) = \frac{20}{s}$$

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$$168\, sV_c(s) + V_c(s) = \frac{20}{s}$$

$$V_c(s) = \frac{20}{s(168s+1)}$$

$$V_c(s) = \frac{20}{168s\left(s+\frac{1}{168}\right)}$$

$$V_c(s) = \frac{20/168}{s\left(s+\frac{1}{168}\right)}$$

Apply partial fractions

$$\frac{20/168}{s\left(s+\frac{1}{168}\right)} = \frac{A}{s} + \frac{B}{s+\frac{1}{168}}$$

$$\frac{A}{\cancel{s}} = \frac{\frac{20}{168}}{\cancel{s}\left(s+\frac{1}{168}\right)}\Bigg|_{s=0}$$

$$\boxed{A = 20}$$

$$\frac{B}{s+\cancel{\frac{1}{168}}} = \frac{20/168}{s\left(s+\cancel{\frac{1}{168}}\right)}\Bigg|_{s=-\frac{1}{168}}$$

$$B = -20$$

$$A = 20$$

lly,

$$B = -20$$

$$V_c(s) = \frac{20}{s} - \frac{20}{s+\frac{1}{168}}$$

$$V_c(s) = 20\left[\frac{1}{s} - \frac{1}{s+\frac{1}{168}}\right]$$

Apply inverse laplace transform

$$V_c(t) = 20\left[1 - e^{-\frac{t}{168}}\right] \text{ volts}$$

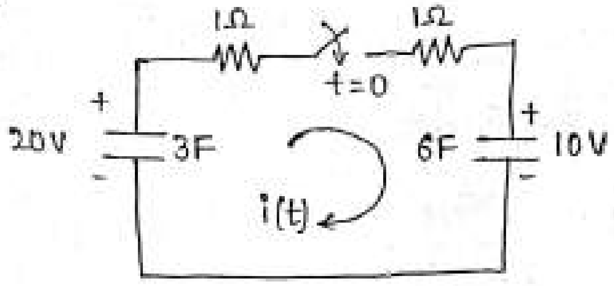
$$i(t) = C\frac{dV_c(t)}{dt}$$

$$= 14\times 20\, \frac{d}{dt}\left[1 - e^{-\frac{t}{168}}\right]$$

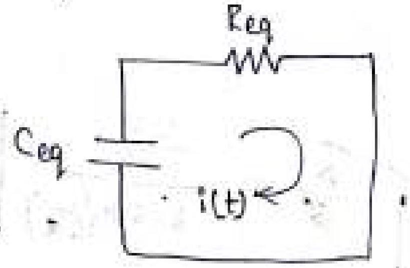
$$= 280\, e^{-\frac{t}{168}}\left[\frac{-1}{168}\right]$$

$$i(t) = -1.666\, e^{-\frac{t}{168}} \text{ amp}$$

2. Find i(t) and V(t) for the following figure in which 3F capacitor initially charged to 20V and 6F capacitor is charged to 6V. Assume initial capacitor voltage is 9V.



Sol. By closing switch



$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$, $R_{eq} = R_1 + R_2$

$= \frac{3 \times 6}{3+6}$ $= 1+1$

$= 2F$ $= 2\Omega$

$V_{eq} = 20 - 10 = 10V$

Initial conditions $t = 0$, $V_c(0) = 9V$

Apply KVL to the circuit

$i_R + V_c(t) = 0$

$C \frac{dV_c(t)}{dt} R + V_c(t) = 0$

$2 \frac{dV_c(t)}{dt} R + V_c(t) = 0$

Apply laplace transform.

$4 [sV_c(s) - V_c(0)] + V_c(s) = 0$

$4sV_c(s) - 4 \times 9 + V_c(s) = 0$

$V_c(s) = \frac{36}{4s+1}$

$V_c(s) = \frac{9}{s + \frac{1}{4}}$

inverse Laplace Transform

$V_c(t) = 9e^{-\frac{t}{4}}$ volts

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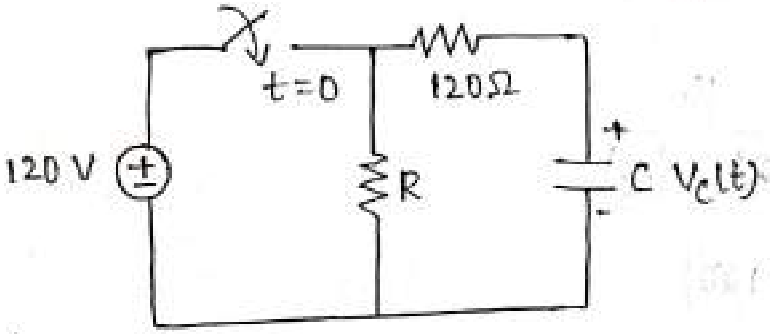
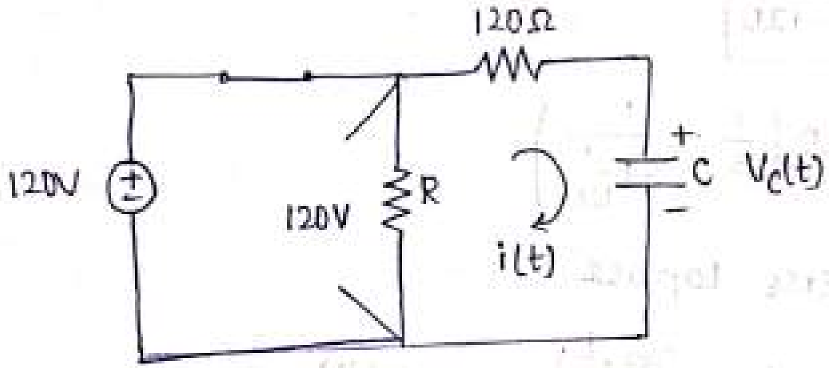
3. In the circuit shown in the fig. the switch is open for long time and closing the switch at $t=0$, the capacitor voltage rises to 70V in 10ms. After steady state is reached and the switch is opened again and capacitor voltage is 90V in 0.5s. Find the values of R&C.



Sol: Initial conditions

$$t=0,\ V_c(0)=0 \longrightarrow (1)$$

Now, switch is closed at $t=0$



Apply KVL to the circuit

$$120 - i(t)\,120 - V_c(t) = 0$$

$$i(t)(120) + V_c(t) = 120$$

$$C\frac{dV_c(t)}{dt}(120) + V_c(t) = 120 \longrightarrow (2)$$

Apply laplace transform

$$120C\left[sV_c(s) - V_c(0)\right] + V_c(s) = \frac{120}{s}$$

$$V_c(s)\left[cs(120)+1\right] = \frac{120}{s}$$

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$$V_c(s) = \frac{120/s}{cs(120)+1}$$

$$= \frac{120}{120c\,s\left(s+\frac{1}{120c}\right)}$$

$$= \frac{1/c}{s\left(s+\frac{1}{120c}\right)} \longrightarrow (3)$$

Partial fractions,

$$\frac{1/c}{s\left(s+\frac{1}{120c}\right)} = \frac{A}{s} + \frac{B}{s+\frac{1}{120c}}$$

at $s=0$, $A = \frac{1/c}{(s+\frac{1}{120c})}$

$$\boxed{A = 120}$$

$$B = \frac{1/c}{s}\Big|_{s=\frac{-1}{120c}}$$

$$\boxed{B = -120}$$

$$V_c(s) = 120\left(\frac{1}{s} - \frac{1}{s+\frac{1}{120c}}\right)$$

Apply inverse laplace

$$V_c(t) = 120\left(1-e^{-\frac{1}{120c}t}\right) \longrightarrow (4)$$

from problem, $t = 10\,msec = 10^{-2}$ sec

$V_c = 70\,V$

from eqn (4)

$$70 = 120\left(1-e^{\frac{-10^{-2}}{120c}}\right)$$

$$-50 = -120\,e^{\frac{-10^{-2}}{120c}}$$

$$0.416 = e^{\frac{-10^{-2}}{120c}}$$

Apply ln on both sides

$$\ln(0.416) = \ln\left(e^{\frac{-10^{-2}}{120C}}\right)$$

$$-0.87 = \frac{-10^{-2}}{120C}$$

$$120C \times 0.87 = 10^{-2}$$

$$104.4\,C = 10^{-2}$$

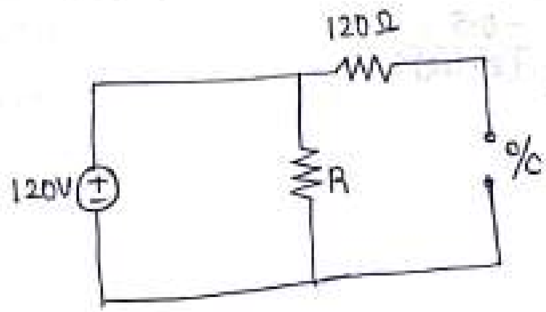
$$C = \frac{10^{-2}}{104.4}$$

$$= 9.57 \times 10^{-3} \times 10^{-2}$$

$$= 95.7 \times 10^{-6}\,F$$

$$\boxed{C = 95.7\,\mu F}$$

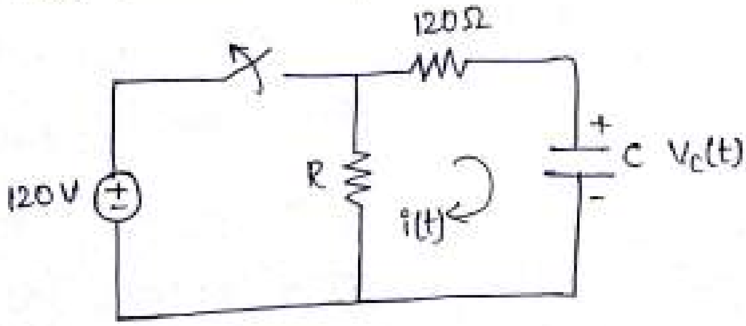
After steady state is reached the switch is again opened then the circuit is



Initial conditions,

$t = 0,\ V_c(0) = 120V \longrightarrow (5)$

Now switch is open



Apply KVL to the circuit

$$(R+120)\, i(t) + V_c(t) = 0$$

$$(R+120)\left[C\frac{dV_c(t)}{dt}\right] + V_c(t) = 0 \longrightarrow (6)$$

Apply Laplace transform

$$(R+120) \times C\left[sV_c(s) - V_c(0)\right] + V_c(s) = 0$$

$$V_c(s)\left[(R+120)C s + 1\right] = 120 \times (R+120)C$$

$$V_c(s) = \frac{120 \times (R+120)C}{(120+R)C\left[s + \frac{1}{(R+120)C}\right]}$$

$$V_c(s) = \frac{120}{s + \frac{1}{(R+120)C}} \longrightarrow (7)$$

Apply inverse Laplace transform

$$V_c(t) = 120\, e^{\frac{-t}{(R+120)C}} \longrightarrow (8)$$

from problem $V_c = 90V$, $t = 0.5$ sec

$$90 = 120\, e^{\frac{-0.5}{(R+120)C}}$$

$R = 18.03\ k\Omega$

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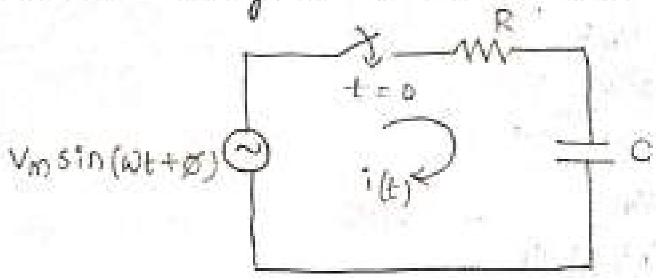
Method 3:

AC transient analysis of series R-C circuit by DE method.



Apply KVL to the circuit,

$$V_m \sin(\omega t+\phi) - i(t)R - \frac{1}{C}\int i\,dt = 0$$

$$V_m \sin(\omega t+\phi) = i(t)R + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

differentiate eqn (1) wrt 't'

$$V_m \omega \cos(\omega t+\phi) = \frac{di}{dt}R + \frac{1}{C}i$$

$$\frac{di}{dt} + \frac{1}{RC}i = \frac{V_m\omega}{R}\cos(\omega t+\phi) \longrightarrow (2)$$

It is like non-homogeneous eqn.

Solution is $i(t) = i_{CF} + i_{PI} \longrightarrow (3)$

$$i_{CF} = ke^{-Pt} = ke^{-\frac{1}{RC}t} \longrightarrow (4).$$

consider,

$$i_{PI} = A\cos(\omega t+\phi) + B\sin(\omega t+\phi) \longrightarrow (5)$$

differentiate wrt 't'

$$\frac{di}{dt} = -A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi) \longrightarrow (6)$$

Substitute eqn (5) & (6) in eqn (2)

$$-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi) + \frac{1}{RC}\left(A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right) = \frac{V_m\omega}{R}\cos(\omega t+\phi) \longrightarrow (7)$$

Compare, $\sin(\omega t+\phi)$ terms

$$-A\omega + \frac{B}{RC} = 0 \longrightarrow (8)$$

Compare $\cos(\omega t+\phi)$ terms

$$B\omega + \frac{A}{RC} = \frac{V_m\omega}{R} \longrightarrow (9)$$

from eqn (8) $A = \dfrac{B}{\omega RC} \longrightarrow (10)$

Substitute eqn (10) in eqn (9)

$$B\omega + \frac{B}{\omega R^2C^2} = \frac{V_m\omega}{R}$$

$$B = \frac{V_m \omega^2 C^2 R}{\omega^2 R^2 C^2 + 1}$$

$$\boxed{B = \frac{V_m R}{R^2 + \left(\frac{1}{\omega C}\right)^2}} \longrightarrow (11)$$

$$A = \frac{V_m}{\omega C\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)} \longrightarrow (12)$$

substitute eqn(11) & eqn(12) in eqn (5)

$$i_{PI} = \underbrace{\frac{\left(\frac{V_m}{\omega C}\right)}{R^2 + \left(\frac{1}{\omega C}\right)^2}}_{K_1 \sin\theta} \cos(\omega t + \phi) + \underbrace{\frac{V_m R}{R^2 + \left(\frac{1}{\omega C}\right)^2}}_{K_1 \cos\theta} \sin(\omega t + \phi) \longrightarrow (13)$$

$$= K_1 \sin\theta \cos(\omega t + \phi) + K_1 \cos\theta \sin(\omega t + \phi)$$

$$K_1^2(\sin^2\theta + \cos^2\theta) = \frac{\left(\left(\frac{1}{\omega C}\right)^2 + R^2\right) V_m^2}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)^2}$$

$$\boxed{K_1 = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}} \qquad \tan\theta = \frac{1}{\omega C R}$$

$$\theta = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$

$$i_{PI} = K_1 \sin(\omega t + \phi + \theta)$$

$$= \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\left(\frac{1}{\omega C R}\right)\right) \longrightarrow (14)$$

substitute eqn (14) & eqn (4) in eqn(3)

$$i(t) = Ke^{-\frac{t}{RC}} + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\frac{1}{\omega C R}\right) \longrightarrow (15)$$

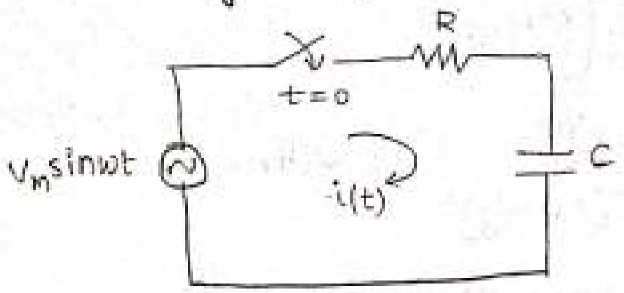
to find K, use initial conditions $t=0, i(t)=0$

$$0 = Ke^0 + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left[\omega t + \phi + \tan^{-1}\left(\frac{1}{\omega C R}\right)\right]$$

$$K = \frac{-V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left[\omega t + \phi - \tan^{-1}\left(\frac{1}{\omega C R}\right)\right]$$

Method 4 :

AC Transient analysis of series RC circuit by LT method :



Initial conditions.

$$t=0,\ i=0 \longrightarrow (1)$$

Apply KVL to the circuit

$$V_m \sin \omega t = i(t)R + \frac{1}{C}\int i\,dt \longrightarrow (2)$$

Apply laplace transform

$$\frac{V_m \omega}{s^2+\omega^2} = I(s)R + \frac{1}{C}\left(\frac{I(s)}{s} - \frac{I(0)}{0}\right)$$

$$I(s)\left[R+\frac{1}{Cs}\right] = \frac{V_m \omega}{s^2+\omega^2}$$

$$I(s) = \frac{V_m \omega}{\left(R+\frac{1}{Cs}\right)(s^2+\omega^2)}$$

$$= \frac{V_m \omega}{\frac{R}{s}\left(s+\frac{1}{CR}\right)(s^2+\omega^2)}$$

$$= \frac{\left(\frac{V_m \omega s}{R}\right)}{\left(s+\frac{1}{CR}\right)(s^2+\omega^2)} \longrightarrow (3)$$

Apply partial fractions

$$\frac{\left(\frac{V_m \omega s}{R}\right)}{\left(s+\frac{1}{RC}\right)(s^2+\omega^2)} = \frac{A}{s+\frac{1}{RC}} + \frac{Bs+D}{s^2+\omega^2} \longrightarrow (4)$$

$$A = \left.\frac{\frac{V_m \omega s}{R}}{s^2+\omega^2}\right|_{s=\frac{-1}{RC}} = \frac{\frac{V_m \omega}{R} \times \frac{-1}{RC}}{\left(\frac{1}{RC}\right)^2 + \omega^2}$$

$$= \frac{\left(\frac{-V_m \omega}{R^2 C}\right)}{\omega^2\left[1+\frac{1}{R^2C^2\omega^2}\right]}$$

$$A = \frac{\left(-\frac{V_m \omega}{R^2 C}\right)}{\frac{\omega^2}{R^2}\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}$$

$$A = \frac{\frac{-V_m}{\omega C}}{R^2+\left(\frac{1}{\omega C}\right)^2} \longrightarrow (5)$$

To find B & D, cross multiply eqn (4)

$$\frac{V_m \omega s}{R} = A(s^2+\omega^2)+(Bs+D)\left(s+\frac{1}{RC}\right)$$

$$\frac{V_m \omega s}{R} = As^2+A\omega^2+Bs^2+\frac{Bs}{RC}+Ds+\frac{D}{RC} \longrightarrow (6)$$

Compare s^2 terms,

$$A+B=0$$

$$B=-A$$

$$B = \frac{\left(\frac{V_m}{\omega C}\right)}{R^2+\left(\frac{1}{\omega C}\right)^2} \longrightarrow (7)$$

Compare constant terms

$$A\omega^2+\frac{D}{RC}=0$$

$$\frac{D}{RC} = -A\omega^2$$

$$D = \frac{V_m \omega R}{R^2+\left(\frac{1}{\omega C}\right)^2} \longrightarrow (8)$$

Substitute A, B & D values in eqn (3)

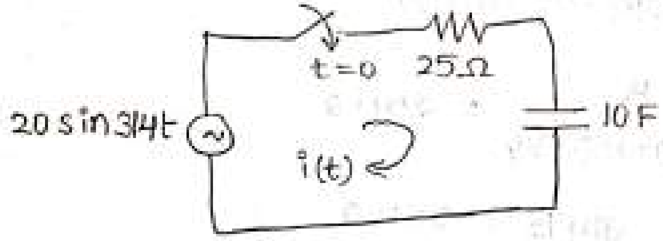
$$I(s) = \frac{\left(\frac{-V_m}{\omega C}\right)}{\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]\left(s+\frac{1}{RC}\right)} + \frac{\frac{V_m s}{\omega C}+V_m \omega R}{\left[R^2+\left(\frac{1}{\omega C}\right)^2\right](s^2+\omega^2)} \longrightarrow (9)$$

Apply inverse Laplace transform.

$$i(t) = \frac{\left(\frac{-V_m}{\omega C}\right)}{R^2+\left(\frac{1}{\omega C}\right)^2} e^{-\frac{t}{RC}} + \frac{\left(\frac{V_m}{\omega C}\right)}{R^2+\left(\frac{1}{\omega C}\right)^2}\cos \omega t + \frac{V_m R}{R^2+\left(\frac{1}{\omega C}\right)^2}\sin \omega t.$$

1. A sinusoidal voltage 20 sin 314t having R = 25Ω & C = 10F. Calculate the current flowing through the capacitor after switch is closed at t = 0 using
(a) DE method.
(b) Laplace transform method.

Sol:



(a) DE method:

Using initial conditions

at $t=0, i=0 \longrightarrow (1)$

Apply KVL to the circuit

$$20 \sin 314t = 25\, i(t) + \frac{1}{10}\int i\,dt \longrightarrow (2)$$

differentiate wrt 't'

$$6280 \cos \omega t = \frac{di}{dt}25 + \frac{1}{10} i$$

$$\frac{di}{dt} + \frac{1}{250} i = \frac{6280}{25} \cos \omega t \longrightarrow (3)$$

It is like non-homogeneous eqn

solution is $i(t) = i_{CF} + i_{PI} \longrightarrow (4)$

$$i_{CF} = Ke^{-\frac{t}{250}} \longrightarrow (5)$$

Consider, $i_{PI} = A \cos \omega t + B \sin \omega t \longrightarrow (6)$

differentiate wrt 't'

$$\frac{di}{dt} = -A\omega \sin \omega t + B\omega \cos \omega t \longrightarrow (7)$$

$$\frac{di}{dt} = -A\,314 \sin 314t + B\,314 \cos 314t \longrightarrow (7)$$

Substitute eqn (6) & (7) in eqn (3)

$$-A\,314 \sin 314t + B\,314 \cos 314t + \frac{1}{250}(A \cos \omega t + B \sin \omega t) = \frac{6280}{25}\cos \omega t \longrightarrow (8)$$

compare sin 314t terms.

$$-314A + \frac{B}{250} = 0 \longrightarrow (9)$$

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Compare $\cos 314t$ terms

$$314B + \frac{A}{250} = \frac{6280}{25} \longrightarrow (10)$$

from eqn (9) $A = \frac{B}{250 \times 314} \longrightarrow (11)$

Substitute eqn (11) in (10)

$$314B + \frac{B}{250 \times 250 \times 314} = 251.2$$

$$314B = 251.2$$

$$\boxed{B = 0.8} \longrightarrow (12)$$

$$A = \frac{0.8}{250 \times 314}$$

$$\boxed{A = 1.019 \times 10^{-5}} \longrightarrow (13)$$

Substitute eqn (12) & (13) in eqn (6)

$$i_{PI} = (1.019 \times 10^{-5}) \cos 314t + 0.8 \sin 314t \longrightarrow (14)$$

Substitute eqn (5) & eqn (14) in eqn (4)

$$i(t) = Ke^{\frac{-t}{250}} + 0.00001 \cos 314t + 0.8 \sin 314t.$$

To find K, use initial conditions.

at $t = 0, i = 0$

where,

$$K = -0.00001 \cos 314t + 0.8 \sin 314t.$$

(b) Laplace Transform Method :

Initial condition $t = 0, i = 0 \longrightarrow (1)$

Apply KVL to the circuit.

$$20 \sin 314t = 25 i(t) + \frac{1}{10} \int i \, dt \longrightarrow (2)$$

Apply Laplace transform

$$\frac{20 \times 314}{s^2 + 314^2} = 25 I(s) + \frac{1}{10} \frac{I(s)}{s}$$

$$\frac{20 \times 314}{s^2 + 314^2} = I(s) \left[25 + \frac{1}{10s} \right]$$

$$I(s) = \frac{20\times 314}{(s^2+314^2)\left(25+\frac{1}{10s}\right)}$$

$$= \frac{20\times 314}{\frac{25}{s}\left(s+\frac{1}{250}\right)(s^2+314^2)}$$

$$I(s) = \frac{\left(\frac{20\times 314\times s}{25}\right)}{\left(s+\frac{1}{250}\right)(s^2+314^2)} \longrightarrow (3)$$

Apply partial fractions

$$\frac{\left(\frac{20\times 314\times s}{25}\right)}{\left(s+\frac{1}{250}\right)(s^2+314^2)} = \frac{A}{s+\frac{1}{250}} + \frac{Bs+C}{s^2+314^2}$$

$$A = \left.\frac{\left(\frac{20\times 314\times s}{25}\right)}{s^2+314^2}\right|_{at\ s=\frac{-1}{250}} = \frac{\frac{20\times 314}{25}\times\frac{-1}{250}}{\left(\frac{-1}{250}\right)^2+314^2}$$

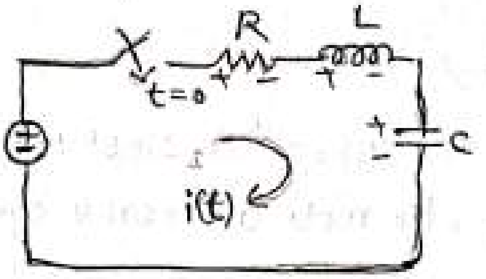
$$= \frac{-\frac{20\times 314}{25\times 25\times 10}}{314^2\left[1+\frac{1}{250\times 314^2}\right]}$$

$$=$$

Series R-L-C circuit

Method 1 :

DC transient analysis of series R-L-C circuit using DE method



Apply KVL to the circuit, $-V + Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i\,dt = 0$

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

$V_R = iR$
$V_L = L\frac{di}{dt}$
$V_C = \frac{1}{C}\int i\,dt$

differentiate eqn (1) w.r.t "t"

$$R\frac{di}{dt} + \frac{d^2i}{dt^2}L + \frac{1}{C}i = 0 \longrightarrow (2)$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

Let $\frac{d}{dt} = s$

$$s^2 i + \frac{R}{L}si + \frac{i}{LC} = 0 \longrightarrow (3)$$

from eqn (3)

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \longrightarrow (4)$$

It is a second order DE. So, the solution is

General eq.
$[as^2 + bs + c = 0]$
$a = 1,\ b = \frac{R}{L},\ c = \frac{1}{LC}$

$$s_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{+R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

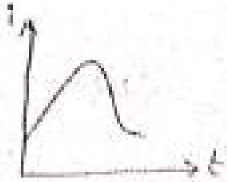
$$= \frac{1}{2}\left[\frac{-R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}\right]$$

$$s_{12} = \underbrace{\frac{-R}{2L}}_{\alpha} \pm \underbrace{\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}_{\beta} \longrightarrow (5)$$

$$s_{12} = \alpha \pm \beta$$

<u>Case (i)</u>: If $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$. The roots are real & unequal. The response is over damped.

The solution is $i(t) = e^{\alpha t}[K_1 e^{\beta t} + K_2 e^{-\beta t}]$.



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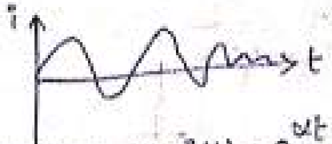
Case 2: Similarly, if $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, the roots complex & conjugate, response is under damped.

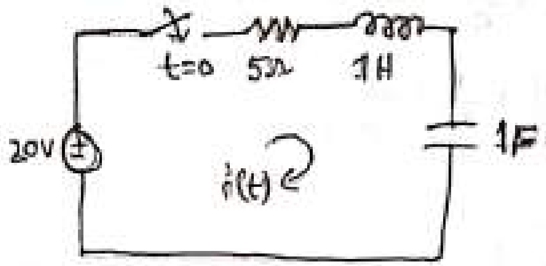


The solution is $i(t) = e^{\alpha t}[k_2 \sin\beta t + k_1 \cos\beta t]$

Case 3: If $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$, the roots are real & equal. The response critically damped.

The solution is $i(t) = e^{\alpha t}[k_1 + k_2 t]$

1. In a series R-L-C circuit, $R = 5\Omega$, $L = 1$ H & $C = 1F$. A DC voltage of 20V is applied to a circuit at $t=0$. Obtain $i(t)$.



Sol: Apply KVL to the circuit, $-20 + 5i(t) + 1 \cdot \frac{di}{dt} + \frac{1}{1}\int i\,dt$

$$20 = 5i + \frac{di}{dt} + \int i\,dt \longrightarrow (1)$$

differentiate eqn (1)

$$5\frac{di}{dt} + \frac{d^2i}{dt^2} + i = 0$$

$$\frac{d^2i}{dt^2} + 5\frac{di}{dt} + i = 0$$

Let $\frac{d}{dt} = s$

$$s^2 + 5s + 1 = 0$$

$$s_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25-4}}{2}$$

$$= \frac{-5 \pm \sqrt{21}}{2} = -0.21, -4.79 \longrightarrow (2)$$

Case (i): $25 > 4$, The roots are real and unequal. The response is over damped.

The solution is

$$i(t) = e^{-\frac{5}{2}t}\left[k_1 e^{\frac{\sqrt{21}}{2}t} + k_2 e^{-\frac{\sqrt{21}}{2}t}\right]$$

$$i(t) = e^{-2.5t}\left[k_1 e^{2.29t} + k_2 e^{-2.29t}\right] \longrightarrow (3)$$

To find k:

use initial condition $t=0, i=0$.

$$i(0) = e^0\left[k_1+k_2\right]$$

$$0 = k_1+k_2$$

$$\boxed{k_1 = -k_2} \longrightarrow (4)$$

again use $\frac{di}{dt}$ for eqn (3)

$$\left[e^{-2.5+2.29} = e^{-0.21}\right]$$

$$\left[e^{-2.5-2.29} = e^{-4.79}\right]$$

$$\frac{di}{dt} = -0.21k_1 e^{-0.21t} - 4.79k_2 e^{-4.79t} \longrightarrow (5)$$

again put initial conditions in eqn (5).

$$\frac{di}{dt} = -0.21k_1 - 4.79k_2 \longrightarrow (6)$$

Substitute initial conditions in eqn (1)

$$20 = 5(0) + 1\frac{di}{dt} + 1\cdot 0$$

$$\frac{di}{dt} = 20 \longrightarrow (7)$$

put eqn (7) in eqn (6)

$$20 = -0.21k_1 - 4.79k_2 \longrightarrow (8)$$

solve eqn (4) & eqn (8)

$$20 = 0.21k_2 - 4.79k_2$$

$$20 = -4.58k_2$$

$$\boxed{\begin{aligned} k_2 &= -4.366 \\ k_1 &= 4.366 \end{aligned}}$$

$$\therefore i(t) = e^{-2.5t}\left[4.36e^{2.29t} - 4.36e^{-2.29t}\right].$$

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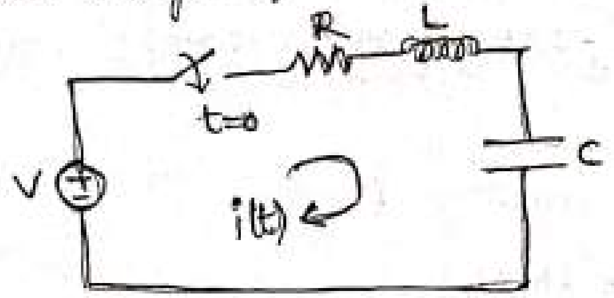
Method 2:

DC transient analysis of series RLC circuit by Laplace method



$L\left(\frac{di}{dt}\right) = sI(s) - i$

Apply KVL to the circuit

$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

apply Laplace $\quad \frac{V}{s} = I(s)R + LsI(s) + \frac{1}{C}\frac{I(s)}{s} \longrightarrow (2)$

$$\frac{V}{s} = I(s)\left[R + sL + \frac{1}{Cs}\right]$$

$$I(s) = \frac{(V/s)}{R + sL + \frac{1}{Cs}}$$

$$= \frac{(V/s)}{L\left(\frac{R}{L} + s + \frac{1}{LCs}\right)} = \frac{\frac{V}{s}}{\frac{L}{s}\left[\frac{Rs}{L} + s^2 + \frac{1}{LC}\right]}$$

$$I(s) = \frac{V/L}{s^2 + \frac{Rs}{L} + \frac{1}{LC}} \longrightarrow (3)$$

take critical resistance $R_{cr} = 2\sqrt{\frac{L}{C}}$

$$(\text{zeta})\ \xi = \frac{R}{R_{cr}} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

(natural frequency) $\omega_n^2 = \frac{1}{LC}$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$2\xi\omega_n = 2 \times \frac{R}{2}\sqrt{\frac{C}{L}} \times \frac{1}{\sqrt{LC}}$$

$$2\xi\omega_n = \frac{R}{L}$$

Now, eqn (3) becomes

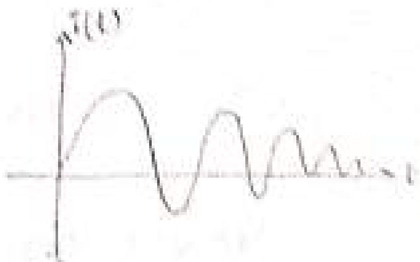
$$I(s) = \frac{V/L}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{V/L}{-\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}} \longrightarrow (4)$$

Case(i): If $\xi<1$, roots of the eqn are complex conjugate the roots are underdamped.

$$i(t) = \frac{V}{Lw_d} e^{-\xi w_n t} \sin w_d t$$

where $w_d = w_n\sqrt{1-\xi^2}$



case(ii): If $\xi=1$, then the I(s) is given by

$$I(s) = \frac{(V/L)}{s^2+2\xi w_n s+w_n^2} = \frac{(V/L)}{(s+w_n)^2}$$

By applying inverse laplace,

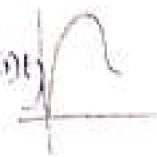
$$L\left[te^{-at}\right] = \frac{1}{(s+a)^2}$$

$$i(t) = \frac{V}{L}te^{-w_n t}$$

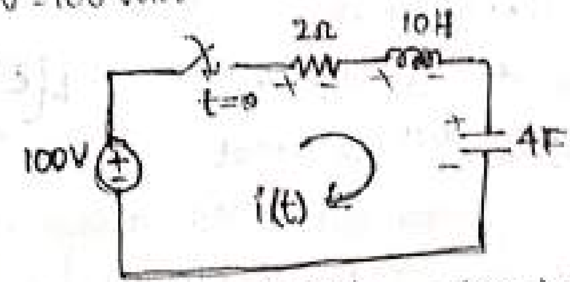
case(iii): If $\xi>1$, then damped is over damped.

The solution of the eqn is

$$i(t) = \frac{V}{2Lw_n\sqrt{\xi^2-1}} e^{-\xi w_n t}\left(e^{(w_n\sqrt{\xi^2-1})t} + e^{-(w_n\sqrt{\xi^2-1})t}\right)$$



1. In RLC Series circuit $R=2\Omega$, $L=10H$, $C=4F$ and the switch is closed at $t=0$, find the expression for current by using LT. where $V=100$ Volts.



Apply KVL to the circuit, $-100+2i(t)+10\frac{di}{dt}+\frac{1}{4}\int i\,dt=0$

$$100 = 2i(t)+10\frac{di(t)}{dt}+\frac{1}{4}\int i\,dt \longrightarrow (1)$$

Apply Laplace to the eqn (1)

$$\frac{100}{s} = 2I(s)+10\left[sI(s)-I(0)\right]+\frac{1}{4}\frac{I(s)}{s}$$

$$I(s)\left[2+10s+\frac{1}{4s}\right] = \frac{100}{s}$$

$$I(s) = \frac{100}{s\left[2+10s+\frac{1}{4s}\right]}$$

$$= \frac{100}{2s+10s^2+\frac{1}{4}}$$

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$$I(s) = \frac{100}{10\left(s^2+\frac{2}{10}s+\frac{10}{40}\right)}$$

$$I(s) = \frac{10}{s^2+0.2s+0.025} \longrightarrow (2)$$

$$s^2+0.2s+0.025 \Rightarrow s_{12} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-0.2\pm\sqrt{0.2^2-4(0.025)}}{2}$$

$$= -0.1\pm 0.122i$$

$$I(s) = \frac{10}{s^2+2\times0.1\times s+(0.1)^2-(0.1)^2+0.025}$$

$$= \frac{10}{(s+0.1)^2+0.015}$$

$$I(s) = \frac{10}{(s+0.1)^2+(0.122)^2} \longrightarrow (3)$$

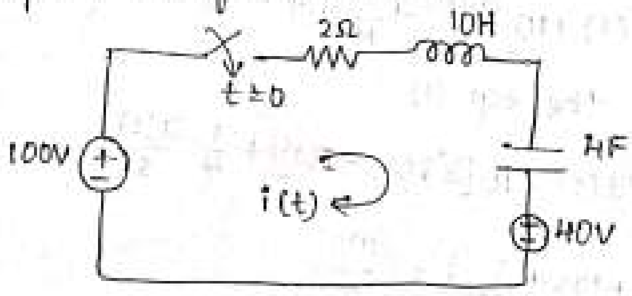
$$I(s) = \frac{10}{(s+0.1)^2+(0.122)^2} \times \frac{0.122}{0.122} \longrightarrow (4)$$

Apply inverse laplace to the eqn (4).

$$L\left[e^{-at}\sin\omega t\right] = \frac{\omega}{(s+a)^2+\omega^2}$$

$$i(t) = 81.96\, e^{-0.1t}\sin 0.122t.$$

2. In RLC circuit, the capacitor has initial voltage of 40V & the switch is closed at t=0. Find the expression for current by using Laplace transform method.



Apply KVL to the circuit,

$$100 = 2i(t)+10\frac{di}{dt}+\frac{1}{4}\int i\,dt+40$$

$$60 = 2i(t)+10\frac{di}{dt}+\frac{1}{4}\int i\,dt \longrightarrow (1)$$

Apply Laplace transform,

$$\frac{60}{s} = 2I(s)+10sI(s)+\frac{1}{4}\frac{I(s)}{s}$$

$$I(s)\left[2+10s+\frac{1}{4s}\right] = \frac{60}{s}$$

$$I(s) = \frac{60}{s\left[2+10s+\frac{1}{4s}\right]}$$

$$= \frac{60}{10s^2+2s+\frac{1}{4}}$$

$$= \frac{6}{s^2+\frac{1}{5}s+\frac{1}{40}}$$

$$I(s) = \frac{6}{s^2+0.2s+0.025} \longrightarrow (2)$$

$$s^2+0.2s+0.025 \Rightarrow s_{12} = \frac{-0.2 \pm \sqrt{(0.2)^2-4(0.025)}}{2}$$

$$= -0.1 \pm 0.122i$$

$$I(s) = \frac{6}{s^2+2\times0.1\times s+(0.1)^2-0.1^2+0.025}$$

$$= \frac{6}{(s+0.1)^2+0.015}$$

$$= \frac{6}{(s+0.1)^2+(0.122)^2} \longrightarrow (3)$$

$$= \frac{6}{(s+0.1)^2+(0.122)^2} \times \frac{0.122}{0.122}$$

$$= \frac{6}{0.122}\left[\frac{0.122}{(s+0.1)^2+(0.122)^2}\right]$$

apply inverse laplace

$$i(t) = \frac{6}{0.122}\left[e^{-0.1t}\sin 0.122t\right] \text{ amp}$$

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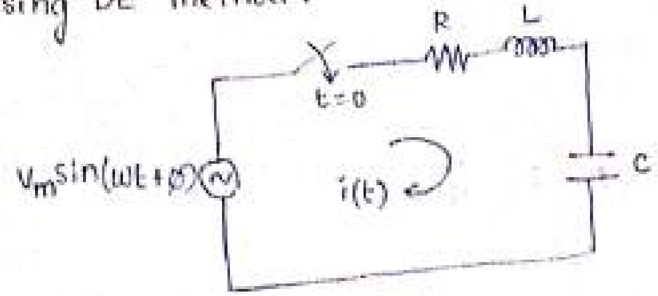
Method 3 :

AC transient analysis of series RLC series circuit by using DE method :



Apply KVL to the circuit.

$$V_m \sin(\omega t+\phi) = i(t)R + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

differentiate wrt 't'.

$$V_m \omega \cos(\omega t+\phi) = \frac{di}{dt}R + L\frac{d^2i}{dt^2} + \frac{1}{C}i \longrightarrow (2)$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{V_m\omega}{L}\cos(\omega t+\phi) \longrightarrow (3)$$

It is a second order differential equation.

Let us consider, the solution is

$$i(t) = i_{CF} + i_{PI} \longrightarrow (4)$$

and take

$$i_{PI} = A\cos(\omega t+\phi) + B\sin(\omega t+\phi) \longrightarrow (5)$$

$$\frac{di}{dt} = -A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi) \longrightarrow (6)$$

$$\frac{d^2i}{dt^2} = -A\omega^2\cos(\omega t+\phi) - B\omega^2\sin(\omega t+\phi) \longrightarrow (7)$$

substitute eqns (5), (6), (7) in eqn (3)

$$\left[-A\omega^2\cos(\omega t+\phi) - B\omega^2\sin(\omega t+\phi)\right] + \frac{R}{L}\left[-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi)\right]$$

$$+\frac{1}{LC}\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] = \frac{V_m\omega}{L}\cos(\omega t+\phi) \longrightarrow (8)$$

Compare $\cos(\omega t+\phi)$ terms

$$-A\omega^2 + \frac{B\omega R}{L} + \frac{A}{LC} = \frac{V_m\omega}{L} \longrightarrow (9)$$

Compare $\sin(\omega t+\phi)$ terms

$$-B\omega^2 - \frac{AR\omega}{L} + \frac{B}{LC} = 0 \longrightarrow (10)$$

from eqn (10) $\quad B\left(\frac{1}{LC} - \omega^2\right) - \frac{AR\omega}{L} = 0$

$$B = \frac{-(\omega R A/L)}{\omega^2 - \frac{1}{LC}} \longrightarrow (11)$$

from eqn (9)

$$A\left(\frac{1}{LC} - \omega^2\right) + \frac{B\omega R}{L} = \frac{V_m\omega}{L} \longrightarrow (12)$$

$$A\left(\frac{1}{LC} - \omega^2\right) + \left(\frac{AR\omega/L}{\frac{1}{LC} - \omega^2}\right)\left(\frac{\omega R}{L}\right) = \frac{V_m\omega}{L}$$

$$A\left(\frac{1}{LC} - \omega^2 + \frac{R^2\omega^2}{\frac{1}{LC} - \omega^2}\right) = \frac{V_m\omega}{L}$$

$$A\left(\frac{1}{LC} - \omega^2 + \frac{LCR^2\omega^2}{1 - LC\omega^2}\right) = \frac{V_m\omega}{L}$$

$$A\left(\frac{1 - LC\omega^2}{LC} + \frac{LCR^2\omega^2}{1 - LC\omega^2}\right) = \frac{V_m\omega}{L}$$

substitute eqn (11) in eqn (12)

$$A\left(\omega^2 - \frac{1}{LC}\right) + A\left(\frac{\omega R}{L}\right)^2 \times \frac{1}{\omega^2 - \frac{1}{LC}} = \frac{-V_m\omega}{L}$$

$$A\left[\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{\omega R}{L}\right)^2\right] = \frac{-V_m\omega}{L}\left(\omega^2 - \frac{1}{LC}\right)$$

$$A = \frac{\frac{-V_m\omega}{L}\left(\omega^2 - \frac{1}{LC}\right)}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{\omega R}{L}\right)^2}$$

$$A = \frac{\frac{-V_m\omega}{L} \times \frac{\omega}{L}\left[\omega L - \frac{1}{\omega C}\right]}{\frac{\omega^2}{L^2}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

$$A = \frac{-V_m\left[\omega L - \frac{1}{\omega C}\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \longrightarrow (13)$$

Substitute eqn (13) in eqn (11)

$$\boxed{B = \frac{V_m R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

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Substitute A & B in eqn (5)

$$i_{PI} = \underbrace{\frac{-V_m(\omega L - \frac{1}{\omega C})}{R^2 + (\omega L - \frac{1}{\omega C})^2}}_{K_1 \sin\theta} \cos(\omega t + \phi) + \underbrace{\frac{V_m R}{R^2 + (\omega L - \frac{1}{\omega C})^2}}_{K_1 \cos\theta} \sin(\omega t + \phi) \longrightarrow (14)$$

$$K_1^2(\sin^2\theta + \cos^2\theta) = \frac{V_m^2(\omega L - \frac{1}{\omega C})^2}{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]^2} \times \frac{R^2 V_m^2}{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]^2}$$

$$K_1^2 = \frac{V_m^2 \cancel{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]}}{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]^{\cancel{2}}}$$

$$K_1^2 = \frac{V_m^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$K_1 = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-V_m(\omega L - \frac{1}{\omega C})}{V_m R}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)}$$

then eqn(14) becomes

$$i_{PI} = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \left[-\sin\theta \cdot \cos(\omega t + \phi) + \cos\theta \cdot \sin(\omega t + \phi)\right]$$

$$i_{PI} = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \sin(\omega t + \phi - \theta) \longrightarrow (15)$$

<u>To find i_{CF}</u>:

Consider $\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$

take $\frac{di}{dt} = s$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) i = 0$$

$$s_{1,2} = \underbrace{\frac{-R}{2L}}_{\alpha} \pm \underbrace{\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}_{\beta}$$

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Case 1: $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ (over damped)

The roots are real and unequal.

Then the solution is

$$i_{CF} = e^{\alpha t}\left[K_2 e^{\beta t} + K_3 e^{-\beta t}\right]$$

Now the complete solution is

$$i(t) = e^{\frac{-R}{2L}t}\left[K_2 e^{\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\,t} + K_3 e^{-\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\,t}\right] + \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi - \theta)$$

Case 2: $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ (under damped)

The roots are complex conjugate

$$i_{CF} = e^{\alpha t}\left[k_2 \cos\beta t + k_3 \sin\beta t\right]$$

$$i(t) = e^{\frac{-R}{2L}t}\left[k_2 \cos\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\,t + k_3 \sin\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\,t\right] + \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi - \theta)$$

Case 3: $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ (critically damped)

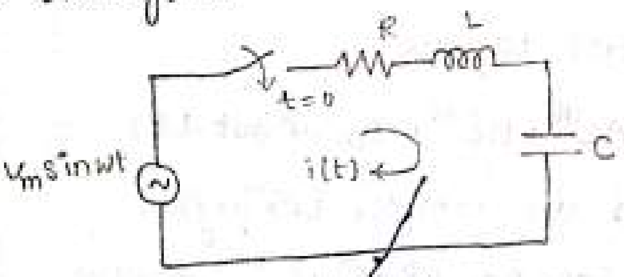
The roots are real and equal.

$$i_{CF} = e^{\alpha t}\left[K_2 + K_3 t\right]$$

$$i(t) = e^{-\frac{R}{2L}t}\left[K_2 + K_3 t\right] + \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi - \theta)$$

Method 4:

AC transient analysis of series RLC circuit by Laplace transform method.



Apply KVL to the circuit

$$V_m \sin \omega t = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \longrightarrow (1)$$

Apply Laplace transform

$$\frac{V_m \omega}{S^2 + \omega^2} = R\,I(s) + SL\,I(s) + \frac{I(S)}{CS} \longrightarrow (2)$$

$$I(s) = \frac{V_m \omega}{(S^2 + \omega^2)\left(R + SL + \frac{1}{CS}\right)}$$

$$I(s) = \frac{V_m w s}{(s^2+w^2)\, L\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}$$

$$I(s) = \frac{V_m w s/L}{(s^2+w^2)\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)} \longrightarrow (3)$$

Case 1 : Roots are real and equal, then

$$s^2+\frac{R}{L}s+\frac{1}{LC} = (s+a)^2$$

$$I(s) = \frac{\left(\frac{V_m w s}{L}\right)}{(s^2+w^2)(s^2+a^2)^2} \longrightarrow (4)$$

Apply partial fractions

$$I(s) = \frac{A}{s+a}+\frac{B}{(s+a)^2}+\frac{Cs+D}{s^2+w^2}$$

By inverse laplace transform

$$i(t) = Ae^{-at}+Bte^{-at}+I_m\sin(wt+\theta)$$

Case 2 :

The root are real and unequal,

$$s^2+\frac{R}{L}s+\frac{1}{LC} = (s+a)(s+b)$$

$$I(s) = \frac{\left(\frac{V_m w s}{L}\right)}{(s^2+w^2)(s+a)(s+b)}$$

$$I(s) = \frac{A}{s+a}+\frac{B}{s+b}+\frac{Cs+D}{s^2+w^2}$$

Apply inverse laplace

$$i(t) = Ae^{-at}+Be^{-bt}+I_m\sin(wt\pm\theta)$$

Case 3 : The roots are complex conjugate.

$$s^2+\frac{R}{L}s+\frac{1}{LC} = \left[s^2+\frac{R}{L}s+\left(\frac{R}{2L}\right)^2\right]+\left(\frac{1}{LC}-\left(\frac{R}{2L}\right)^2\right)$$

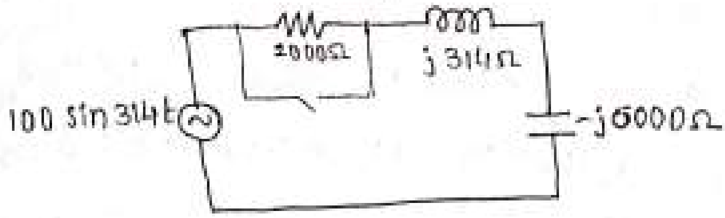
$$= (s+a)^2+w_d^2.$$

$$I(s) = \frac{(V_m w s/L)}{(s^2+w^2)\left[(s+a)^2+w_d^2\right]} = \frac{As+B}{s^2+w^2}+\frac{Cs+D}{(s+a)^2+w_d^2}$$

Inverse laplace,

$$i(t) = I_m\sin(wt\pm\theta)+I_0e^{-at}\sin(wt\pm\theta).$$

1. In the figure the switch is open, steady state is reached with $V = 100 \sin 314t$. The switch is closed at $t=0$, the circuit is allowed to come to steady state again. Determine the steady state current and complete transient current.



Sol: Case 1: Switch is open

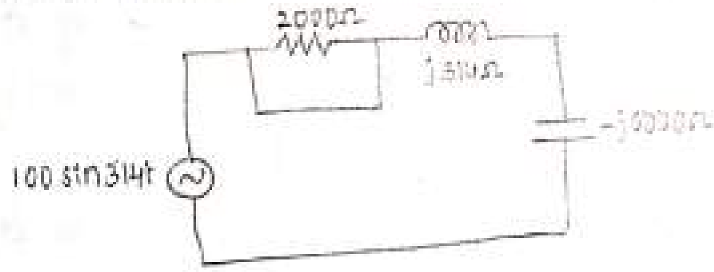
$$z = 2000 + j(314 - 6000)$$

$$z = 6027.48 \angle -70.62$$

$$I = \frac{V}{z}$$

$$I = 0.017 \angle 70.62 \longrightarrow (1)$$

Case 2: switch is closed.



$$z = j(314 - 6000).$$

$$z = 5686 \angle -90$$

$$|I| = \frac{V}{z}$$

$$I = 0.018 \angle 90$$

Steady state current (I_{ss})

$$I_{ss} = 0.018 \sin(314t - 90^\circ) \longrightarrow PI$$

To find transient current (complementary function)

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) i = 0$$

$$S_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

put $R = 0$

$$S_{1,2} = \pm\sqrt{\frac{-1}{LC}}$$

$$= \pm j\,0.001$$

roots are complex conjugate.

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$$i_{CF} = e^{\alpha t}(k_1 \cos\beta t + k_2 \sin\beta t)$$

$$= k_1 \cos\beta t + k_2 \sin\beta t$$

$$= k_1 \cos 0{\cdot}001t + k_2 \sin 0{\cdot}001t$$

complete current $i(t) = i_{CF} + i_{PI}$

$$i(t) = k_1 \cos 0{\cdot}001t + k_2 \sin 0{\cdot}001t + 0{\cdot}018 \sin(314t - 90)$$

$$= k_1 \cos 0{\cdot}001t + k_2 \sin 0{\cdot}001t - 0{\cdot}018 \cos(\cancel{90} - 314t) \rightarrow (2)$$

Use initial conditions

$$t = 0,\ i = 0$$

$$0{\cdot}017 = k_1 - 0{\cdot}0175$$

$$k_1 = 0{\cdot}0365$$

UNIT-III

FOURIER TRANSFORM

Fourier theorem — Trigonometric form and exponential form of fourier series — Conditions of symmetry — line spectra and phase angle spectra — Analysis of electrical circuit excited by non-sinusoidal sources of periodic-wave forms — fourier integrals and fourier transforms — properties of fourier transform and applications of electrical circuit.

Fourier Series :

→ Fourier series means a non sinusoidal wave form like triangular ~~plus~~, rectangular plus (or) square plus etc are applied to the inputs to the electrical circuit.

→ It is represented by $f(t)$.

→ In general fourier series expansion is given below

$$f(t) = a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \dots + a_n\cos n\omega t + b_1\sin\omega t + \dots$$

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n\sin n\omega t] \longrightarrow (1)$$

Exponential form of fourier series :

We know,

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n\sin n\omega t] \longrightarrow (1)$$

$$= a_0 + \sum_{n=1}^{\infty}\left(\sqrt{a_n^2+b_n^2}\right)\left[\frac{a_n}{\sqrt{a_n^2+b_n^2}}\cos n\omega t + \frac{b_n}{\sqrt{a_n^2+b_n^2}}\sin n\omega t\right]$$

$$= a_0 + \sum_{n=1}^{\infty} A_n\cos(n\omega t - \theta_n) \longrightarrow (2)$$

Using Euler's Identity Expansion

$$f(t) = a_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty} C_n e^{jn\omega t} \longleftrightarrow f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} \longrightarrow (3)$$

Multiplying both sides by $e^{-jk\omega t}$ & integrating with t_1 to t_1+T

$$\int_{t_1}^{t_1+T} f(t)e^{-jk\omega t}\,dt = \int_{t_1}^{t_1+T}\sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} e^{-jk\omega t}\,dt$$

$$(\text{for } n=k) = C_n T_0 \longrightarrow (4) \quad \text{where } \int_{t_1}^{t_1+T}\sum_{n=-\infty}^{\infty} e^{j(n-k)\omega t}\,dt = T_0$$

$$= (t)_{t_1}^{t_1+T}$$

$$= t_1+T-t_1$$

$$= T$$

$$\boxed{C_n = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\, e^{-jn\omega t}\, dt}$$

Trigonometric Form:

We know,

$$C_n = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\, e^{-jn\omega t}\, dt \longrightarrow (1)$$

multiplied by '2' on both sides

$$2C_n = \frac{2}{T_0}\int_{t_1}^{t_1+T} f(t)\, e^{-jn\omega t}\, dt \longrightarrow (2)$$

$$a_n - jb_n = \frac{2}{T_0}\int_{t_1}^{t_1+T} f(t)\, [\cos n\omega t - j\sin n\omega t]\, dt$$

$$a_n - jb_n = \frac{2}{T_0}\int_{t_1}^{t_1+T} f(t)\cos n\omega t\, dt - j\frac{2}{T_0}\int_{t_1}^{t_1+T} f(t)\sin n\omega t\, dt$$

on comparing,

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T} f(t)\cos n\omega t\, dt \longrightarrow (3)$$

$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T} f(t)\sin n\omega t\, dt \longrightarrow (4)$$

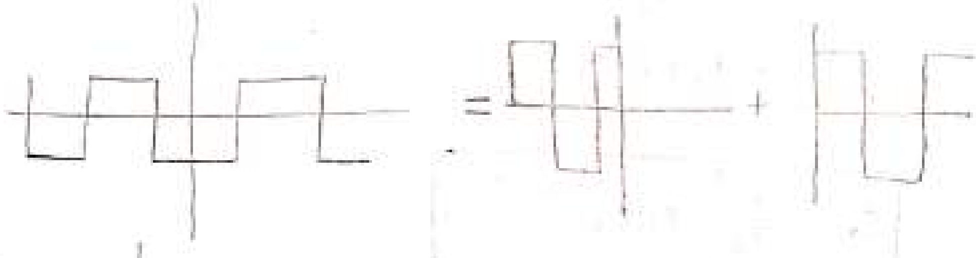
$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\, dt \longrightarrow (5)$$

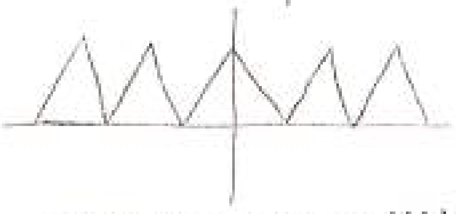
Symmetry in Fourier Series:

There are three types of symmetry. They are:

1. Even
2. Odd
3. Half wave

1. Even Symmetry :





condition $f(-t) = f(t)$

Taking $a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\,dt$, $t_1 = \frac{-T_0}{2}$

$$a_0 = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{-\frac{T_0}{2}+T_0} f(t)\,dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\,dt \qquad f(-t) = f(t) \;,\; d(-t) = dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{0} f(t)\,dt + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{0} f(-t)(-dt) + \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} f(t)\,dt$$

$$\boxed{a_0 = \frac{2}{T_0}\int_{0}^{\frac{T_0}{2}} f(t)\,dt}$$

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\cos n\omega t\,dt \quad , \text{ put } t_1 = \frac{-T_0}{2}$$

$$= \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cos n\omega t\,dt$$

$$= \frac{2}{T_0}\left[\int_{-T_0/2}^{0} f(t)\cos n\omega t\,dt + \int_{0}^{\frac{T_0}{2}} f(t)\cos n\omega t\,dt\right]$$

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put $f(-t) = f(t)$

$$a_n = \frac{2}{T_0}\left[\int_{-T_0/2}^{0} f(-t)\cos[n\omega(-t)](-dt) + \int_{0}^{T_0/2} f(t)\cos n\omega t\, dt\right]$$

$$= \frac{2}{T_0}\left[\int_{-T_0/2}^{0} f(t)\cos n\omega t\,(-dt) + \int_{0}^{T_0/2} f(t)\cos n\omega t\, dt\right]$$

$$\boxed{a_n = \frac{4}{T_0}\int_{0}^{T_0/2} f(t)\cos n\omega t\, dt}$$

$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\sin \omega t\, dt \qquad \text{put } t_1 = -\frac{T_0}{2}$$

$$= \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\sin \omega t\, dt$$

$$= \frac{2}{T_0}\left[\int_{-T_0/2}^{0} f(t)\sin \omega t\, dt + \int_{0}^{T_0/2} f(t)\sin \omega t\, dt\right]$$

$$= \frac{2}{T_0}\left\{\int_{0}^{\frac{T_0}{2}} f(t)\sin \omega t\, dt + \int_{0}^{T_0/2} f(t)\sin \omega t\, dt\right\}$$

$$\boxed{b_n = \frac{4}{T_0}\int_{0}^{\frac{T_0}{2}} f(t)\sin \omega t\, dt}$$

2. Odd Symmetry:

$$f(-t) = -f(t)$$

(i) $a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)\,dt$

putting $t_1 = -\frac{T_0}{2}$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\,dt$$

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{0} f(t)\,dt + \int_{0}^{T_0/2} f(t)\,dt\right]$$

put $f(-t) = -f(t)$ in 1st term

$$= \frac{1}{T_0}\int_{-T_0/2}^{0} f(-t)\,d(-t) + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

$$= \frac{-1}{T_0}\int_{-T_0/2}^{0} f(t)[-d(t)] + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

$$= -\frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\cos n\omega t\,dt$$

put $t_1 = \frac{-T_0}{2}$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cos n\omega t\,dt$$

$$= \frac{2}{T_0}\int_{-T_0/2}^{0} f(t)\cos n\omega t\,dt + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos n\omega t\,dt$$

put $f(-t) = -f(t)$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{0} f(-t)\cos n\omega(-t)\, d(-t) + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos n\omega t\, dt$$

$$= -\frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos n\omega t\, dt + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos n\omega t\, dt$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T_0}\int_{t}^{t+T_0} f(t)\sin n\omega t\, dt$$

put $t_1 = -T_0/2$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\sin n\omega t\, dt$$

$$= \frac{2}{T_0}\int_{-T_0/2}^{0} f(t)\sin n\omega t\, dt + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega t\, dt$$

put $f(-t) = -f(t)$, $d(-t) = -dt$

$$= \frac{2}{T_0}\int_{-T_0/2}^{0} f(-t)\sin n\omega(-t)\, d(-t) + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega(t)\, dt$$

$$= \frac{-2}{T_0}\int_{-T_0/2}^{0} f(t)\sin n\omega t\, dt + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega t\, dt$$

$$= \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega t\, dt + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega t\, dt$$

$$\boxed{b_n = \frac{4}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega t\, dt}$$

3. Half Wave Symmetry:

$$f(t) = -f(t - T_0/2)$$

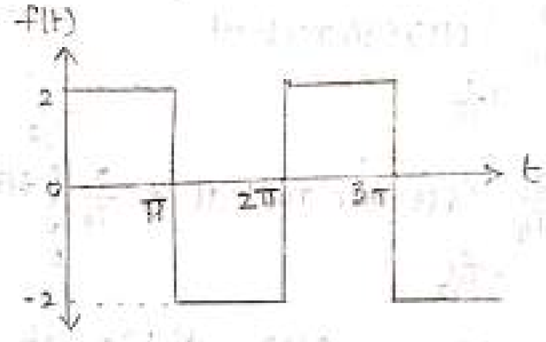
If n is even, $a_0 = 0$, $a_n = 0$, $b_n = 0$

If n is odd, $a_0 = 0$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos n\omega t\, dt$$

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\sin n\omega t\, dt$$

1. Obtain Fourier series for the following wave form



Ans: We know that Fourier Series Expansion is

$$f(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos n\omega t + b_n \sin n\omega t]$$

$$= a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \dots + b_1\sin\omega t + b_2\sin 2\omega t +$$

One cycle $T = 2\pi = T_0$

(1) $a_0 = \frac{1}{T_0}\int_0^{2\pi} f(t)\, dt$

$$= \frac{1}{T_0}\left[\int_0^{\pi} f(t)\, dt + \int_{\pi}^{2\pi} f(t)\, dt\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 2\,dt + \int_{\pi}^{2\pi}(-2)\, dt\right]$$

$$= \frac{1}{2\pi}\left[2(t)_0^{\pi} - 2(t)_{\pi}^{2\pi}\right]$$

$$= \frac{2}{2\pi}\left[(\pi - 0) - (2\pi - \pi)\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{T_0}\int_0^{2\pi} f(t)\cos n\omega t\, dt$$

Here $\omega = 2\pi f = 2\pi \frac{1}{T}$.

$\omega = \frac{2\pi}{2\pi} = 1$

$$= \frac{2}{T_0}\left[\int_0^{\pi} f(t)\cos nt\, dt + \int_{\pi}^{2\pi} f(t)\cos nt\, dt\right]$$

$$= \frac{2}{2\pi}\left[\int_0^{\pi} 2\cos nt\, dt + \int_{\pi}^{2\pi}(-2)\cos nt\, dt\right]$$

$$= \frac{2}{\pi}\left[\left(\frac{\sin nt}{n}\right)_0^{\pi} - \left(\frac{\sin nt}{n}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{2}{\pi}\left[\frac{\sin n\pi}{n} - \frac{\sin 0}{n} - \frac{\sin 2n\pi}{n} + \frac{\sin n\pi}{n}\right]$$

for $n = 1, 2, 3 \ldots$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T_0}\int_0^{2\pi} f(t)\sin n\omega t\, dt$$

$$= \frac{2}{2\pi}\left[\int_0^{\pi} f(t)\sin n\omega t\, dt + \int_{\pi}^{2\pi} f(t)\sin n\omega t\, dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} 2\sin n\omega t\, dt + \int_{\pi}^{2\pi} -2\sin n\omega t\, dt\right]$$

$$= \frac{2}{\pi}\left[\int_0^{\pi} \sin nt\, dt + \int_{\pi}^{2\pi} -\sin nt\, dt\right]$$

$$= \frac{2}{\pi}\left[\left[\frac{-\cos nt}{n}\right]_0^{\pi} + \left(\frac{\cos nt}{n}\right)_{\pi}^{2\pi}\right]$$

$$b_n = \frac{2}{\pi}\left[\frac{-\cos n\pi}{n} + \frac{\cos 0}{n} + \frac{\cos 2n\pi}{n} - \frac{\cos n\pi}{n}\right]$$

For $n = 1$, $\quad b_1 = \frac{2}{\pi}\left[-(-1) + 1 + 1 + 1\right]$

$$\boxed{b_1 = \frac{8}{\pi}}$$

$$b_2 = \frac{2}{\pi}\left[-\frac{1}{2}+\frac{1}{2}+\frac{1}{2}-\frac{1}{2}\right]$$

$$\boxed{b_2 = 0}$$

$$b_3 = \frac{2}{\pi}\left[\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right]$$

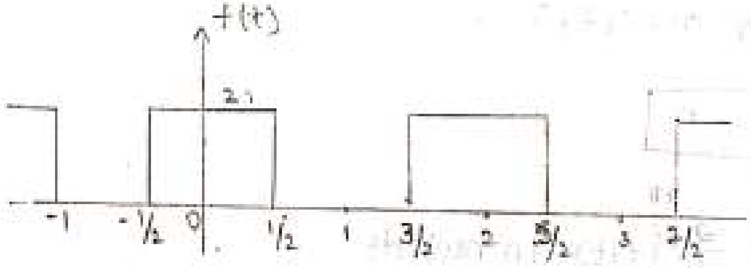
$$b_3 = \frac{8}{3\pi}$$

Fourier series:

$$f(t) = b_1 \sin\omega t + b_3 \sin 3\omega t + \cdots$$

$$= \frac{8}{\pi}\sin\omega t + \frac{8}{3\pi}\sin 3\omega t + \cdots$$

2. Obtain Fourier series



T = 2

Ans: Fourier Series is

$$f(t) = a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \cdots + b_1\sin\omega t + b_2\sin\omega t +$$

(i) $a_0 = \frac{1}{T_0}\int_0^2 f(t)\,dt$

$$= \frac{1}{2}\int_0^{1/2} f(t)\,dt + \frac{1}{2}\int_{3/2}^{2} f(t)\,dt$$

$$= \frac{1}{2}\int_0^{1/2} 2\,dt + \frac{1}{2}\int_{3/2}^{2} 2\,dt$$

$$= (t)_0^{1/2} + (t)_{3/2}^{2}$$

$$= 1/2 - 0 + 2 - \frac{3}{2}$$

$$= \frac{5}{2} - \frac{3}{2}$$

$$\boxed{a_0 = 1}$$

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(ii) $a_n = \frac{2}{T_0}\int_0^2 f(t)\cos n\omega t\,dt$

$$= \frac{2}{2}\left[\int_0^{1/2} 2\cos n\omega t\,dt + \int_{3/2}^{2} 2\cos n\omega t\,dt\right]$$

$$= 2\int_0^{1/2}\cos n\omega t\,dt + \int_{3/2}^{2}\cos n\omega t\,dt$$

$$= 2\left[\frac{\sin n\pi t}{n\pi}\right]_0^{1/2} + \left[\frac{\sin n\pi t}{n\pi}\right]_{3/2}^{2}$$

$$= 2\left[\frac{\sin\frac{n\pi}{2}}{n\pi} - \sin 0 + \frac{\sin 2n\pi}{n\pi} - \frac{\sin\frac{3n\pi}{2}}{n\pi}\right]$$

$$= 2\left[\frac{\sin\frac{n\pi}{2}}{n\pi} - \frac{\sin\frac{3n\pi}{2}}{n\pi}\right]$$

For $n=1$, $a_1 = \left[\frac{1}{\pi} + \frac{1}{\pi}\right]2$

$$\boxed{a_1 = \frac{4}{\pi}}$$

For $n=2$, $a_2 = 0$

For $n=3$, $a_3 = 2\left[\frac{-1}{3\pi} - \frac{1}{3\pi}\right]$

$$\boxed{a_3 = \frac{-4}{3\pi}}$$

(iii) $b_n = \frac{2}{T_0}\int_0^2 f(t)\sin n\omega t\,dt$

$$= \frac{2}{2}\left[\int_0^{1/2} 2\sin n\omega t\,dt + \int_{3/2}^{2} 2\sin n\omega t\,dt\right]$$

$$= 2\left[\left[\frac{-\cos n\pi t}{n\pi}\right]_0^{1/2} + \left[-\frac{\cos n\pi t}{n\pi}\right]_{3/2}^{2}\right]$$

$$= 2\left[-\frac{\cos\frac{n\pi}{2}}{n\pi} + \frac{\cos 0}{n\pi} - \frac{\cos 2n\pi}{n\pi} + \frac{\cos\frac{3n\pi}{2}}{n\pi}\right]$$

$b_1 = 2\left(0 + \frac{1}{\pi} + \frac{1}{\pi}\right) - \frac{1}{\pi} = \frac{0}{\pi} = 0$

$b_2 = 2[0] = 0$

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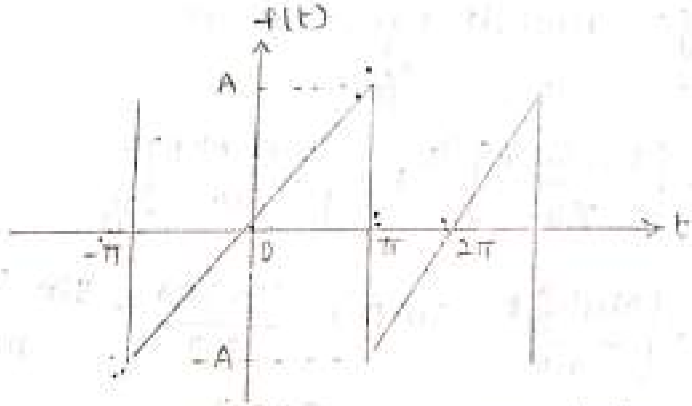
Fourier series is

$$f(t) = 1 + \frac{4}{\pi}\cos \omega t - \frac{4}{3\pi}\cos 3\omega t + \cdots$$

3. Obtain Fourier series for the following wave form.



* For the above problem. we can take function in 2 ways. 1. splitting $0-2\pi$ into $(0-\pi)+(\pi-2\pi)$.

$$\int_0^{2\pi} f(t)\,dt = \int_0^{\pi} f(t) + \int_{\pi}^{2\pi} f(t)$$

2. Take only one function with limits $(-\pi$ to $\pi)$

$$\int_{-\pi}^{\pi} f(t)\,dt.$$

(i) T = one cycle $= \pi-(-\pi) = 2\pi = T_0$

$$a_0 = \frac{1}{T_0}\int_{-\pi}^{\pi} f(t)\,dt$$

$$\omega = \frac{2\pi}{2\pi} = 1$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{y_2-y_1}{x_2-x_1}\right)t\,dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{A-(-A)}{\pi-(-\pi)}\,t\,dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{2A}{2\pi}\,t\,dt$$

$$= \frac{A}{2\pi^2}\left[\frac{t^2}{2}\right]_{-\pi}^{\pi}$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi}\left(\frac{A}{\pi}\right)t\cos n\omega t\,dt$$

$$= \frac{A}{\pi^2}\int_{-\pi}^{\pi} t\cos n\omega t\,dt$$

$t \searrow \cos nt$, $1 \uparrow \frac{\sin nt}{n}$

$$= \frac{A}{\pi^2}\int_{-\pi}^{\pi} t\cos nt\,dt$$

$$= \frac{A}{\pi^2}\left[\left(\frac{t\sin nt}{n}\right)_{-\pi}^{\pi} - \frac{1}{n}\int_{-\pi}^{\pi}\sin nt\,dt\right]$$

$$= \frac{A}{\pi^2}\left[\frac{-1}{n}\left(-\frac{\cos nt}{n}\right)_{-\pi}^{\pi}\right]$$

$$= \frac{-A}{n^2\pi^2}\left[-\cos n\pi + \cos n\pi\right]$$

$$= \frac{2A}{n^2\pi^2}\left[\cos n\pi - \cos n\pi\right]$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi}\left(\frac{A}{\pi}\right)t\sin n\omega t\,dt$$

$$= \frac{A}{\pi^2}\int_{-\pi}^{\pi} t\sin nt\,dt$$

$$= \frac{A}{\pi^2}\left[\left(-\frac{t\cos nt}{n}\right)_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi}\cos nt\,dt\right]$$

$$= \frac{A}{\pi^2}\left[\left(-\frac{t\cos nt}{n}\right)_{-\pi}^{\pi} + \frac{1}{n^2}\left(\sin nt\right)_{-\pi}^{\pi}\right]$$

$$= \frac{A}{\pi^2}\left[\frac{-\pi\cos n\pi}{n} - \frac{\pi\cos n\pi}{n} + \frac{1}{n^2}\left[\sin n\pi + \sin n\pi\right]\right]$$

$$= \frac{-2A\pi\cos n\pi}{\pi^2 n}$$

$$\boxed{b_n = \frac{-2A}{n\pi}\cos n\pi}$$

$$b_1 = \frac{2A}{\pi},\quad b_2 = \frac{-A}{\pi},\quad b_3 = \frac{2A}{3\pi}$$

Fourier expansion is

$$f(t) = \frac{2A}{\pi}\sin\omega t - \frac{A}{\pi}\sin 2\omega t + \frac{2A}{3\pi}\sin 3\omega t - \cdots\cdots$$

Fourier Transform:

→ Fourier transform also used to find out the solution for electrical circuit by using non-sinusoidal, non-periodic and periodic wave forms.

→ It is represented by symbol $f(j\omega)$. Then,

$$\boxed{f(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt}$$

properties:

1. Linearity: If $f(j\omega)$ is the fourier transform of time function $f(t)$. Then it satisfies the following relation $a_1 f_1(t) + a_2 f_2(t) = a_1 F_1(j\omega) + a_2 F_2(j\omega)$.

2. Time Scaling property: If $f(t)$ is the time function and $f(j\omega)$ is the fourier transform function, Then, $F[f(at)] = \frac{1}{|a|}\left(\frac{F(j\omega)}{a}\right)$

3. Time Shifting property: If $f(t)$ is the time function and it is shifted to t_0 and $F(j\omega)$ is the fourier transform, Then $F[f(t-t_0)] = e^{-j\omega t_0} F(j\omega)$.

4. Frequency shifting property: $F(j\omega)$ is the fourier transform function and $f(t)$ is the time function, and w_0 is the frequency shifting. Then

$$F[f(t)\, e^{j\omega_0 t}] = F[j(\omega-\omega_0)]$$

5. Modulation property: $F(j\omega)$ is the fourier transform function and $f(t)$ is the time function, Then modulation property $F[f(t)\cos\omega_0 t] = \frac{1}{2}[F(j(\omega-\omega_0)) + F(j(\omega+\omega_0))]$

lly $F[f(t)\sin\omega_0 t] = \frac{1}{2j}[F(j(\omega-\omega_0)) - F(j(\omega+\omega_0))]$.

6. Time differentiation property: If $F(j\omega)$ is the fourier transform function and $f(t)$ is the time function, Then $F\left[\frac{d}{dt} f(t)\right] = (j\omega) F(j\omega)$.

7. Frequency differentiation property: If $F(j\omega)$ is the Fourier transform function and $f(t)$ is the time function. Then,

$$\frac{d}{dt} F(j\omega) = F\left[(-jt) f(t)\right]$$

8. Complex Translation property: If $F(j\omega)$ is the fourier transform function and $f(t)$ is the time function, Then,

$$F\left[f(t)e^{at}\right] = F(j\omega - a)$$

$$\text{lly} \quad F\left[f(t)e^{-at}\right] = F(j\omega + a)$$

9. Convolution property: If $F(j\omega)$ is the fourier transform function and $f(t)$ is the time function, Then,

$$F\left[f_1(t) \cdot f_2(t)\right] = F_1(j\omega) \times F_2(j\omega).$$

1. Find the Fourier transform of a single triangular pulse as shown in the figure.

$$F(j\omega) = \int_{-T/2}^{T/2} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T/2}^{0} f(t)\, e^{-j\omega t}\, dt + \int_{0}^{T/2} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T/2}^{0} \frac{y_2 - y_1}{x_2 - x_1} t\; e^{-j\omega t} dt + \int_{0}^{T/2} \frac{y_2 - y_1}{x_2 - x_1} t\; e^{-j\omega t} dt$$

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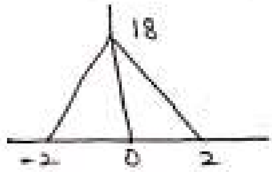
Q. Find the fourier transform of a single triangular pulse as shown in the figure.



Sol: $F(j\omega) = \int_{-2}^{2} f(t) e^{-j\omega t} dt$

$$= \int_{-2}^{0} f(t) e^{-j\omega t} dt + \int_{0}^{2} f(t) e^{-j\omega t} dt$$

$$= \int_{-2}^{0} \left(\frac{y_2-y_1}{x_2-x_1}\right)(t+2) e^{-j\omega t} dt + \int_{0}^{2} \left(\frac{y_2-y_1}{x_2-x_1}\right)(t-2) e^{-j\omega t} dt$$

$$= \int_{-2}^{0} \frac{18}{2}(t+2) e^{-j\omega t} dt + \int_{0}^{2} \frac{-18}{2}(t-2) e^{-j\omega t} dt$$

$$= 9\left\{ \int_{-2}^{0} (t+2) e^{-j\omega t} dt + \int_{0}^{2} -(t-2) e^{-j\omega t} dt \right.$$

$$= 9\left\{ \left[(t+2)\frac{e^{-j\omega t}}{-j\omega} - \int \frac{e^{-j\omega t}}{-j\omega} dt \right]_{-2}^{0} - \left[(t-2)\frac{e^{-j\omega t}}{-j\omega} - \int \frac{e^{-j\omega t}}{-j\omega} \right]_{0}^{2} \right\}$$

$$= 9\left\{ \left[(t+2)\frac{e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{(-j\omega)^2} \right]_{-2}^{0} - \left[(t-2)\frac{e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{(-j\omega)^2} \right]_{0}^{2} \right\}$$

$$= 9\left\{ \left(\frac{-2}{j\omega} - \frac{1}{(j\omega)^2} + \frac{e^{j2\omega}}{(j\omega)^2} \right) - \left(-\frac{e^{-j2\omega}}{(j\omega)^2} - \frac{2}{j\omega} + \frac{1}{(j\omega)^2} \right) \right\}$$

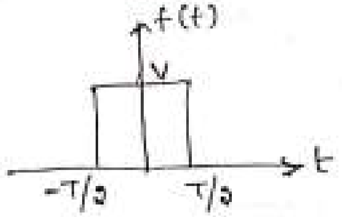
$$= 9\left\{ \frac{e^{j2\omega}}{(j\omega)^2} + \frac{e^{-j2\omega}}{(j\omega)^2} - \frac{2}{(j\omega)^2} \right\}$$

$$= 9\left\{ \frac{e^{j2\omega} + e^{-j2\omega}}{(j\omega)^2} - \frac{2}{(j\omega)^2} \right\}$$

$$= 9\left\{ \frac{(e^{j2\omega} + e^{-j2\omega}) - 2}{(j\omega)^2 \times \frac{4}{4}} \right\}$$

$$= 36\left\{ \frac{e^{-j2\omega} + e^{j2\omega}}{(j2\omega)^2} - \frac{2}{(j2\omega)^2} \right\}$$

3. Find the fourier transform for the following wave form.



$$\sin w = \frac{e^{jw} - e^{-jw}}{2j}$$

$$\cos \omega = \frac{e^{j\omega} + e^{-j\omega}}{2}$$

Sol: $F(j\omega) = \int_{-T/2}^{T/2} f(t) e^{-j\omega t} dt$

$= \int_{-T/2}^{0} f(t) e^{-j\omega t} + \int_{0}^{T/2} f(t) e^{-j\omega t} dt$

$= \int_{-T/2}^{0} V e^{-j\omega t} dt + \int_{0}^{T/2} V e^{-j\omega t} dt$

$= V \left\{ \left(\frac{e^{-j\omega t}}{-j\omega} \right)_{-T/2}^{0} + \left(\frac{e^{-j\omega t}}{-j\omega} \right)_{0}^{T/2} \right\}$

$= V \left\{ \frac{-1}{j\omega} - \frac{e^{j\frac{T}{2}\omega}}{-j\omega} + \frac{e^{-j\omega\frac{T}{2}}}{-j\omega} + \frac{1}{j\omega} \right\}$

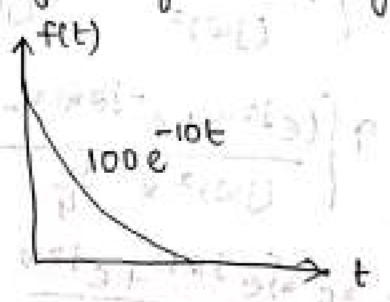
$= V \left\{ \frac{e^{j\omega\frac{T}{2}}}{j\omega} - \frac{e^{-j\omega\frac{T}{2}}}{j\omega} \right\}$

$= V \left\{ \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\omega \times \frac{2}{2}} \right\}$

$= 2V \left\{ \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j\omega} \right\}$

$= 2V \left(\frac{\sin \omega T/2}{\omega} \right)$

4. Obtain Fourier transform of following figure.



Sol:
$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} f(t)\, e^{-j\omega t}\, dt + \int_{0}^{\infty} f(t) e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} (0)\, e^{-j\omega t}\, dt + \int_{0}^{\infty} 100\, e^{-10t}\, e^{-j\omega t}\, dt$$

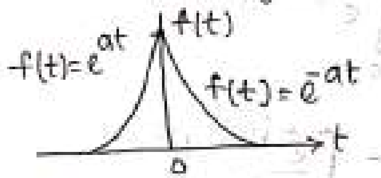
$$= 100 \int_{0}^{\infty} e^{-(10+j\omega)t}\, dt$$

$$= 100 \left[\frac{e^{-(10+j\omega)t}}{-(10+j\omega)}\right]_0^{\infty}$$

$$= 100 \times \left[0 - \left(\frac{-1}{10+j\omega}\right)\right]$$

$$\boxed{F(j\omega) = \frac{100}{10+j\omega}}$$

4. Find the fourier transform of the following signal.



Sol: From the figure,
$$f(t) = e^{-at}, \quad 0 < t < \infty$$
$$= e^{at}, \quad -\infty < t < 0$$

$$F(j\omega) = \int_{-\infty}^{0} f(t)\, e^{-j\omega t}\, dt + \int_{0}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{(-a-j\omega)t}\, dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{a-j\omega}\right]_{-\infty}^{0} + \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty}$$

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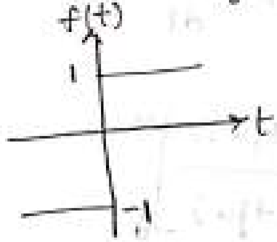
$$f(j\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$f(j\omega) = \frac{a+j\omega+a-j\omega}{a^2+\omega^2}$$

$$\boxed{f(j\omega) = \frac{2a}{a^2+\omega^2}}$$

5 Determine the fourier transform of the following signal.



Sol: Here, $f(t) = 1, \quad 0 < t < \alpha$

$\qquad = -1, \quad -\alpha < t < 0$

$$F(j\omega) = \int_{-\infty}^{0} f(t)\, e^{-j\omega t} dt + \int_{0}^{\infty} f(t)\, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{-j\omega t} dt - \int_{0}^{\infty} e^{-j\omega t} dt$$

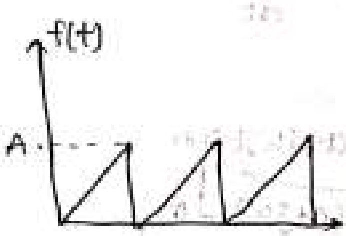
$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\infty}^{0} - \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{0}^{\infty}$$

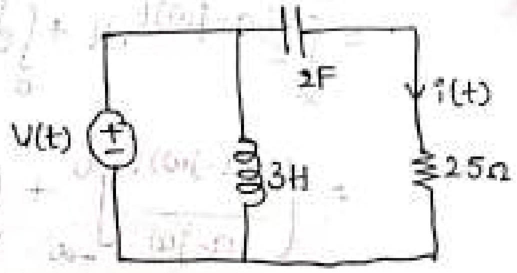
$$= \left(\frac{1}{-j\omega} + \left(\frac{1}{-j\omega}\right)\right)$$

$$= \frac{2}{j\omega}.$$

Fourier transform application to Electrical Circuits:

6. Voltage having the waveform shown in the figure is applied to a electrical circuit which is also shown in the figure. Then determine the current flowing through the resistor.





Sol: $i(t) = \dfrac{V(t)}{25 + \frac{1}{j\omega C}}$

$= \dfrac{V(t)}{25 + \frac{1}{2j\omega}}$

$V(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \dots + b_1 \sin \omega t + b_2 \sin 2\omega t + \dots$

$a_0 = \dfrac{1}{T_0} \int_0^T f(t)\, dt$

$= \dfrac{1}{T} \int_0^T \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(t-0)\, dt$

$= \dfrac{1}{T} \int_0^T \left(\dfrac{A-0}{T-0}\right) t\, dt$

$= \dfrac{A}{T^2} \int_0^T t\, dt$

$= \dfrac{A}{T^2} \left(\dfrac{t^2}{2}\right)_0^T$

$= \dfrac{A}{T^2} \left[\dfrac{T^2}{2}\right]$

$\boxed{a_0 = \dfrac{A}{2}}$

$a_n = \dfrac{2}{T_0} \int_0^T f(t) \cos n\omega t\, dt$ $\omega = 2\pi f = \dfrac{2\pi}{T}$

$= \dfrac{2}{T} \int_0^T \dfrac{A}{T} t \cos \dfrac{2n\pi}{T} t\, dt$ $t = 1$

$\cos \frac{2n\pi}{T}, \dfrac{\sin \frac{2n\pi}{T}}{\left(\frac{2n\pi}{T}\right)}$

$= \dfrac{2A}{T^2} \int_0^T t \cos \dfrac{2n\pi}{T} t\, dt$

$= \dfrac{2A}{T^2} \left[\dfrac{t \cdot T}{2n\pi} \sin \dfrac{n2\pi}{T} t - \dfrac{T}{2n\pi} \int \sin \dfrac{2n\pi}{T} t\, dt\right]_0^T$

$= \dfrac{2A}{T^2} \left[\dfrac{tT}{2n\pi} \sin \dfrac{2n\pi}{T} t + \left(\dfrac{T}{2n\pi}\right)^2 \cos \dfrac{2n\pi}{T} t\right]_0^T$

$= \dfrac{2A}{T^2} \left[0 + \left(\dfrac{T}{2n\pi}\right)^2 \cos 2n\pi - \left(\dfrac{T}{2n\pi}\right)^2 \cos 0\right]$

For $n=1$, $a_1 = \frac{2A}{T^2}\left[\left(\frac{T}{2\pi}\right)^2 - \left(\frac{T}{2\pi}\right)^2\right]$

$= 0$

For $n=2$, $a_2 = 0$.

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$$

$$= \frac{2}{T}\int_0^T \frac{A}{T} t \sin \frac{2n\pi}{T} t\, dt$$

$$= \frac{2A}{T^2}\int_0^T t \sin \frac{2n\pi}{T} t\, dt$$

$$= \frac{2A}{T^2}\left[t\left(\frac{-\cos\frac{2n\pi}{T}t}{\left(\frac{2n\pi}{T}\right)}\right) + \int \frac{\cos\frac{2n\pi}{T}t}{\frac{2n\pi}{T}}dt\right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{tT}{2n\pi}\left(-\cos\frac{2n\pi}{T}t\right) + \left(\frac{T}{2n\pi}\right)^2 \sin\frac{2n\pi}{T}t\right]_0^T$$

$$= \frac{2A}{T^2}\left[\frac{T^2}{2n\pi}(-1)\right]$$

$$= \frac{-A}{n\pi}$$

For $n=1$, $b_1 = -\frac{A}{\pi}$; For $n=2$, $b_2 = -\frac{A}{2\pi}$

$$V(t) = \frac{A}{2} - \frac{A}{\pi}\sin\omega t - \frac{A}{2\pi}\sin 2\omega t$$

$$i(t) = \frac{V(t)}{25 + \frac{1}{2j\omega}}$$

$$= \frac{\frac{A}{2} - \frac{A}{\pi}\sin\omega t - \frac{A}{2\pi}\sin 2\omega t}{25 + \frac{1}{2j\omega}}$$

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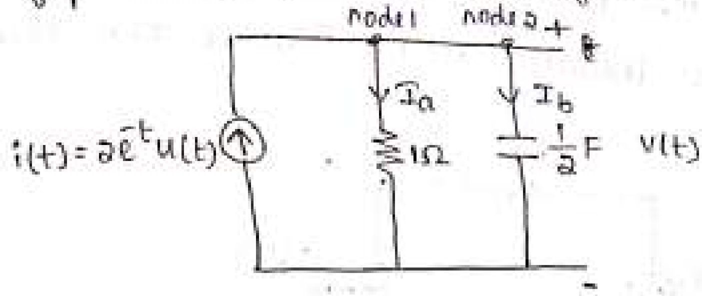
7. Find the response voltage in the network shown in the figure. Use Fourier transform method.



Using KCL,

$$i(t) = i_a + i_b$$

$$2e^{-t}u(t) = \frac{v(t)}{1} + \frac{v(t)}{X_C}$$

$$2e^{-t}u(t) = v(t) + \frac{v(t)}{\left(\frac{1}{j\omega C}\right)}$$

$$2e^{-t}u(t) = v(t) + j\omega C\, v(t)$$

$$2e^{-t}u(t) = v(t)[1 + j0.5\omega]$$

Apply Fourier transform

$$2 \times \frac{1}{j\omega+1} = V(j\omega)[1 + j0.5\omega]$$

$$V(j\omega) = \frac{2}{(j\omega+1)(1+j0.5\omega)}$$

$$V(j\omega) = \frac{2}{(1+j\omega)(\frac{1}{2})(2+j\omega)}$$

$$V(j\omega) = \frac{4}{(1+j\omega)(2+j\omega)}$$

Using partial fractions.

$$V(j\omega) = \frac{A}{1+j\omega} + \frac{B}{2+j\omega}$$

$A = 4$, $B = -4$

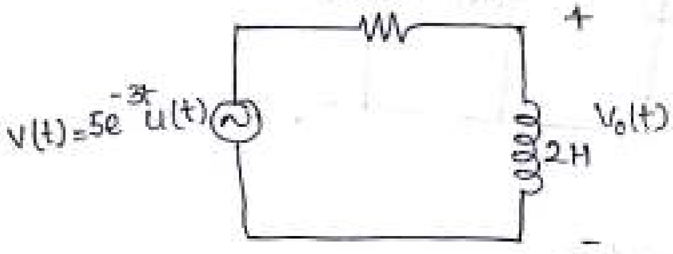
$$V(j\omega) = \frac{4}{1+j\omega} - \frac{4}{2+j\omega}$$

Inverse ~~Lapla~~ Fourier transform

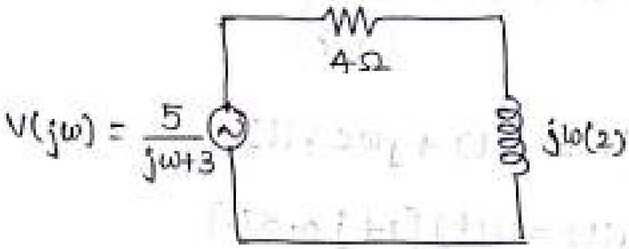
$$v(t) = 4e^{-t}u(t) - 4e^{-2t}u(t).$$

8. For input voltage $v(t) = 5e^{-3t}u(t)$ is applied to a series R-L circuit with 4Ω resistor & $L = 2H$. Find the output voltage $V_0(t)$ across inductor using Fourier transform method.

Sol:



For the above circuit first convert into Fourier transform circuit, then apply voltage divison method.



Using voltage division rule,

voltage across inductor $[V_0(t)] = \dfrac{j(2\omega)}{4+j(2\omega)} \times \dfrac{5}{j\omega+3}$

$$= \frac{j10\omega}{[4+j(2\omega)](j\omega+3)}$$

Using partial fractions,

$$V_0(j\omega) = \frac{A}{4+j2\omega} + \frac{B}{j\omega+3}.$$

$A = -20$, $B = 20$

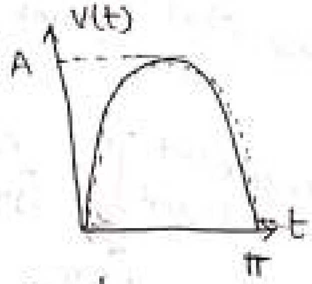
$$V_0(j\omega) = \frac{-20}{4+j2\omega} + \frac{20}{j\omega+3}$$

$$V_0(j\omega) = \frac{-10}{2+j\omega} + \frac{20}{j\omega+3}$$

Using inverse Fourier transform

$$V_0(t) = 10e^{-2t}u(t) + 20e^{-3t}u(t).$$

Q. Find the fourier transform of the sine pulse shown in the figure and sketch the magnitude & phase spectra.



Sol: Magnitude spectra:

We know that,

exponential form $C_n = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\, e^{-jn\omega t}\, dt$

$$C_n = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\,[\cos n\omega t - j\sin n\omega t]\, dt$$

$$Re[C_n] + Img[C_n] = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\cos n\omega t\, dt - \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\sin n\omega t\, dt$$

$$Re[C_n] = \frac{1}{T_0}\int_{t_1}^{t_1+T} f(t)\cos n\omega t\, dt$$

$$Img[C_n] = \frac{-1}{T_0}\int_{t_1}^{t_1+T} f(t)\sin n\omega t\, dt$$

Magnitude spectra $|C_n| = \sqrt{Re[C_n]^2 + Img[C_n]^2}$

Phase spectra:

$$\phi = Tan^{-1}\left(\frac{Img[C_n]}{Re[C_n]}\right)$$

$$= Tan^{-1}\left(\frac{-\int_{t_1}^{t_1+T} f(t)\sin n\omega t\, dt}{\int_{t_1}^{t_1+T} f(t)\cos n\omega t\, dt}\right)$$

Now,

$$V(t) = \frac{1}{T}\int_0^T f(t)\, e^{-j\omega t}\, dt = I$$

$$= \frac{1}{\pi}\int_0^{\pi} f(t)\, e^{-j\omega t}\, dt \quad \text{if } \omega = 1$$

$$= \frac{A}{\pi}\int^{\pi} \sin n\omega t\, e^{-j\omega t}\, dt$$

$$v(t) = \frac{A}{\pi}\left[\sin t \frac{e^{-j\omega t}}{-j\omega} - \int \cos t \frac{e^{-j\omega t}}{-j\omega}\, dt\right]_0^\pi$$

$$I = \frac{A}{\pi}\left[\sin t \frac{e^{-j\omega t}}{-j\omega} - \cos t \frac{e^{-j\omega t}}{(-j\omega)^2} + \int -\sin t \frac{e^{-j\omega t}}{(-j\omega)^2}\right]_0^\pi$$

$$I = \frac{A}{\pi}\left[\sin t \frac{e^{-j\omega t}}{-j\omega} - \cos t \frac{e^{-j\omega t}}{(-j\omega)^2}\right]_0^\pi - \frac{A}{(j\omega^2)\pi}\int_0^\pi \sin t\, e^{-j\omega t} dt$$

$$I = \frac{A}{\pi}\left[\sin t \frac{e^{-j\omega t}}{-j\omega} - \cos t \frac{e^{-j\omega t}}{(j\omega)^2}\right]_0^\pi - \frac{1}{(j\omega)^2} I$$

$$I = \frac{A}{\pi}\left[\frac{e^{-j\omega\pi}}{(j\omega)^2} + \frac{1}{(j\omega)^2}\right] + \frac{I}{\omega^2}$$

$$I\left(1-\frac{1}{\omega^2}\right) = \frac{A}{\pi}\left(\frac{e^{-j\omega\pi}+1}{(j\omega)^2}\right)$$

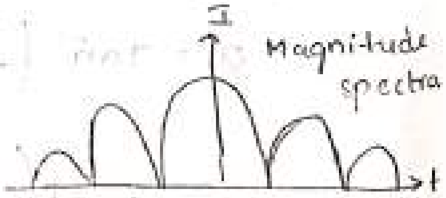
$$I = \frac{A}{\pi}\left(\frac{1+e^{-j\omega\pi}}{-\omega^2\left(\frac{\omega^2-1}{\omega^2}\right)}\right)$$

$$I = \frac{A}{\pi}\left(\frac{1+e^{-j\omega\pi}}{(1-\omega^2)}\right)$$

Magnitude spectra:

$$(I) = \frac{A}{\pi(1-\omega^2)}(1+\cos\omega\pi - j\sin\omega\pi)$$

$$|I| = \sqrt{\frac{A}{\pi(1-\omega^2)}\left[(1+\cos\omega\pi)^2+(\sin\omega\pi)^2\right]}$$

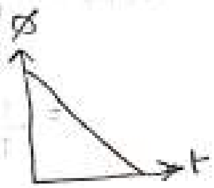


Phase spectra:

$$\phi = \tan^{-1}\left(\frac{-\sin\omega\pi}{\frac{1+\cos\omega\pi \times 2}{2}}\right)$$

$$= \tan^{-1}\left(\frac{-2\sin\frac{\omega\pi}{2}\cos\frac{\omega\pi}{2}}{\cos^2\frac{\omega\pi}{2}\cdot 2}\right)$$

$$= \tan^{-1}\left(\frac{-\sin\frac{\omega\pi}{2}}{\cos\frac{\omega\pi}{2}}\right)$$



$$= \tan^{-1}\left(-\tan\frac{\omega\pi}{2}\right)$$

$$\boxed{\phi = -\frac{\omega\pi}{2}}$$

NSS Unit-5

Fourier Transforms

Fourier Theorem – Trigonometric form and exponential form of fourier Series – conditions of symmetry – line spectra and phase angle spectra – Analysis of electrical circuits excited by non-sinusoidal sources of periodic waveforms – Fourier Integrals and Fourier transforms – Properties of Fourier transforms and application to electrical circuits.

x ——— x

Jean Baptiste Joseph Fourier (1768–1830) developed a technique of analyzing non sinusoidal waveforms applicable to a wide range of Engineering problems. Later it was applied to analyze electric circuits excited by non sinusoidal wave forms.

Fourier Theorem

Fourier Theorem which applies to all the functions that normally arise in physical and engineering application to which Fourier Series can be obtained as follows.

A function f(t) can be written as

$$f(t) = \frac{a_0}{2} + a_1 \cos\omega t + a_2 \cos 2\omega t + \dots + b_1 \sin\omega t + b_2 \sin 2\omega t + \dots$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t\right) \quad \text{— (1)}$$

and the coefficient a_n and b_n are given by

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt \quad \text{— (2)}$$

$$a_n = \frac{2}{T}\int_0^T f(t) \cos n\omega t\,dt \quad \text{— (3)}$$

$$b_n = \frac{2}{T}\int_0^T f(t) \sin n\omega t \quad \text{— (4)}$$

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It is to be noted that the Fourier series can be written in a more compact form either in sin or cos term by combining the sin and cos term as follows.

Let us consider the n^{th} harmonic term

$$f_n(t) = a_n \cos n\omega t + b_n \sin n\omega t$$

$$= \sqrt{a_n^2+b_n^2}\left[\frac{a_n}{\sqrt{a_n^2+b_n^2}}\cos n\omega t + \frac{b_n}{\sqrt{a_n^2+b_n^2}}\sin n\omega t\right]$$

$$= A_n\left[\cos\theta_n \cos n\omega t + \sin\theta_n \sin n\omega t\right]$$

$$= A_n\left[\cos(n\omega t - \theta_n)\right]$$

∴ Fourier series can be written as [in terms of cos form]

$$\boxed{f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t - \theta_n)}$$

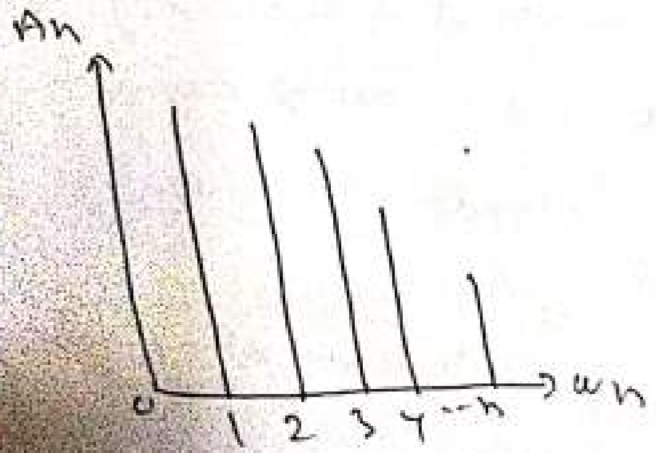
where $\theta = \tan^{-1}\frac{b_n}{a_n}$

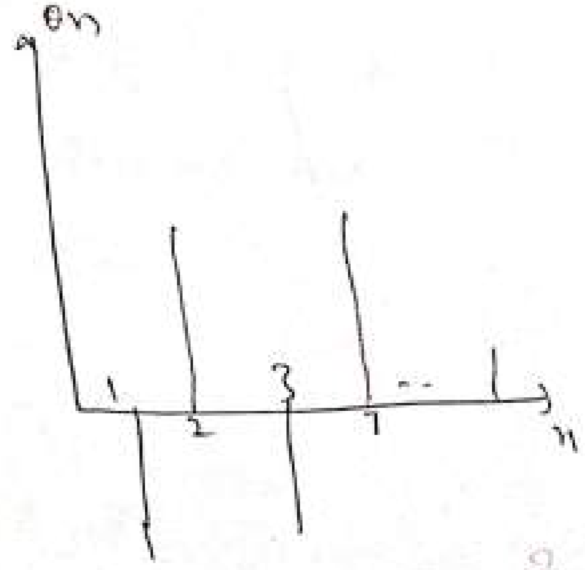
∴ Fourier series in terms of sin form

$$\boxed{f(t) = A_0 + \sum A_n \sin(n\omega t - \theta_n)}$$

where $\theta_n = \tan^{-1}\frac{a_n}{b_n}$

where $\omega = \frac{2\pi}{T}$





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Exponential form of Fourier series

Fourier series can be written as

$$f(t) = a_0 + \sum_{n=1}^{\alpha} A_n \cos(n\omega_0 t - \theta_n)$$

where $\omega = \frac{2\pi}{T_0}$

$n=1$, one cycle covers T seconds while $A_1 \cos(\omega_0 t - \theta_1)$ is termed as fundamental harmonic

$n=2$, Two cycle covers T_0 seconds while $A_2 \cos(2\omega_0 t - \theta_2)$ is termed as 2nd harmonic

In the continued fashion

$n=K$, K cycle covers within T_0 seconds while $A_K \cos(K\omega_0 t - \theta_K)$ is termed as K^{th} harmonic.

Using Euler identity

$$f(t) = a_0 + \sum_{\substack{n=-\alpha \\ n \neq 0}}^{\alpha} C_n e^{jn\omega_0 t}$$

where C_n = complex Fourier coefficients

$$= a_0 + \sum_{n=1}^{\alpha} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

Fourier coefficient C_n can be evaluated as follows

$$\therefore f(t) = \sum_{n=-\alpha}^{\alpha} C_n e^{jn\omega_0 t}$$

multiplying both sides by $e^{-jK\omega_0 t}$ and integrating over the interval t_1 to $t_1 + T_0$, we get

$$\int_{t_1}^{t_1+T_0} f(t)\, e^{-jK\omega_0 t}\, dt = \int_{t_1}^{t_1+T_0} \left(\sum_{n=-\alpha}^{\alpha} C_n e^{jn\omega_0 t} \right) e^{-jK\omega_0 t}\, dt$$

$$= C_k T_0$$

Since $\int_{t_1}^{t_1+T_0} e^{j(n-k)\omega_0 t} dt = 0$ for $n \neq k$
$= T_0$ for $n = k$

∴ The fourier coefficient are defined by the expression

$$C_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t) e^{-jn\omega_0 t} dt \quad — ①$$

The above expression is the exponential form of fourier series.

Trigonometric form of fourier series

From eq ①, we can write

$$2C_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) e^{-jn\omega_0 t} dt$$

$$= \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \left[\cos n\omega_0 t - j\sin n\omega_0 t\right] dt$$

$$= \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \cos n\omega_0 t \, dt - j \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \sin n\omega_0 t \, dt \quad — ②$$

However C_n being a complex coefficient

$$2C_n = a_n - j b_n \quad — ②$$

comparing eq ① & ②

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \cos n\omega_0 t \, dt \quad — ③, \qquad b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \sin n\omega_0 t \, dt \quad — ④$$

Also from eq ①, c_0 being written here as a_0 Then

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)\,dt \quad — (5)$$

Symmetry in Fourier Series

There are three types of symmetry

1. Even function symmetry :

A function is said to be even if

$$f(t) = f(-t)$$

An even function is symmetrical about the vertical axis.

In order to determine the fourier coefficients, the condition of symmetry being applied

Let $t_1 = -\frac{T_0}{2}$

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)\,dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{-\frac{T_0}{2}+T_0} f(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{\frac{T_0}{2}} f(t)\,dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{0} f(t)\,dt + \frac{1}{T_0}\int_{0}^{T/2} f(t)\,dt$$

Now change the variable of the first integral so that $t = -x$ in the first integral. Then and $x = \frac{T_0}{2}$ to 0

$$f(-x) = f(x) \quad , \quad dt = d(-x) = -dx$$

$\therefore$

$$a_0 = \frac{1}{T_0}\int_{\frac{T_0}{2}}^{0} f(x)(-dx) + \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} f(t)\,dt$$

$$= \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} f(x)\,dx + \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} f(t)\,dt$$

$$\boxed{a_0 = \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\,dt}$$

Similarly

$$a_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{0} f(t)\cos n\omega_0 t\,dt + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\int_{\frac{T_0}{2}}^{0} f(x)\cos(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(x)\cos(n\omega_0 x)\,dx + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\cos n\omega_0 t\,dt$$

$$\boxed{a_n = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} f(t)\cos n\omega_0 t\,dt}$$

Also

$$b_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{0} f(t)\sin n\omega_0 t\,dt + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\sin n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\int_{\frac{T_0}{2}}^{0} f(x)\sin(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\sin n\omega_0 t\,dt$$

$$= -\frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(x)\sin n\omega_0 x\,dx + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\sin n\omega_0 t\,dt$$

$$\boxed{b_n = 0}$$

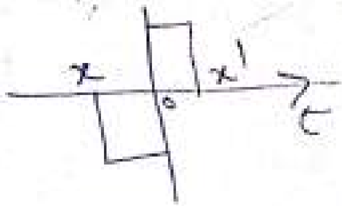
(2) <u>Odd function Symmetry</u>

A function is said to be odd if

$$f(t) = \cancel{f(t)} - f(-t)$$

$$a_0 = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{0} f(t)\,dt + \frac{1}{T_0}\int_0^{\frac{T_0}{2}} f(t)\,dt$$

$$a_0 = \frac{1}{T_0}\int_{\frac{T_0}{2}}^{0} -f(x)\,(-dx) + \frac{1}{T_0}\int_0^{\frac{T_0}{2}} f(t)\,dt$$



$$= -\frac{1}{T_0}\int_0^{\frac{T_0}{2}} f(x)\,dx + \frac{1}{T_0}\int_0^{\frac{T_0}{2}} f(t)\,dt$$

$$\boxed{a_0 = 0}$$

IIly

$$a_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{0} -f(x)\cos(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\cos n\omega_0 t\,dt$$

$$= -\frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(x)\cos n\omega_0 x\,dx + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\cos n\omega_0 t\,dt$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{0} -f(x)\sin(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\sin n\omega_0 t\,dt$$

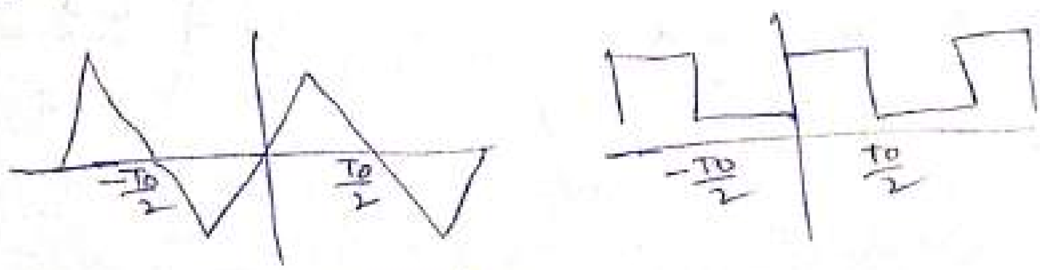
$$= \frac{2}{T_0}\int_0^{\frac{T_0}{2}} + f(x)\sin(n\omega_0 x)\,dx + \frac{2}{T_0}\int_0^{\frac{T_0}{2}} f(t)\sin n\omega_0 t\,dt$$

$$\boxed{b_n = \frac{4}{T_0}\int_0^{\frac{T_0}{2}} f(t)\sin n\omega_0 t\,dt}$$

(3) Half wave Symmetry

A function is said to be halfwave symmetry if

$$f(t) = -f\left(t - \frac{T_0}{2}\right)$$



half Symmetry

here

$$t - \frac{T_0}{2} = x$$

$$t = \frac{T_0}{2} + x$$

$a_0 = 0$

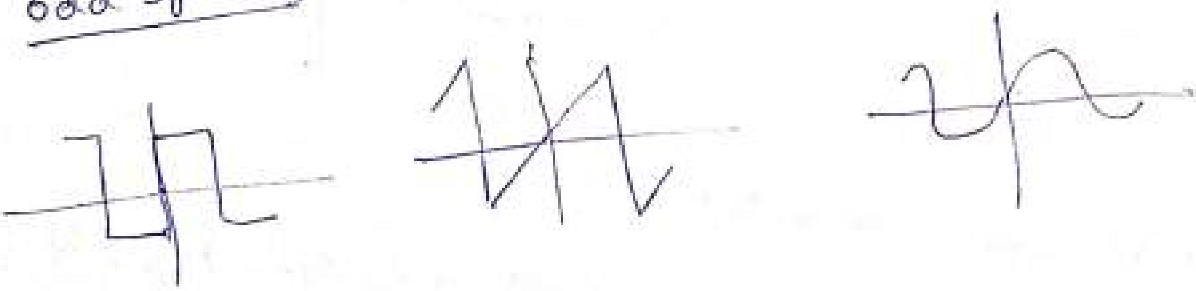
$b_n = a_n = 0$ when n is even

and $a_n = \frac{4}{T_0} \int_0^{\frac{T_0}{2}} f(t) \cos n\omega_0 t \, dt$ when n is odd

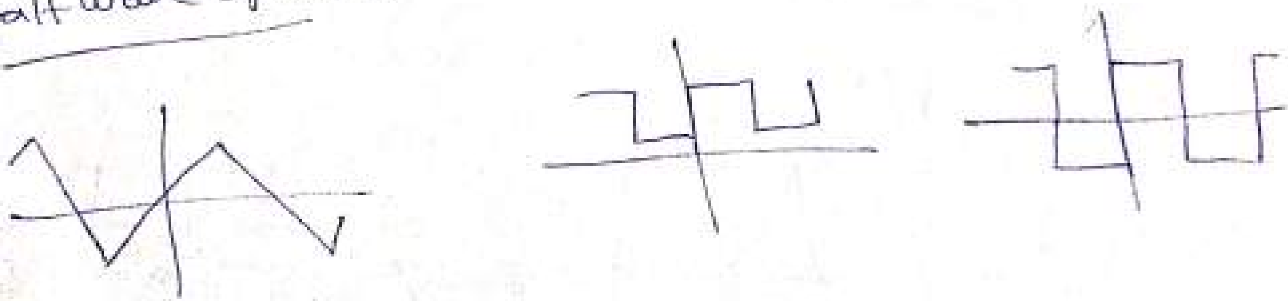
$b_n = \frac{4}{T_0} \int_0^{\frac{T_0}{2}} f(t) \sin n\omega_0 t \, dt$ when n is odd.

Even symmetry

odd symmetry



Halfwave symmetry



Frequency spectrum

- The frequency spectrum of the function f(t) expressed as a Fourier series consists of a plot of the amplitude of the harmonics vs frequency (commonly known as amplitude spectrum.
- The plot of phase of harmonics vs frequency which we call the phase spectrum
- However, the frequency components being discrete, the spectra are called line spectra and it illustrates the frequency content of the signal. Hence $c_0 = a_0$ and $c_n = \sqrt{a_n^2 + b_n^2}$

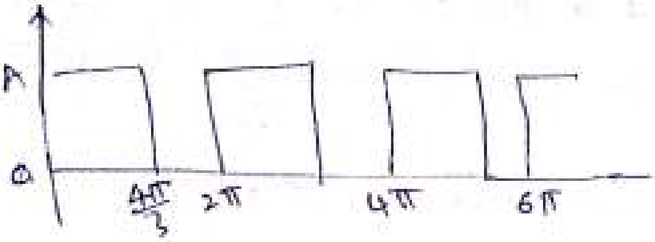
Properties of Fourier analysis

① For an even waveform, all the terms of the Fourier series are cosine terms ie
(terms with a_0 coefficients do exist)
and no sine terms are present since $b_n = 0$.

② For an odd wave form, the Fourier series contains only sine terms. and there is no avg value $a_0 = 0$ and no cosine terms since $a_n = 0$.

③ If the given waveform is of halfwave symmetry, only odd harmonics are present in the series. The series would contain both sine and cosine terms when n is odd. The average value is zero.

(1) obtain the coefficients of the exponential Fourier series for the waveform shown in fig.



$$C_n = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

Sol

$$C_n = \frac{1}{T_0}\int_0^{T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} f(t)\, e^{-jn\omega_0 t}\, dt$$

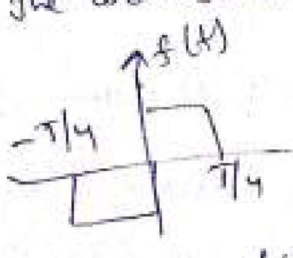
$$= \frac{1}{2\pi}\int_0^{\frac{4\pi}{3}} A\cdot e^{-jn\left(\frac{2\pi}{T_0}\right)t}\, dt$$

$$= \frac{1}{2\pi}\int_0^{4\pi/3} A\, e^{-jnt}\, dt$$

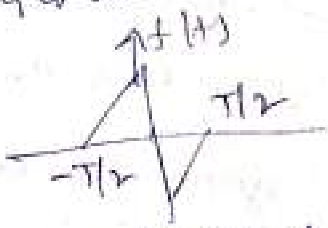
$$= \frac{jA}{2\pi n}\left[e^{-jn\frac{4\pi}{3}} - 1\right]$$

$$= \frac{jA}{2\pi n}\left[e^{-j\left(\frac{4n\pi}{3}\right)} - 1\right]$$

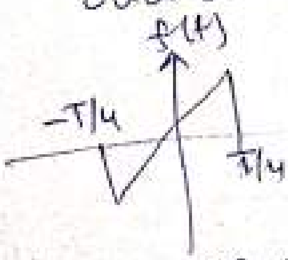
(2) Find the waveform symmetry of the following fig.



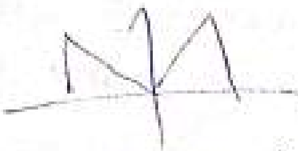
odd function



odd function

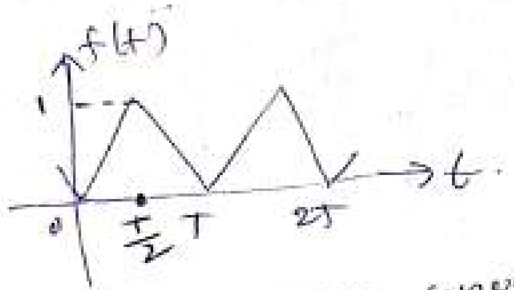


odd function



Even function

(3) Find the Amplitude of the 5th harmonic of the given waveform



Slop $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1}{T/2} = \frac{2}{T}$

Sol Given wave form is a even wave form, so only cosine terms would be present $a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos\left(\frac{2\pi}{T}\right)5t\,dt$

$$a_n = \frac{4}{T}\int_0^{T/2} \frac{2t}{T}\cos\left(\frac{2\pi}{T}\right)(5t)\,dt$$

$$= \frac{4}{T}\times\frac{2}{T}\int_0^{T/2} t\cos\frac{10\pi t}{T}\,dt$$

$$= \frac{8}{T^2}\left\{\left[t\,\frac{\sin\frac{10\pi t}{T}}{\left(\frac{10\pi}{T}\right)}\right]_0^{T/2} - \int_0^{T/2} 1\cdot\frac{\sin\frac{10\pi t}{T}}{\frac{10\pi}{T}}\,dt\right\}$$

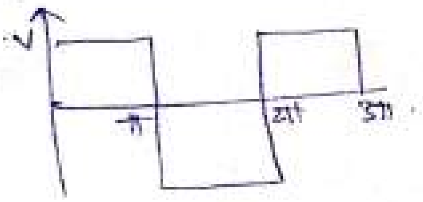
$$= \frac{8}{T^2}\left[\left(\frac{T}{2}\,\frac{\sin\frac{10\pi}{T}\times\frac{T}{2}}{\frac{10\pi}{T}} - 0\right) + \left(\frac{\cos\frac{10\pi t}{T}}{\frac{100\pi^2}{T^2}}\right)_0^{T/2}\right]$$

$$= \frac{8}{T^2}\left[\frac{T^2}{20\pi}(0) + \left(\frac{T^2}{100\pi^2}\times\cos\frac{10\pi T}{2T} - \frac{\cos 0}{\frac{100\pi^2}{T^2}}\right)\right]$$

$$= \frac{8}{T^2}\left[\frac{T^2}{100\pi^2}\left(\cos 5\pi - 1\right)\right]$$

$$= \frac{4}{25\pi^2}\left(\cos 5\pi - 1\right) = \frac{-8}{25\pi^2}$$

(4) A square wave form is shown in fig. obtain the fourier series



Sol here $0<\omega t<\pi$, $f(t)=V$, $\pi<\omega t<2\pi$, $f(t)=-V$

Hence given waveform posesses symmetry i.e odd symmetry.

The fourier series does not contain any cosine term and the average value would be zero.

$\therefore a_n=0, a_0=0$

$$a_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{0} f(t)\cos n\omega_0 t\,dt + \frac{2}{T_0}\int_{0}^{\frac{T_0}{2}} f(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{2\pi}\int_{0}^{\pi} V\cos n\omega t\, d(\omega t) + \frac{2}{2\pi}\int_{\pi}^{2\pi} -V\cos n\omega t\, d(\omega t)$$

$$= \frac{V}{\pi}\left[\left(\frac{1}{n}\sin n\omega t\right)_0^{\pi} - \left(\frac{1}{n}\sin n\omega t\right)_{\pi}^{2\pi}\right.$$

$$= \frac{V}{\pi}\left(\frac{1}{n}\sin n\pi - \frac{1}{n}\sin n(2\pi) + \frac{1}{n}\sin n\pi\right]$$

$$= 0$$

$$b_n = \frac{1}{\pi}\left[\int_{0}^{\pi} V\sin n\omega t\, d(\omega t) + \int_{\pi}^{2\pi} -V\sin n\omega t\, d(\omega t)\right.$$

$$= \frac{V}{\pi}\left[\left(\frac{-\cos n\omega t}{n}\right)_0^{\pi} + \left(\frac{1}{n}\cos n\omega t\right)_{\pi}^{2\pi}\right]$$

$$= \frac{V}{\pi n}\times\left(-\cos n\pi + \cos 0 + \cos n2\pi - \cos n\pi\right)$$

$$= \frac{2V}{n\pi}(1-\cos n\pi)$$

$n=1 \Rightarrow b_n = \frac{4V}{\pi}$

$n=2 \quad b_n = \frac{4V}{2\pi}$

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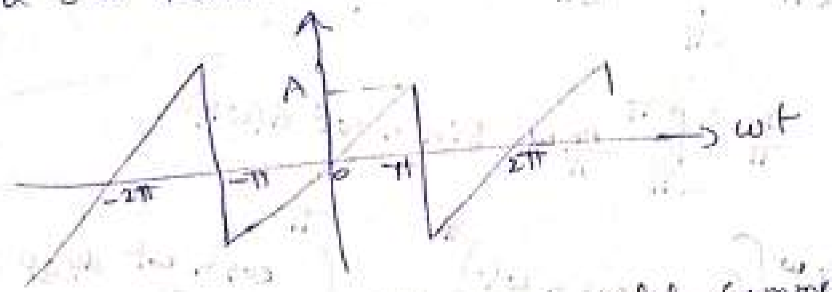
$\cos n\pi = -1$ for $n =$ odd
$= -1$ for $n =$ even

$$f(t) = \frac{4V}{\pi}\sin\omega t + \frac{4V}{3\pi}\sin 3\pi t + \frac{4V}{5\pi}\sin 5\pi t + \cdots$$

(5) Find the Fourier expansion of the given wave form



Sol: By inspection, it is a odd symmetry.

For odd symmetry. If the period 0 to 2π is considered, i.e. 0 to π & π to 2π.

Other way is
0 to π
π to 2π

$a_0 = 0$,

$a_n = 0$,

$$b_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\sin n(\omega t)\, d(\omega t)$$

$$= \frac{4}{2\pi}\int_0^{\pi}\frac{A}{\pi} t \sin n(\omega t)\, d(\omega t)$$

$$= \frac{4A}{2\pi}\times\frac{A}{\pi}\left[\left(-t\cdot\frac{\cos n(\omega t)}{n}\right)_0^{\pi} + \int_0^{\pi}\frac{\cos n\omega t}{n}\, d(\omega t)\right]$$

0

$$= -\frac{2A}{\pi^2}\times\frac{\pi}{n}\cos n\pi$$

$$b_n = -\frac{2A}{n\pi}\cos n\pi$$

$n = 1$, $b_1 = \frac{2A}{\pi}$, $n = 2$, $b_2 = -\frac{2A}{2\pi}$, $n = 3$, $b_3 = \frac{2A}{3\pi}$

Fourier series becomes

$$f(t) = \frac{2A}{\pi}\sin\omega t - \frac{2A}{2\pi}\sin 2\omega t + \frac{2A}{3\pi}\sin 3\omega t - \frac{2A}{4\pi}\sin 4\omega t + \cdots$$

(or)

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) \sin n\omega_0 t \, d(\omega t)$$

$$= \frac{2}{2\pi} \int_{-\pi}^{\pi} \frac{2A}{2\pi} t \sin n\omega t \, d(\omega t)$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{A}{\pi} \omega t \sin n \omega t \, d(\omega t)$$

$$= \frac{A\omega}{\pi^2} \left[\left(-t \cdot \frac{\cos n\omega t}{n} \right)_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{\cos n\omega t}{n} \, d(\omega t) \right]$$

$$= \frac{A\omega}{\pi^2} \left[-\pi \frac{\cos n\pi}{n} - \pi \frac{\cos n\pi}{n} + \left(\frac{\sin n\omega t}{n^2} \right)_{-\pi}^{\pi} \right]$$

$$\left(\frac{\sin n\omega t}{n^2} \right)_{-\pi}^{\pi} \rightarrow 0$$

$$= \frac{-2A\omega\pi \cos n\pi}{\pi^2 \times n} \qquad \left(\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \right)$$

$$= \frac{-2A}{\pi n} (\cos n\pi)$$

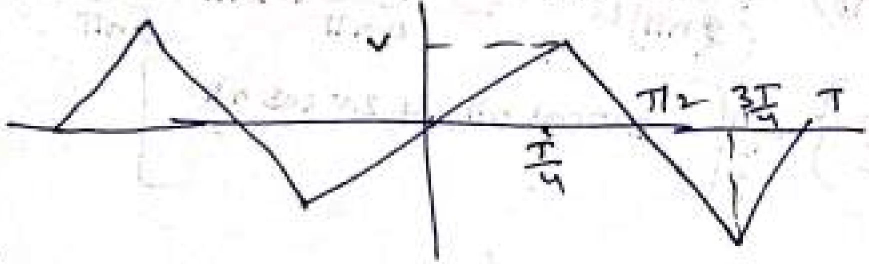
for $n = 1$, $b_1 = \frac{2A}{\pi}$

$n = 2$, $b_2 = \frac{-2A}{2\pi}$

$n = 3$, $b_3 = +\frac{2A}{3\pi}$ ...

$$f(t) = \frac{2A}{\pi} \left(\sin\omega t - \frac{1}{2} \sin 2\omega t + \frac{1}{3} \sin 3\omega t + \cdots \right)$$

→ obtain fourier series



Given function is odd function so only b_n appear

Sol

$$f(t) = \frac{4V}{T}t \ , \quad 0 < t < \frac{T}{4}$$

$$= -\frac{4V}{T}t + 2V \ , \quad \frac{T}{4} < t < \frac{T}{2}$$

$$b_n = \frac{4}{T}\left[\int_0^{T/4} \frac{4V}{T} t \sin n\omega t + \int_{T/4}^{T/2} -\frac{4V}{T} t \sin n\omega t\,dt + \int_{T/4}^{T/2} 2V \sin n\omega t\,dt\right]$$

but $\omega = \frac{2\pi}{T}$

$$= \frac{16V}{T^2}\left[\left(t \cdot \frac{-\cos n\omega t}{n\omega}\right)_0^{T/4} + \left(\frac{\sin n\frac{2\pi}{T}t}{\left(n \times \frac{2\pi}{T}\right)^2}\right)_0^{T/4}\right] + \left(t\frac{\cos n\omega t}{n\omega}\right)_{T/4}^{T/2} - \left(\frac{\sin n\frac{2\pi}{T}t}{\left(n\times\frac{2\pi}{T}\right)^2}\right)_{T/4}^{T/2} + \left[2V\frac{\cos n\omega t}{n\omega}\right]_{T/4}^{T/2}$$

$$= \frac{16V}{T^2}\left(-\frac{T}{4}\left(\frac{\cos n \cdot \frac{2\pi}{T}\frac{T}{4}}{n \times \frac{2\pi}{T}}\right) - (0)\frac{\cos 0}{n\times\frac{2\pi}{T}}\right) + \frac{\sin n\frac{2\pi}{T}\frac{T}{4}}{\left(n\times\frac{2\pi}{T}\right)^2}$$

$$+ \frac{T}{2}\frac{\cos n\frac{2\pi}{T}\frac{T}{2}}{n\times\frac{2\pi}{T}} - \frac{T}{4}\frac{\cos n\frac{2\pi}{T}\cdot\frac{T}{4}}{n\cdot\frac{2\pi}{T}}$$

$$+ \frac{\sin n\frac{2\pi}{T}\times\frac{T}{2}}{\left(n\times\frac{2\pi}{T}\right)^2} - \frac{\sin n\frac{2\pi}{T}\cdot\frac{T}{4}}{n\times\frac{2\pi}{T}}\Bigg]$$

$$-\frac{1}{n\frac{2\pi}{T}}\left(2V\cos n\frac{2\pi}{T}\frac{T}{2} + 2V\cos n\frac{2\pi}{T}\cdot\frac{T}{4}\right)$$

$$= \frac{16V}{T^2}\left[\left\{\frac{-T^2}{8n\pi}\left(\cos\frac{n\pi}{2} - 0\right) + \frac{T^2}{(2n\pi)^2}\sin\frac{n\pi}{2} + \frac{T^2}{4n\pi}\cos n\pi - \frac{T^2}{8n\pi}\cos\frac{n\pi}{2}\right.\right.$$

$$\left.\left. + \frac{T^2}{(2n\pi)^2}\sin\frac{n\pi}{2}\right\} + \frac{4}{T}\left[-2V\cos n\pi + 2V\cos\frac{n\pi}{2}\right]\right]$$

$$= \frac{\cancel{16}^{4}V}{T^2} \times \frac{T^2}{4n\pi}\left[-\frac{1}{2}\left(\cos\frac{n\pi}{2} - 0\right) + \frac{1}{n\pi}\sin\frac{n\pi}{2} + \cos n\pi - \frac{1}{2}\cos\frac{n\pi}{2} + \frac{1}{n\pi}\sin\frac{n\pi}{2}\right] - \frac{T}{2n\pi} \times \frac{4}{T}2V\left[\cos n\pi - \cos\frac{n\pi}{2}\right]$$

$$= \frac{4V}{n\pi}\left[-\cos\frac{n\pi}{2} + \frac{2}{n\pi}\sin\frac{n\pi}{2} + \cos n\pi\right] - \frac{4V}{n\pi}\left[\cos n\pi - \cos\frac{n\pi}{2}\right]$$

$$= \cancel{\frac{4V}{n\pi}\left[\frac{1}{2}\right] = \frac{2V}{n\pi}} \quad \frac{4V}{n\pi}\cdot\frac{2}{n\pi}\sin\frac{n\pi}{2} = \frac{8V}{(n\pi)^2}\sin\frac{n\pi}{2}$$

Fourier series is

$$\cancel{f(t) = \frac{8V}{\pi^2}\sin\omega t + \frac{2V}{2\pi}\sin 2\omega t + \frac{2V}{3\pi}\sin 3\omega t + \cdots}$$

$$f(t) = \frac{8V}{\pi^2}\sin\omega t - \frac{8V}{9\pi^2}\sin 3\omega t + \frac{8V}{25\pi^2}\sin 5\omega t + \cdots$$

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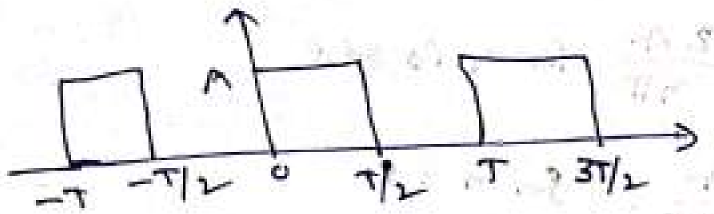
→ obtain the fourier series



Sol By inspection

$$f(t) = A \quad , \quad 0 < t < T/2$$
$$= 0 \quad , \quad \frac{T}{2} < t < T$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T}\int_0^{T/2} f(t)\,dt + \frac{1}{T}\int_{T/2}^{T} f(t)\,dt$$

$$= \frac{1}{T}\cdot A + \frac{1}{T}\cdot 0$$

$$= \frac{A}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega t + \int_{T/2}^{T} 0.\cos n\omega t\,dt\right]$$

$$= \frac{2A}{T}\cdot\left(\frac{\sin n\omega t}{n\omega}\right)_0^{T/2}$$

$$= \frac{2A}{nT\omega}\sin\frac{n\omega T}{2}$$

$$= \frac{2A}{n.T\left(\frac{2\pi}{T}\right)}\sin n\frac{2\pi T}{2T}$$

$$= \frac{A}{n\pi}\sin n\pi$$

$$= 0 \text{ for any value of } n$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt = -\frac{A}{n\pi}(\cos n\pi - 1)$$

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for any value (even) $b_n = 0$

$$b_n = \frac{2A}{n\pi} \text{ for } n \text{ is odd}$$

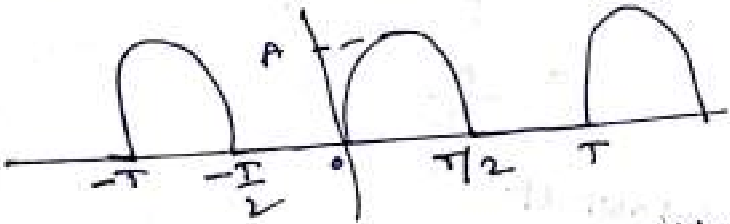
$$= \frac{2A}{\pi} \text{ for } n = 1$$

$$= \frac{2A}{3\pi} \text{ for } n = 3$$

Fourier Series

$$f(t) = \frac{A}{2} + \frac{2A}{\pi}\sin\omega t + \frac{2A}{3\pi}\sin 3\omega t + \cdots$$

→ obtain Fourier Series



Sol By inspection $f(t) = A\sin\omega t \quad 0 \le t < \frac{T}{2}$

$= 0 \quad \frac{T}{2} < t < T$

$$a_0 = \frac{1}{T}\int_0^{T/2} A\sin\omega t \, dt$$

$$= \frac{1}{T}\int_0^{T/2} A\sin\frac{2\pi t}{T} \, dt$$

$$= -\frac{A}{T}\left(\frac{\cos\frac{2\pi}{T}\frac{T}{2}}{\frac{2\pi}{T}} - \frac{1}{\frac{2\pi}{T}}\right)$$

$$= \frac{-A}{2\pi}(\cos\pi - 1)$$

$$= \frac{A}{\pi}$$

$$a_n = \frac{2}{T} \int_0^{T/2} A \sin \frac{2\pi t}{T} \cos n \frac{2\pi}{T} t \, dt$$

$$= \frac{2A}{T} \int_0^{T/2} \frac{1}{2} \left(\sin\left(\frac{2\pi}{T}(1+n)t\right) + \sin\left(\frac{2\pi}{T}(1-n)t\right)\right) dt$$

$$= \frac{-2A}{2T} \left[\frac{T}{2\pi(1+n)} \cos \frac{2\pi}{T}(1+n)t + \frac{T}{2\pi(1-n)} \cos\left(\frac{2\pi}{T}(1-n)t\right) \right]_0^{T/2}$$

$$= -\frac{A}{2\pi} \left[\frac{1}{1+n} \cos\left(\pi(1+n) - 1\right) + \frac{1}{1-n} \cos\left(\pi(1-n) - 1\right)\right.$$

For n odd

$$\cos(\pi(1+n)) = \cos(\pi(1-n)) = 1$$

For n even

$$\cos(\pi(1+n)) = \cos(\pi(1-n)) = -1$$

$a_n = 0$ for n is odd

$$= \frac{2A}{\pi(1-n^2)} \quad n \text{ is even}$$

11^{ly}

$$b_n = \frac{2}{T} \int_0^{T/2} A \sin \frac{2\pi}{T} t \sin \frac{2\pi}{T} n t \, dt$$

$$= \frac{2A}{T} \int_0^{T/2} \frac{1}{2} \left(\cos\left(\frac{2\pi}{T}(1-n)t\right) - \cos \frac{2\pi}{T}(1+n)t \right] dt$$

$$= \frac{A}{n\pi} \left[\frac{1}{1-n} \sin \pi(1-n) - \frac{1}{1+n} \sin \pi(1+n) \right]$$

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For all n, Except n=1

$$\sin\pi(1-n) = \sin\pi(1+n) = 0$$

$$b_n = 0$$

for n=1, $b_1 = \frac{2A}{T}\int_0^{T/2} \sin^2\frac{2\pi t}{T}\,dt$

$$= \frac{2A}{T}\int_0^{T/2}\left(1-\cos\frac{4\pi t}{T}\right)dt$$

$$= \frac{A}{2} - \frac{A}{4\pi}\left(\sin\frac{4\pi t}{T}\right)_0^{T/2}$$

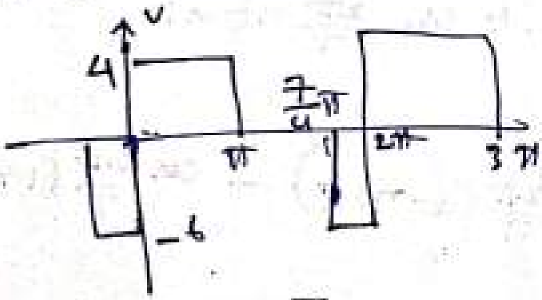
$$= \frac{A}{2}$$

Fourier Series

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin\frac{2\pi t}{T} - \frac{2A}{3\pi}\cos\frac{4\pi t}{T} - \frac{2A}{15\pi}\cos\frac{8\pi t}{T} + \cdots$$

$$= \frac{A}{\pi} + \frac{A}{2}\sin\omega t - \frac{2A}{3\pi}\cos 2\omega t - \frac{2A}{15\pi}\cos 4\omega t + \cdots$$

→ Find Fourier coefficients



$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{\pi} f(t)\,dt + \frac{1}{2\pi}\int_{\frac{7}{4}\pi}^{2\pi} f(t)\,dt$$

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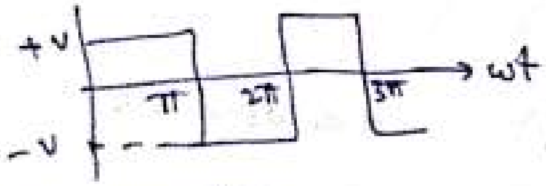
$$= \frac{1}{2\pi} \int_0^{\pi} 4 (\omega t) \, d(\omega t)$$

→ obtain exponential Fourier series



3) By inspection

$$f(t) = +V, \quad 0 < \omega t < \pi$$
$$= -V, \quad \cancel{\omega t <} \ \pi < \omega t < 2\pi$$

Given is odd symmetry $a_0 = 0$

(avg value of the wave is zero)

$$C_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t) \, e^{-jn\omega t} \, dt$$

$$\frac{1}{2\pi} \int_{-\pi}^{0} V e^{-jn\omega t}$$

$$= \frac{1}{2\pi} \int_0^{\pi} V e^{-jn\omega t} d(\omega t) - \frac{1}{2\pi} \int_{\pi}^{2\pi} V e^{-jn\omega t} d(\omega t)$$

$$= \frac{1}{2\pi} \left[\frac{jV}{n} \left(e^{-jn\omega t} \right)_0^{\pi} - j\frac{V}{n} \left(e^{-jn\omega t} \right)_{+\pi}^{2\pi} \right.$$

$$= \frac{jV}{2\pi n} \left[\left(e^{-jn\pi} - e^{0} \right) - e^{0} + e^{jn\pi} \right]$$

$n = -3, -1, 1, 3$

$$= \frac{jV}{n\pi} \left(e^{jn\pi} - 1 \right)$$

For even $= n$, $e^{jn\pi} = 1$ & $C_n = 0$

$n = \text{odd}$ $\quad e^{jn\pi} = -1$ & $C_n = -j\frac{2V}{n\pi}$

Fourier series

$$f(t) = \text{——} + j\frac{2V}{3\pi} e^{-j3\omega t} + j\frac{2V}{\pi} e^{-j\omega t} - j\frac{2V}{\pi} e^{j\omega t} - j\frac{2V}{3\pi} e^{j3\omega t} \text{——}$$

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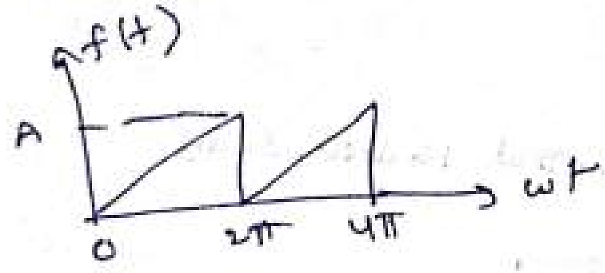
→ Find exponential Fourier series



Sol By inspection

$$f(t) = \frac{A}{2\pi}(\omega t) \qquad 0 < \omega t < 2\pi$$

However $$C_n = \frac{1}{T_0}\int_0^{T_0} f(t)\, e^{-jn\omega t}\, dt$$

In this problem $T_0 = 2\pi$

$$C_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{A}{2\pi}(\omega t)\, e^{-jn\omega t}\, dt$$

$$= \frac{A}{(2\pi)^2}\left[\omega t \cdot \left(\frac{e^{-jn\omega t}}{-jn}\right)_0^{2\pi} - \int_0^{2\pi} 1 \cdot \frac{e^{-jn\omega t}}{-jn}\, d\omega t\right]$$

$$= \frac{A}{(2\pi)^2}\left[\omega t \cdot \frac{e^{-jn\omega t}}{-jn} \times \frac{(-jn)}{(-jn)} - \frac{e^{-jn\omega t}}{(-jn)^2}\right]_0^{2\pi}$$

$$= \frac{A}{(2\pi)^2}\left(\frac{e^{-jn\omega t}}{(-jn)^2}\left[-jn\omega t - 1\right]\right)_0^{2\pi}$$

$$= \frac{A}{(2\pi)^2}\left(\frac{e^{-j2\pi n}}{(-jn)^2}\left(-jn(2\pi) - 1\right) + \frac{e^{-0}}{(-jn)^2}\right)$$

$$= \cancel{-\frac{jA}{2\pi n}} \frac{A}{(2\pi)^2}\left(\frac{-jn2\pi(-1)}{(-jn)^2} + \cancel{\frac{1}{(-jn)^2}} \cancel{\frac{1}{(-jn)^2}}\right)$$

$$\left(e^{-j2\pi n} = -1 \text{ for all } n\right)$$

$$= -\frac{jA}{2\pi n}$$

$$f(t) = \sum_{n=-\alpha}^{\alpha} c_n e^{jn\omega t}$$

$$= \sum_{n=-\alpha}^{\alpha} \left(\frac{-jA}{2\pi n}\right) e^{jn\omega t}$$

$$= \ldots + j\frac{A}{4\pi} e^{-j2\omega t} + j\frac{A}{2\pi} e^{-j\omega t}$$

$$+ \frac{A}{2} - j\frac{A}{2\pi} e^{j\omega t} - j\frac{A}{4\pi} e^{j2\omega t} + \ldots$$

Here avg value is $\frac{A}{2}$ which can easily be derived.

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{A}{2\pi} t \, dt$$

$$= \frac{1}{2\pi}\frac{A}{2\pi}\left(\frac{t^2}{2}\right)_0^{2\pi}$$

$$= \frac{A}{(4\pi^2)} \frac{(2\pi)^2}{2}$$

$$= \frac{A}{2}$$

→ obtain Fourier Series.

$$f(t) = -K \quad -\pi < t < 0$$
$$= +K \quad 0 < t < \pi$$

This is odd symmetry

$$a_n = a_0 = 0.$$

$$b_n = \frac{2}{T_0}\int_{-\pi}^{\pi} f(t) \sin n\omega t \, dt$$

$$= \frac{2}{2\pi}\int_{-\pi}^{0} -K \sin n\omega t + \frac{2}{2\pi}\int_0^{\pi} K \sin n\omega t$$

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$$= \frac{-K}{\pi} \times \left(-\frac{\cos n\omega t}{n\omega}\right)_{-\pi}^{0} + \frac{K}{\pi}\left(-\frac{\cos n\omega t}{n\omega}\right)_{0}^{\pi}$$

$$= \frac{K}{n\omega\pi}\left[\cos 0 - \cos(-n\omega\pi) - \cos n\omega\pi + \cos 0\right]$$

$$= \frac{K}{n\pi \times 1}\left[\cos 0 - \cos(-n\pi) - \cos n\pi + \cos 0\right]$$

$$= \frac{K}{n\pi}\left[2 - 2\cos n\pi\right]$$

$$b_n = \frac{2K}{n\pi}\left[1 - \cos n\pi\right]$$

$$= \frac{4K}{n\pi} \quad \text{for } n \text{ is odd}$$

Fourier series

$$f(t) = b_1 \sin\omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \cdots$$

$$= \frac{4K}{\pi}\sin\omega t + \frac{2K}{\pi}\sin 2\omega t + \frac{4K}{3\pi}\sin 3\omega t + \cdots$$

x

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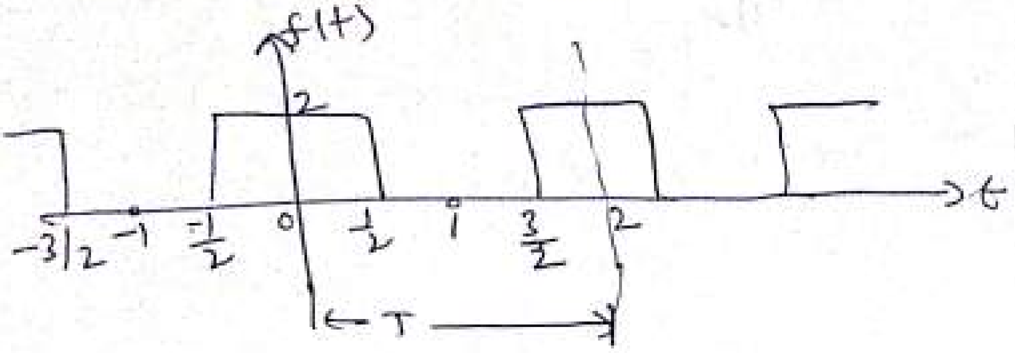
→ Obtain the Fourier series



Sol

In the Given problem, it is not a even or odd -

$T = 2$ sec

$$a_0 = \frac{1}{T}\left\{\int_0^{1/2} 2\,dt + \int_{3/2}^{2} 2\,dt\right\} \qquad [\because T = 2]$$

$$= \frac{1}{2}\left\{2\cdot\frac{1}{2} + 2\cdot\left(2-\frac{3}{2}\right)\right\}$$

$$= \frac{1}{2}\times(1+1) = 1$$

$$\boxed{a_0 = 1}$$

$$a_n = \frac{2}{T}\left\{\int_0^{1/2} 2\cos n\omega t\,dt + \int_{3/2}^{2} 2\cos n\omega t\,dt\right\}$$

$$\boxed{\omega = \frac{2\pi}{T} = \frac{2\pi}{2},\quad \omega = \pi}$$

$$= 2\left\{\left[+\frac{\sin n\omega t}{n\omega}\right]_0^{1/2} + \left[+\frac{\sin n\omega t}{n\omega}\right]_{3/2}^{2}\right\}$$

$$= \frac{2}{n\pi}\left\{\sin\frac{n\pi}{2} + \underbrace{\sin\frac{2n\pi}{1}}_{\to 0} - \sin\frac{3n\pi}{2}\right\}$$

$$= \frac{2}{n\pi}\left\{\sin\frac{n\pi}{2} - \sin\frac{3n\pi}{2}\right\}$$

$n=1,\ a_1 = 4/\pi$

$n=2,\ a_2 = 0$

$n=3,\ a_3 = -4/3\pi$

Fourier Integral (or) Fourier Transform

- For the signal f(t), F(jω) is called a Fourier integral or Fourier transform or continuous spectrum.
- with the help of Fourier integral, the behaviour of the linear circuits can be analyzed.
- Basically, a Fourier integral is one which transforms a function in time domain (t) into function in frequency domain (ω)
- A line or discrete spectrum becomes continuous spectrum if frequency tends to zero as time period T tends to infinity
- Fourier integral is a limiting case of a Fourier series.
- Fourier integral does not exist for a periodic function becau[se] it does not satisfy the condition of absolute integrability.
- Fourier series in time domain, there exist's only one Fourier integral in frequency domain corresponding to it.

→ As $T \to \alpha$, $\omega \to 0$ as $\omega = \frac{2\pi}{T}$. Due to this a single pulse of f(t) is generated. This pulse will not be periodic. Hen[ce] it is necessary to modify the fourier series.

we know the Fourier series is given by

$$f(t) = \sum_{n=-\alpha}^{\alpha} C_n e^{jn\omega t}$$

$$\text{where } C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega t}\, dt$$

where f(t) is a periodic function

suppose if f(t) is a non periodic function

Then a few changes will have to be incorporated i.e

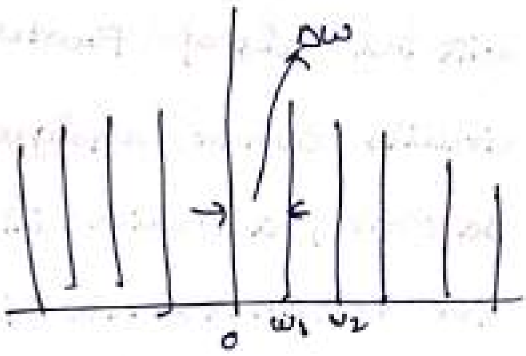
As $T \to \alpha$

$\omega \to 0 \quad \left(\omega = \frac{2\pi}{T}\right)$

and n becomes irrelavent

$n\omega \to \omega$

$T = \frac{2\pi}{\Delta\omega}$



where $\Delta\omega$ is the spacing b/w adjacent component of angular frequency

Then above equation can be reduced to

$$f(t) = \sum_{n=-\alpha}^{\alpha} c' e^{j\omega t} \qquad \omega = 0, \Delta\omega, 2\Delta\omega \ldots \quad — (3)$$

$$\text{where } c' = \frac{\Delta\omega}{2\pi}\int_{-T/2}^{T/2} f(t)\, e^{-j\omega t}\, dt \quad — (4)$$

sub eq (4) in eq (3)

$$f(t) = \sum_{n=-\alpha}^{\alpha}\left[\frac{1}{2\pi}\int_{-T/2}^{T/2} f(t)\, e^{-j\omega t}\, dt\right] \cdot e^{j\omega t}\, \Delta\omega$$

changing summation form to integral form

$$f(t) = \frac{1}{2\pi}\int_{-\alpha}^{\alpha}\left[\int_{-\alpha}^{\alpha} f(t)\, e^{-j\omega t}\, dt\right] e^{j\omega t}\, dt.$$

$$= \frac{1}{2\pi}\int_{-\alpha}^{\alpha} F(j\omega)\, e^{j\omega t}\, dt$$

where $F(j\omega) = \int_{-\alpha}^{\alpha} f(t)\, e^{-j\omega t}\, dt$ is a Fourier integral of f(t).

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Properties

The properties of Fourier transform are as follows

(1) Symmetry property

if $f(t)$ is function in time domain and $F(j\omega)$ is function in frequency domain i.e fourier transform. Then

if $f(t) = F(j\omega)$

Then $F(t) = 2\pi f(-j\omega)$

- This property states that the Fourier transform of a gate function is a sampling function and vice versa.

(2) Linearity property

if $f_1(t)$ & $f_2(t)$ are the two time domain functions and $F_1(j\omega)$ and $F_2(j\omega)$ are the Two Fourier transforms. Then a is a orbitory constant then

$$a f_1(t) + a_2 f_2(t) = a_1 F_1(j\omega) + a_2 F_2(j\omega)$$

(3) Scaling property:

Let $f(t)$ is the time domain function and $F(j\omega)$ is the Fourier transform Then

$$f(at) = \frac{1}{|a|} F\left(\frac{j\omega}{a}\right)$$

where

$f(at)$ is the function $f(t)$ compressed in time scale of factor 'a'

$F\left(\frac{j\omega}{a}\right)$ is the function expanded by frequency scale by the same factor 'a'

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4. Frequency shifting property

Let $f(t)$ is the time domain function
$F(j\omega)$ is the Fourier transform.

if
$f(t) = F(j\omega)$ then

$$f(t)\, e^{j\omega_0 t} = F(j(\omega-\omega_0))$$

Proof:

$$F\left(f(t)\, e^{j\omega_0 t}\right) = \int_{-\infty}^{\infty} f(t)\, e^{j\omega_0 t}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-j(\omega-\omega_0)t}\, dt$$

$$= F\left[(j(\omega-\omega_0))\right]$$

(5) Time shifting property

Let $f(t)$ is the time domain function
$F(j\omega)$ is the Fourier transform

$$f(t-T) = F(j\omega)\, e^{-j\omega T}$$

Proof

$$F\left[f(t-T)\right] = \int_{-\infty}^{\infty} f(t-T)\, e^{-j\omega t}\, dt$$

$t-T = x$, $dt = dx$ and $t = x+T$

Hence

$$F\left[f(t-T)\right] = \int_{-\infty}^{\infty} f(x)\, e^{-j\omega(x+T)}\, dx$$

$$= e^{-j\omega T} \int_{-\infty}^{\infty} f(x)\, e^{-j\omega x}\, dx$$

$$F(f(t-T)] = F(j\omega)\, e^{-j\omega T}$$

6. Differentiation property

if $f(t) \leftrightarrow F(j\omega)$ then

$$\frac{d}{dt}(f(t)) \leftrightarrow j\omega F(j\omega)$$

Proof

By the definition of the inverse fourier transform, we have

$$F^{-1}(F(j\omega)) = f(t) = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} F(j\omega)\, e^{j\omega t}\, dt \quad \text{--- (1)}$$

$$\frac{d}{dt} f(t) = \frac{d}{dt}\left[\frac{1}{2\pi}\int_{-\alpha}^{\alpha} F(j\omega)\, e^{j\omega t}\, dt\right]$$

$$= \frac{1}{2\pi}\int_{-\alpha}^{\alpha} \frac{d}{dt}\left(F(j\omega)\, e^{j\omega t}\right) dt$$

$$= \frac{1}{2\pi}\int_{-\alpha}^{\alpha} F(j\omega)\, e^{j\omega t}(j\omega)\, d\omega$$

$$= \frac{j}{2\pi}\int_{-\alpha}^{\alpha} \omega\, F(j\omega)\, e^{j\omega t}\, d\omega$$

$$= j\, F^{-1}[\omega \cdot F(j\omega)]$$

$$\boxed{\frac{d}{dt} f(t) = F^{-1}(j\omega \cdot F(j\omega))}$$

$$F\left[\frac{d}{dt} f(t)\right] = j\omega \cdot F(j\omega)$$

(7) Integration property

$$F\left[\int_{-\alpha}^{\alpha} f(t)\,dt\right] = \frac{1}{j\omega} F(j\omega)$$

(8) Folding property

Let $f(t)$ & $F(j\omega)$ then

$$F(f(-t)) = F(-j\omega)$$

---

prove time scalling property

Given is $f(at) = \frac{1}{|a|} F\left(\frac{j\omega}{a}\right)$

proof:

$$F(f(t)) = \int_{-\alpha}^{\alpha} f(t)\, e^{-j\omega t}\, dt$$

case (1) when 'a' is +ve

$$F(f(at)) = \int_{-\alpha}^{\alpha} f(at)\, e^{-j\omega t}\, dt \quad \text{— (1)}$$

Let $x = at \Rightarrow t = \frac{x}{a}$

$dt = \frac{dx}{a}$

eq (1) becomes

$$F(f(at)] = \int_{-\alpha}^{\alpha} f(x)\, e^{-j\omega \frac{x}{a}}\, \frac{dx}{a}$$

$$= \frac{1}{a} \int_{-\alpha}^{\alpha} f(x)\, e^{-\frac{j\omega}{a}}\, dx$$

$$\boxed{F(f(at)) = \frac{1}{a} F\left(\frac{j\omega}{a}\right)}$$

Problem

① ~~Fourier Sine & cosine integral~~ Discuss the F.T of Sinusoidal Function

Sol $f(t) = \sin\omega_0 t$ — $\sin\omega_0 t$ is absolute integrability & hence

$$F(j\omega) = \lim_{T\to\infty} \int_{-T/2}^{T/2} f(t)\, e^{-j\omega t}\, dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \sin\omega_0 t\, e^{-j\omega t}\, dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} \right] e^{-j\omega t}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2j} \left[\left(\int_{-T/2}^{T/2} e^{-j(\omega-\omega_0)t}\, dt \right) - \left(\int_{-T/2}^{T/2} e^{-j(\omega+\omega_0)t}\, dt \right) \right]$$

$$= \lim_{T\to\infty} \frac{1}{2j} \left(\frac{e^{-j(\omega-\omega_0)t}}{-j(\omega-\omega_0)} \right)_{-T/2}^{T/2} - \left(\frac{e^{-j(\omega+\omega_0)t}}{-j(\omega+\omega_0)} \right)_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{2j} \left(\frac{e^{j(\omega-\omega_0)T/2} - e^{-j(\omega-\omega_0)\frac{T}{2}}}{j(\omega-\omega_0)} \right) - \left(\frac{e^{j(\omega+\omega_0)\frac{T}{2}} - e^{-j(\omega+\omega_0)\frac{T}{2}}}{j(\omega+\omega_0)} \right)$$

$$= \lim_{T\to\infty} \frac{1}{2j} \left[\frac{2j \sin(\omega-\omega_0)T/2}{j(\omega-\omega_0)} - \frac{2j \sin(\omega+\omega_0)T/2}{j(\omega+\omega_0)} \right]$$

$$= \lim_{T\to\infty} (-j) \left[\frac{\sin(\omega-\omega_0)T/2}{\omega-\omega_0} - \frac{\sin(\omega+\omega_0)T/2}{\omega+\omega_0} \right] \times \frac{\frac{T}{2}}{\frac{T}{2}}$$

From Sampling function

$$\frac{\sin\omega t}{\omega t} = \mathrm{Sinc}(\omega t)$$

$$= \lim_{T\to\infty} (-j) \left[\frac{T}{2} \cdot \operatorname{sinc}(\omega-\omega_0)\frac{T}{2} - \frac{T}{2} \operatorname{sinc}(\omega+\omega_0)\frac{T}{2} \right] \frac{\pi}{\pi}$$

$$= (-j\pi) \left[\lim_{T\to\infty} \frac{T}{2\pi} \operatorname{sinc}(\omega-\omega_0)\frac{T}{2} - \lim_{T\to\infty} \frac{T}{2\pi} \operatorname{sin}(\omega+\omega_0)T/2 \right.$$

$$= (-j\pi) \left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0) \right.$$

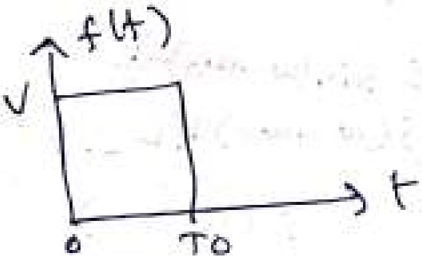
$$\boxed{F(j\omega) = j\pi \left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0) \right]}$$

$$\left(\lim_{T\to\infty} \frac{T}{2\pi} \operatorname{sinc}(T\omega) = \delta(\omega) \right.$$

→ $f(t) = \cos\omega_0 t$

$$F(j\omega) = \pi \left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0) \right]$$

→ ~~f(t) = e~~ Determine the F.T of a pulse of duration T_0 and magnitude V shown in fig.



Sol: $f(t) = V \quad , 0 < t < T_0$

$\quad\quad = 0 \quad , t > T_0$

Fourier transform $F(j\omega)$ is

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{T_0} V e^{-j\omega t}\, dt$$

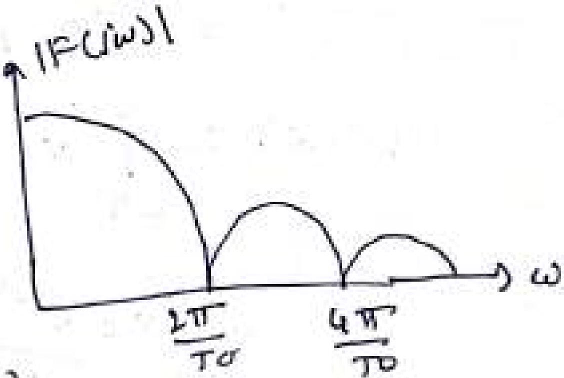
$$= V \left[\frac{e^{-j\omega t}}{-j\omega} \right]_0^{T_0} = \frac{jV}{\omega} \left(e^{-j\omega T_0} - 1 \right)$$

$$= -\frac{2V}{\omega}\left[\frac{e^{-j\omega\frac{T_0}{2}} - e^{j\omega\frac{T_0}{2}}}{2j}\right] e^{-j\omega\frac{T_0}{2}}$$

$$= \frac{2V}{\omega}\sin\frac{\omega T_0}{2}\, e^{-j\omega\frac{T_0}{2}}$$

$$\boxed{F(j\omega) = V T_0 \frac{\sin \omega\frac{T_0}{2}}{\omega\frac{T_0}{2}} e^{-j\omega\frac{T_0}{2}}}$$

The magnitude of $|F(j\omega)|$ is of the form $\frac{\sin x}{x}$ and plot is shown in fig.

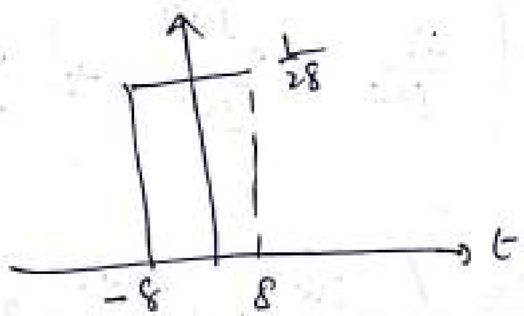


The magnitude is zero when

$$\frac{\omega T_0}{2} = \pi, 2\pi, 3\pi \text{ etc.}$$

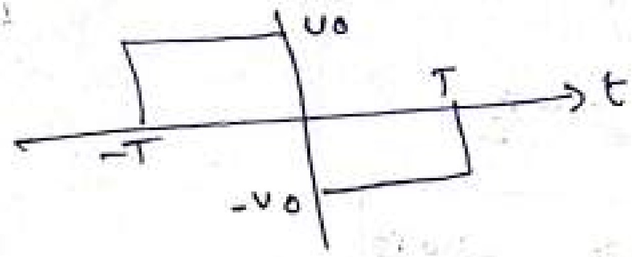
→ F.T of impulse function $\delta(t)$

Sol:



$$\int_{-\alpha}^{\alpha} \delta(t)\,dt = 1$$

→ Determine the F-T of following fig



Sol By inspection

$f(t) = +V_0 \quad -T < t < 0$

$= -V_0 \quad , \quad 0 < t < T$

$$F(j\omega) = \int_{-T}^{0} V_0 e^{-j\omega t} + \int_{0}^{T} V_0 e^{-j\omega t} dt$$

$$= \left(V_0 \frac{e^{-j\omega t}}{-j\omega} \right)_{-T}^{0} + \left(V_0 \cdot \frac{e^{-j\omega t}}{+j\omega} \right)_{0}^{T}$$

$$= V_0 \left(\frac{-1}{j\omega} + \frac{e^{j\omega T}}{j\omega} + \frac{e^{-j\omega T}}{j\omega} - \frac{1}{j\omega} \right)$$

$$= \frac{V_0}{j\omega} \left(e^{j\omega T} + e^{-j\omega T} - 2 \right)$$

$$= \frac{2jV_0}{\omega} \left(1 - \cos\omega T \right)$$

$$\boxed{F(j\omega) = j\omega T^2 V_0 \frac{\sin^2 \frac{\omega T}{2}}{\left(\frac{\omega T}{2}\right)^2}}$$

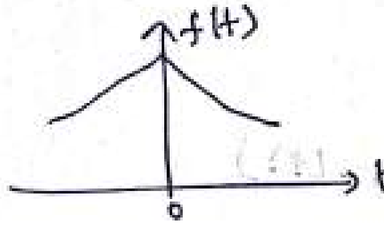
$\therefore \left(\cos\omega T = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \right)$

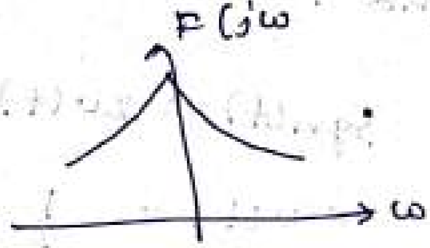
$1 - \cos\omega T$
$= 2 \sin^2 \frac{\omega T}{2}$

$= \frac{2jV_0}{\omega} \times \frac{\omega^2 T^2}{4} \times \frac{2 \sin^2 \frac{\omega T}{2}}{\left(\frac{\omega T}{2}\right)^2}$

$= j\omega T^2 V_0 \frac{\sin^2 \frac{\omega T}{2}}{\left(\frac{\omega T}{2}\right)^2}$

→ Find the F.T of the function





Given function is shown in modulus so fig is like that

Sol here $f(t) = A e^{-a|t|}$, $a > 0$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$-\infty$ to $0 \to t = -t$
0 to $\infty \to t = t$

$$= \int_{-\infty}^{\infty} A e^{-a|t|} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} A e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} A e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} A e^{(a-j\omega)t} dt + \int_{0}^{\infty} A e^{(-a-j\omega)t} dt$$

$$= \frac{A}{a-j\omega} + \frac{A}{a+j\omega} = \frac{2aA}{a^2+\omega^2}$$

→ Determine the F.T of Signum function

Sol Signum function is

$$f(t) = -1 \quad , \quad -\infty < t < 0$$
$$= 1 \quad , \quad 0 < t < \infty$$

$$\text{sgn}(t) = \lim_{a \to 0} e^{-a|t|} \text{sgn}(t)$$

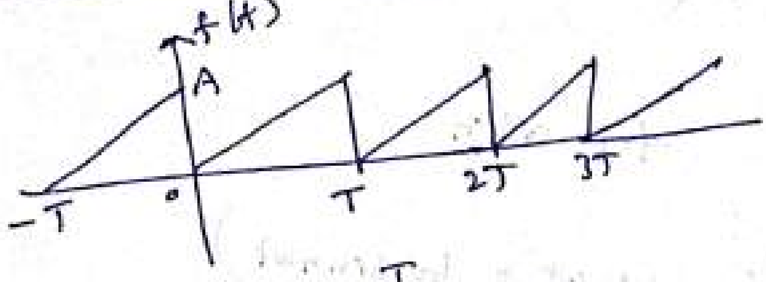
$$F(j\omega) = \int_{-\infty}^{\infty} \lim_{a\to 0} e^{-a|t|} \text{sign}(t) e^{-j\omega t} dt$$

$$= \lim_{a \to 0} \left[\int_{-\infty}^{0} -e^{(a-j\omega)t} dt + \int_{0}^{\infty} e^{-(a+j\omega)} \right]$$

$$= \lim_{a \to 0} \left(\frac{1}{a+j\omega} - \frac{1}{a-j\omega} \right) = \frac{2}{j\omega}$$

Analysis of electrical ckts

(1)



it is a odd function

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{A}{T}t\,dt$$

$$= \frac{A}{T^2}\left(\frac{t^2}{2}\right)_0^T = \frac{A}{T^2}\cdot\frac{T^2}{2} = \frac{A}{2}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt$$

$$= \frac{2}{T}\int_0^T \frac{A}{T}t\cos n\omega t\,dt$$

$$= \frac{2A}{T^2}\left[\left(t\cdot\frac{\sin n\omega t}{n\omega}\right)_0^T + \left(\frac{\cos n\omega t}{(n\omega)^2}\right)_0^T\right]$$

$\rightarrow 0$

$$= \frac{2A}{T^2}\times\left(\frac{\cos n\frac{2\pi}{T}\cdot T}{\left(n\frac{2\pi}{T}\right)^2}\right)_0^T$$

$$= \frac{2A}{T^2}\times\frac{T^2}{(2\pi n)^2}\left(\cos 2\pi n - 1\right)$$

$$= 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt$$

$$= \frac{2A}{T^2}\left[\left(t\ \frac{-\cos n\omega t}{n\cdot\frac{2\pi}{T}}\right)_0^T + \left(\frac{\sin n\omega t}{\left(n\cdot\frac{2\pi}{T}\right)^2}\right)_0^T\right.$$

$\rightarrow 0$

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$$= \frac{2A}{T^2} \times \frac{T^2}{2\pi n} \times (-\cos 2\pi n)$$

$$b_n = \frac{-A}{\pi n} \quad \text{for all } n$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$= \frac{A}{2} - \frac{A}{\pi}\left(\sin\omega t + \frac{A}{2}\sin 2\omega t + \frac{\sin 3\omega t}{3} + \cdots\right)$$

→ The voltage having the waveform shown in fig(1) is applied to ckt shown in fig(2). Determine $i(t)$

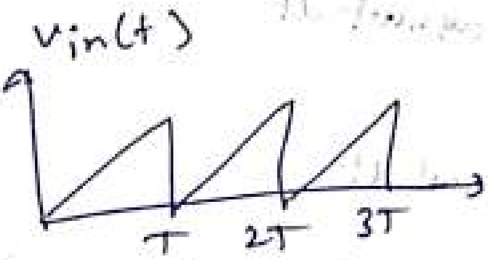

fig (1)

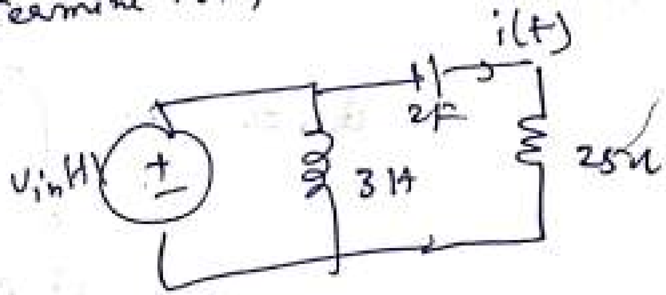

fig (2)

$R + jX_C$

$R + \frac{1}{j\omega C}$

Sol For fig (1)

$$v_{in}(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \left(\frac{-A}{\pi n}\right) \sin n\omega t$$

$$v_{in}(t) = \frac{A}{2} - \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{\sin n\omega t}{n}$$

$\frac{1}{\omega C}$

$\frac{1}{2\pi f C}$

From Fig(b)

$$i(t) = \frac{v_{in}(t)}{25 + \frac{1}{2jn\omega}}$$

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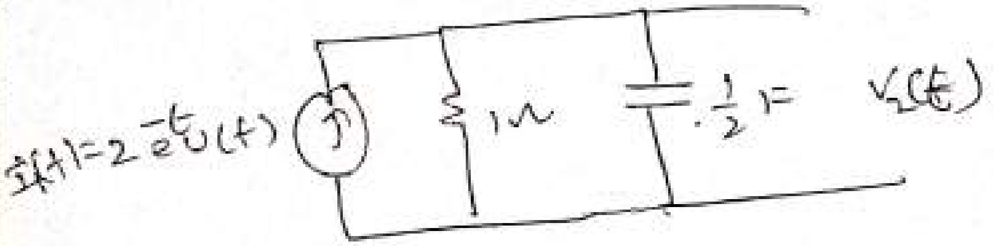
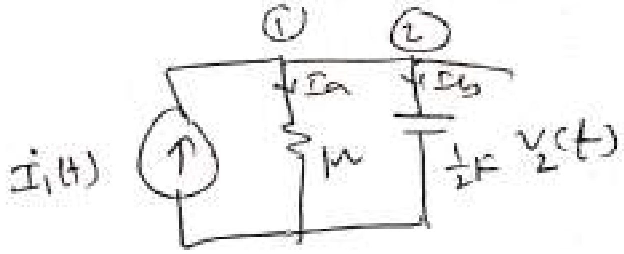
→ Find the response of the voltage in the n/w shown in fig. use F-T method



apply KCL at node 1



$$I_1(t) = I_a + I_b$$

$$2e^{-t}u(t) = \frac{v_2(t)}{R} + j\omega C v_2(t)$$

$$= v_2(t)\left(\frac{1}{R} + j\omega C\right)$$

$$= v_2(t)\left(1 + j\omega\left(\tfrac{1}{2}\right)\right)$$

$$2e^{-t}u(t) = v_2(t)(1 + 0.5(j\omega)) \quad -(1)$$

$$F(Ae^{-at}u(t)) = \frac{A}{a+j\omega}$$

Let $F(v_2(t)) = V_2(j\omega)$

apply F.T on both sides of equation

$$F(2e^{-t}u(t)) = F(v_2(t)(1+j0.5\omega))$$

$$\frac{2}{1+j\omega} = V_2(j\omega)(1+j0.5\omega)$$

$$V_2(j\omega) = \frac{2}{(1+j\omega)(1+j0.5\omega)}$$

$$= \frac{2}{(1+j\omega)\,0.5\left(\frac{1}{0.5}+j\omega\right)}$$

$$= \frac{4}{(1+j\omega)(2+j\omega)} \quad \text{(partial fraction)}$$

$$= \frac{A}{1+j\omega} + \frac{B}{2+j\omega}$$

$A = 4$

$B = -4$

$$V_2(j\omega) = \frac{4}{1+j\omega} - \frac{4}{2+j\omega}$$

taking ~~F~~ inverse F.T, we get

$$F^{-1}(V_2(j\omega)) = F^{-1}\left(\frac{4}{1+j\omega} - \frac{4}{2+j\omega}\right)$$

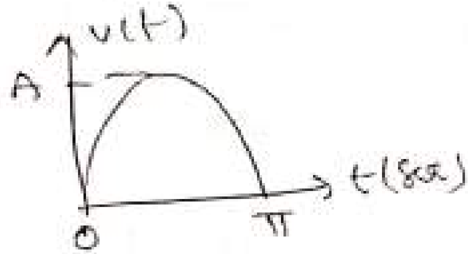
$$v_2(t) = 4e^{-t}u(t) - 4e^{-2t}u(t)$$

$$= 4u(t)\left[e^{-t} - e^{-2t}\right)$$

The response is

$$\boxed{v_2(t) = 4u(t)\left(e^{-t} - e^{-2t}\right)}$$

→ Find the F.T of sine pulse shown in fig and sketch amplitude and phase spectra. If this vol is applied to a series RL ckt with $R = 1\Omega$ & $L = 1H$. Determine Amplitude & phase spectra for the resultant current $i(t)$.



Sol

From the above wave form

$v(t) = A\sin t$

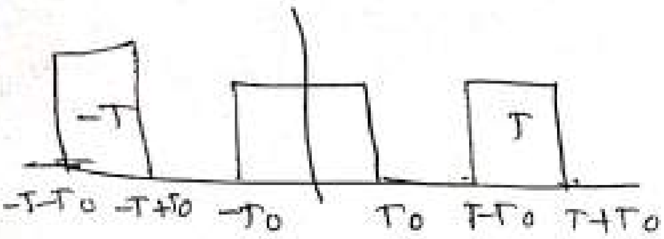
By the definition of F.T, we have

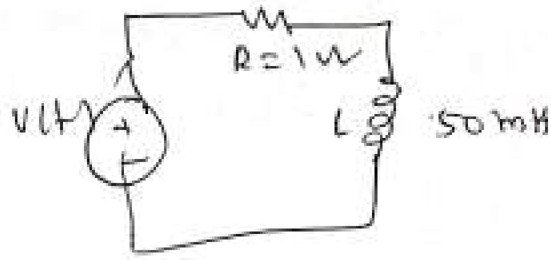
$$V(j\omega) = \int_{-\infty}^{\infty} v(t)e^{-j\omega t}dt$$

$$= \int_0^{\pi} v(t)e^{-j\omega t}dt$$

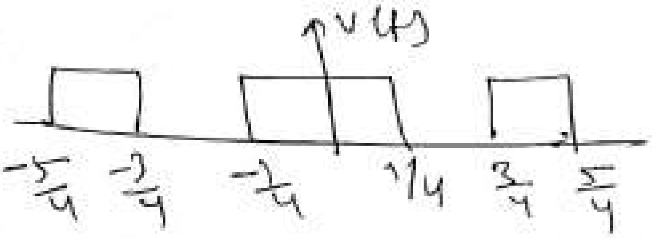
Let $V(j\omega) = A\int_0^{\pi} \sin(t) \cdot e^{-j\omega t}dt = I$ (say)

→ Determine & plot the vol across the resistor in the ckt shown in fig (1) using ckt analysis techniques, if the input is a square wave with $T = 1$, $T_0 = \frac{1}{4}$





Sol Given $T = 1$ sec
$T_0 = \frac{1}{4}$ sec



$$v(t) = 0 \;, \; -\frac{3}{4} < t < -\frac{1}{4}$$
$$= 1 \;, \; -\frac{1}{4} < t < \frac{1}{4}$$
$$= 0 \;, \text{for } \frac{1}{4} < t < \frac{3}{4}$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos n\omega t\,dt$$

$$= 2\left[\int_{-1/4}^{1/4} 1 \cdot \cos n\omega t\,dt\right]$$

$$= 2\left(\frac{\sin n\omega t}{n\omega}\right)_{-1/4}^{1/4}$$

$$= \frac{2}{n\cdot\frac{2\pi}{T}}\left[\sin n\cdot\frac{2\pi}{T}\left(\frac{1/4}{2}\right) + \sin n\cdot\frac{2\pi}{T}\left(\frac{1/4}{2}\right)\right]$$

$$= \frac{T}{n\pi}\left[\sin\frac{n\pi}{2T} + \sin\frac{n\pi}{T}\right]$$

$$= \frac{2T}{n\pi}\sin\frac{n\pi}{2T} = \frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$T = 25\pi$ ms, $R = 15\Omega$,

$\ldots(t) = ?$ and $I_f = ?$

fundamental frequency current

$$a_n = \frac{2}{\pi}, \frac{-2}{3\pi}, \frac{2}{5\pi}, \quad n = 1, 3, 5$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin n\omega t \, dt$$

$$= \frac{2}{1}\int_{-1/4}^{1/4} 1 \cdot \sin n\omega t \, dt$$

$n \cdot \frac{2\pi}{4T}$

$\frac{n\pi}{2}$

$$= 0$$

$$a_0 = \frac{1}{T}\int_{-1/4}^{1/4} v(t)\, dt$$

$$= (t)_{-1/4}^{1/4}$$

$$= \frac{1}{2}$$

$$v(t) = \frac{1}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$$= \frac{1}{2} + \frac{2}{\pi}\cos\omega t - \frac{2}{3\pi}\cos 3\omega t + \frac{2}{5\pi}\cos 5\omega t + \cdots$$

voltage across Resistor

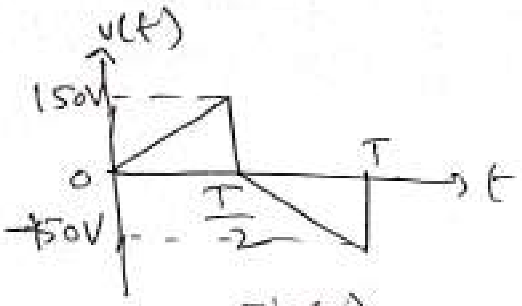
$$v_R(t) = I(t) R$$

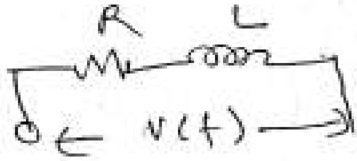
where. $$I(t) = \frac{v(t)}{1 + j2\pi f L} = \frac{v(t)}{1 + j\frac{2\pi}{T}L}$$

$$= \frac{v(t)}{1 + j2\pi L}$$

→ Determine the Fourier Series $T = 25\pi$ ms, $R = 15\Omega$, $L = 0.0438H$. Find upto 7^{th} harmonic $v(t) = ?$ and $I_f = ?$

fundamental frequency current





Fig(a)

given

$T = 25\pi$ ms

$R = 15\Omega$

$L = 0.0438H$

Fig(a) is a half wave symmetry

$$v(t) = -v\left(t - \frac{T}{2}\right)$$

$$v(t) = \frac{150t}{\left(\frac{T}{2}\right)}, \quad \omega = \frac{2\pi}{T}$$

$$= \frac{300t}{T}$$

for n even $\quad a_0 = a_n = b_n = 0$

for n odd $\quad a_n = \frac{4}{T}\int_0^{T/2} v(t)\cos n\omega t\, dt$

$$= \frac{4}{T}\int_0^{T/2} \frac{300t}{T}\cos n\omega t\, dt$$

$$= \frac{1200}{T^2}\int_0^{T/2} t\cos n\omega t\, dt$$

$$= \frac{1200}{T^2}\left[\left(t\,\frac{\sin n\omega t}{n\cdot\frac{2\pi}{T}}\right)_0^{T/2} + \left(\frac{\cos n\frac{2\pi}{T}t}{\left(n\cdot\frac{2\pi}{T}\right)^2}\right)_0^{T/2}\right]$$

$$= \frac{\cancel{1200}^{\,600}}{\cancel{T^2}} \times \frac{\cancel{T^2}}{\cancel{2}n\pi}\left[0 - 0 + \frac{(\cos n\pi - 1)}{2\pi n}\right]$$

$$= \frac{300}{(n\pi)^2} (\cos n\pi - 1)$$ $t \geq 0.$

$$= \frac{-600}{n^2\pi^2} \quad n \text{ is odd.}$$

$$b_n = \frac{4}{T}\int_0^{T/2} v(t)\sin n\omega t\,dt$$

$$= \frac{300}{n\pi}$$

$$v(t) = a_0 + \sum_{n=1}^{\alpha}(a_n \cos n\omega t + b_n \sin n\omega t)$$

$$= \frac{-600}{\pi^2}\cos\omega t + \frac{-600}{9\pi^2}\cos 3\omega t + \frac{-600}{25\pi^2}\cos 5\omega t + \frac{-600}{49\pi^2}\cos 7\omega t +$$

$$\cdots + \frac{300}{\pi}\sin\omega t + \cdots \frac{300}{7\pi}\sin 7\omega t + \cdots$$

Fourier

Fundametal frequency voltage

$$v_f(t) = \frac{-600}{\pi^2}\cos\omega t + \frac{300}{\pi}\sin\omega t$$

$$= \frac{600}{\pi^2}\left(-\cos\omega t + \frac{\pi}{2}\sin\omega t\right)$$

$$= \frac{600}{\pi^2}\sqrt{\left(\frac{\pi}{2}\right)^2+1}\cdot\sin(\omega t - \theta)$$

$$\theta = \tan^{-1}\left(\frac{2}{\pi}\right)$$

$$\left[\because = \tan^{-1}\frac{a_n}{b_n}\right) = \tan^{-1}\left(\frac{1}{\frac{\pi}{2}}\right)\Big]$$

From Fig (b)

$$Z = R + jX_L = R + j\omega L$$

$$= 15 + j\,0.0438 \times \omega$$

where $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{25\pi \times 10^{-3}} = \frac{2000}{25} = 80\,rad/sec$

$$\therefore Z = 15 + j3.504\,\Omega$$

$$= 15.404\angle 13.15^\circ\,\Omega$$

Fundametal freqency curent

$$I_f = \frac{|V_f|}{Z} = \frac{\frac{600}{\pi^2}\sqrt{\left(\frac{\pi}{2}\right)^2+1}}{15.404}$$

$$= 7.35A$$

Rms value of fundamental current

$$I_{rms} = \frac{7.35}{\sqrt{2}} = 5.19A.$$

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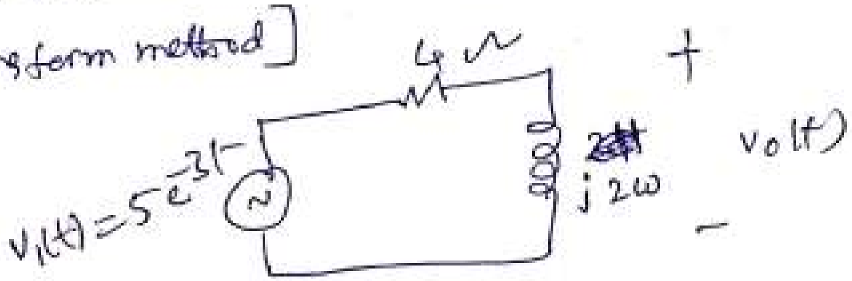
ACO

→ The current source in the fig is $i(t) = 4e^{-t}$ for $t \geq 0$. Find the voltage v_o using Fourier transform method?

→ An Input voltage $v_1(t) = 5e^{-3t}u(t)$ is applied to a series RL ckt with $R = 4\Omega$ and $L = 2H$. Find the o/p voltage $v_o(t)$ across the inductor using frequency domain analysis (Fourier transform method]



Sol

using Fourier transform method.

$$F(v_1(t)) = v_1(j\omega) = FT[5e^{-3t}u(t)]$$

$$\boxed{v_1(j\omega) = \frac{5}{j\omega+3}}$$

using voltage division rules

$$v_o(j\omega) = \frac{j2\omega}{4+j2\omega} \times \frac{5}{j\omega+3}$$

$$= \frac{j\omega}{2+j\omega} \times \frac{5}{j\omega+3}$$

$$\boxed{v_o(j\omega) = \frac{5(j\omega)}{(2+j\omega)(j\omega+3)}}$$

UNIT-V

FILTER DESIGN AND CIRCUIT SIMULATION

Filters - low pass - High pass and band pass - RL, RC filters - derived filters and composite design filters.
Circuit simulation - Description of circuit elements - Nodes and sources, input and output variables - modelling of the above element - DC analysis.

* Filters Introduction :

→ Filters consisting of purely reactive n/w [L-C n/w] that passes desired band of frequencies and attenutes all other frequencies.

Ex: Tuning of radio, selection of channels in TV, communication system in Tele communication engineering.

→ Filters are broadly classified into two types.

1. Active filters
2. Passive filters

1. Active Filters : The filters which consists of active elements such as operation amplifiers, transistors in addition to R, L, C, then it is called Active filters.

Advantages :

1. No resonance issue
2. It is used to reduce harmonic contents
3. It is used for voltage regulation.
4. It is used for reactive power compensation.
5. Reliable operation.

Disadvantages :

1. It requires addition power supply to operate operation amplifie so, it is expensive.

2. Passive filters : The filters which consisting of passive elements like R, L, C, then it is called passive filters.

Advantages :

1. Cheap
2. Designing is easy
3. High efficiency
4. High reliable.

Disadvantages :

1. It is restricted to particular harmonics
2. It is fixed reactive power compensation
3. It is tunning for fixed frequency.
4. In this filter, L is used. These are larger in size and bulky. It needs expensive.

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* Basic definitions:

1. Pass band: The range of frequencies over which attenuation is zero. Then it is called pass band.
2. Stop band: The range of frequencies over which attenuation is infinity, then it is called stop band.
3. Cut off frequency: It is the frequency which seperates pass band from stop band.

* Filter Fundamentals:

→ The complete study of any filter section needs the calculation of (a) characteristic impedance (Z_0 or Z_c).

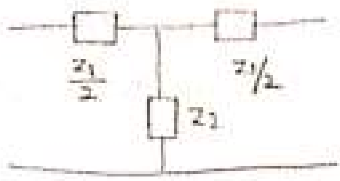
(b) Propagation constant (γ)

(c) Attenuation constant (α)

(d) Phase constant (β).

(a) Characteristic Impedance (Z_0 or Z_c):

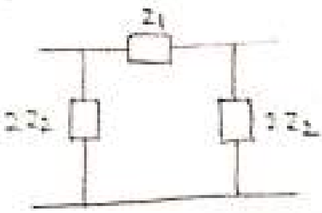
(i) For T-network:



$$Z_{0T} = \sqrt{\left(\frac{Z_1}{2}\right)^2 + Z_1 Z_2}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_{0T}}{Z_2}$$

(ii) For π-network:



$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

$$e^{\gamma} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_1}{Z_{0\pi}}$$

* Classification of filters:

Filters are broadly classified into 3 types.

1. constant K-filters
2. m-derived filters
3. composite filters

1. Constant K-filters:

→ In this type of filters, it satisfies the following relation.

$Z_1 Z_2 = K^2$ or R_0^2 → Designed impedance.

→ Constant K-filters are classified into 4 types:

1. Constant K low pass filters.
2. Constant K high pass filters
3. Constant K band pass filters
4. Constant K band elimination filters.

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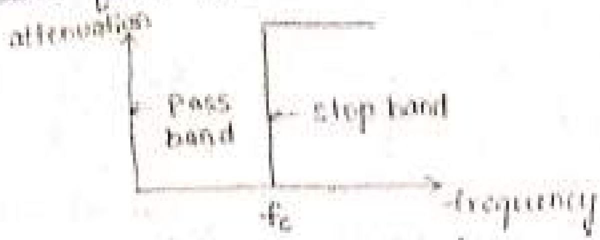
constant K low pass filter:

→ It is the filter which passes the signal upto cut off frequency and attenuates higher than the cut off frequency.

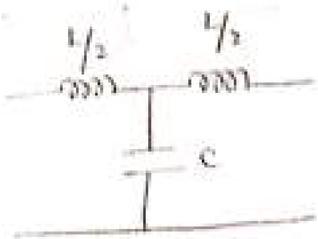
→ The wave form is



In this case, let $Z_1 = j\omega L$; $Z_2 = \frac{1}{j\omega C}$

$\frac{Z_1}{2} = j\omega\left(\frac{L}{2}\right)$, $Z_2 = \frac{1}{j\omega C}$

$\frac{Z_1}{2} \propto \frac{L}{2}$, $Z_2 \propto \frac{1}{C}$

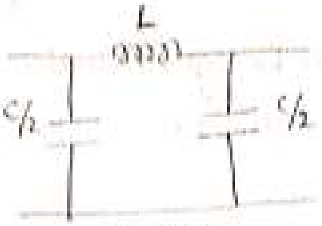


T - N/W

For π N/W

$Z_1 \propto L$, $2Z_2 = \frac{2}{j\omega C}$

$2Z_2 \propto \frac{1}{(C/2)}$



π - N/W

(i) Design Impedance (R_0) :

We know that, $Z_1 Z_2 = R_0^2$

$R_0 = \sqrt{Z_1 Z_2}$

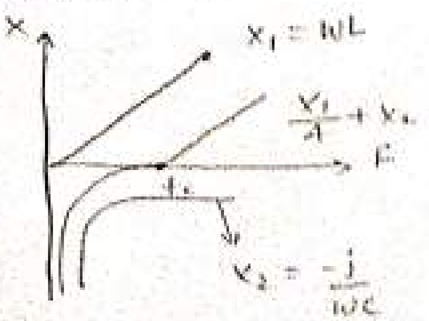
$= \sqrt{(j\omega L) \times \frac{1}{j\omega C}}$

$$\boxed{R_0 = \sqrt{\frac{L}{C}}}$$

(ii) Cut off frequency:

$Z_1 = j\omega L$	$Z_2 = \frac{1}{j\omega C}$
$X_1 = j\omega L$	$X_2 = \frac{-j}{\omega C}$
$X_1 \propto \omega L$	$X_2 \propto -\frac{1}{\omega C}$

Reactance curves:



from the graph,

At f_c: $\frac{X_1}{4} + X_2 = 0$

$\frac{\omega L}{4} - \left(\frac{1}{\omega C}\right) = 0$

$\frac{\omega L}{4} = \frac{1}{\omega C}$

$(2\pi f_c)^2 LC = 4$

$f_c^2 = \frac{1}{\pi^2 LC}$

f_c (cut off frequency) $= \frac{1}{\pi\sqrt{LC}}$

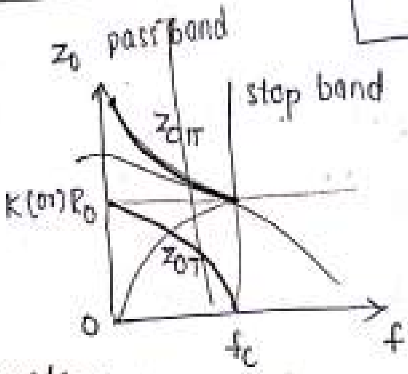
(iii) Z_{0T} & $Z_{0\pi}$

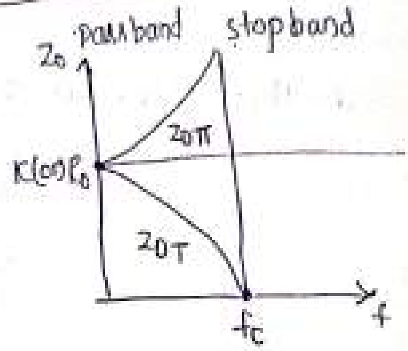
$$Z_{0T} = \sqrt{\left(\frac{Z_1}{2}\right)^2 + Z_1Z_2}$$

$$\boxed{Z_{0T} = R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

$$Z_{0\pi} = \frac{Z_1Z_2}{Z_{0T}} = \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

$$\boxed{Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}}$$





(iv) Design Equations:

We know that, $R_0 = \sqrt{\frac{L}{C}}$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

case(i): $\frac{R_0}{f_c} = \sqrt{\frac{L}{C}} \times \pi\sqrt{LC}$

$= \pi L \Rightarrow \boxed{L = \frac{R_0}{\pi f_c}}$ henry (H)

case(ii) $R_0 f_c = \sqrt{\frac{L}{C}} \times \frac{1}{\pi\sqrt{LC}}$

$= \frac{1}{\pi C} \Rightarrow \boxed{C = \frac{1}{\pi R_0 f_c}}$ farad (F).

1. Design a constant K low pass filter of T and π N/w with a cut off frequency of 2KHz with a load resistance termination of 500Ω.

Sol: Given that,

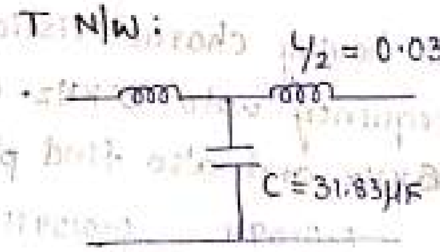
$f = 2000\ Hz$

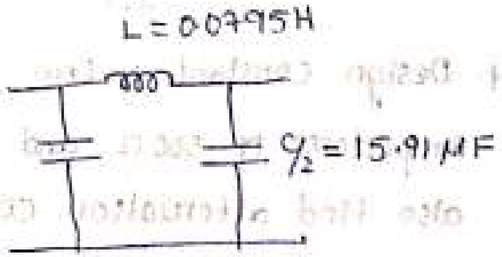
$R_0 = 500\Omega$

$$L = \frac{R_0}{\pi f_c} = 0.0795\ H$$

$$C = \frac{1}{R_0 \pi f_c} = 31.83\mu F.$$

T N/w:





2. A low pass filter of T-section has series inductance of 80 mH and shunt capacitance of 0.022 μF. Determine the cut-off frequency nominal designed Impedance R_0 and also design equivalent T and π networks.

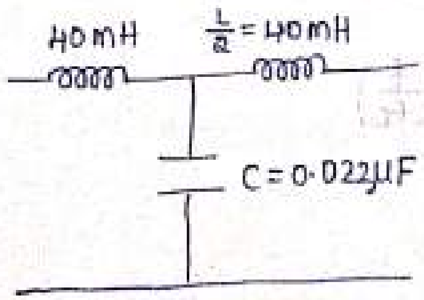
Sol: Given,

$L = 80mH$

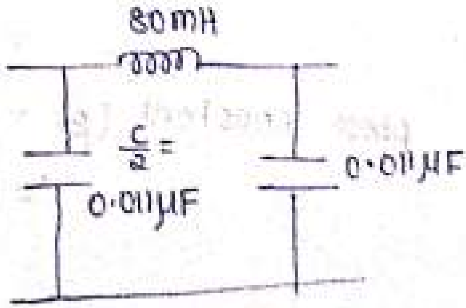
$C = 0.022\mu F$

$$R_0 = \sqrt{\frac{L}{C}} = 1906.93\Omega$$

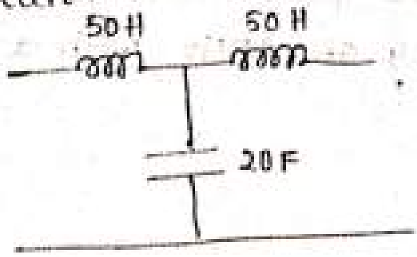
$$f_c = \frac{1}{\pi\sqrt{LC}} = 7587.41\ Hz$$



T n/w



3. Find the designed impedance and cut off frequency for the following circuit.



sol: Given,

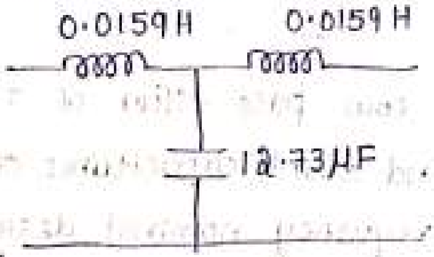
$L = 100\,H$

$C = 20\,F$

$R_0 = \sqrt{\frac{L}{C}} = 2.236\,\Omega$

$f_c = \frac{1}{\pi\sqrt{LC}} = 0.007117\,Hz$

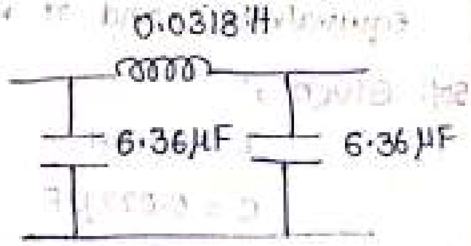
4. Design constant K low pass filter having characteristic impedance of 500Ω and to pass frequency upto 5 KHz. And also find attenuation constant at 6 KHz and also find phase constant at 04 KHz.



sol: Given,

$R_0 = 500\,\Omega$

$f_c = 5\,KHz$



$L = \frac{R_0}{\pi f_c} = 0.0318\,H$

$C = \frac{1}{\pi R_0 f_c} = 12.73\,\mu F$

Attenuation constant $(\alpha) = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$

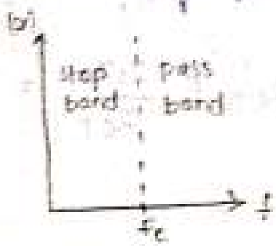
$= 1.244$

phase constant $(\beta) = 2\sin^{-1}\left(\frac{f}{f_c}\right)$

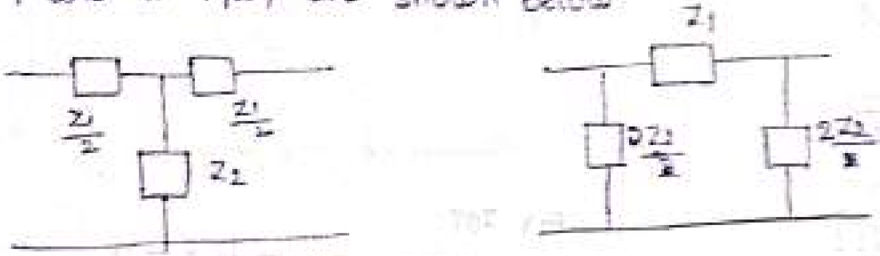
$= 106.26°$

Constant 'k' High pass filter:

→ The filter which attenuates the signal upto cut off frequency and passes the all other frequencies above cut off frequency is called as constant-k high pass filter.



→ In this high pass filters $z_1 = \frac{1}{j\omega C}$; $z_2 = j\omega L$; then the nominal T and π n/w's are shown below.



$z_1 = \frac{1}{j\omega C}$, $\frac{z_1}{2} = \frac{1}{j\omega(2C)}$, $\frac{z_1}{2} \propto \frac{1}{2C}$

$z_2 \propto L$

$2z_2 = j\omega(2L)$

1. Design Impedance:

$z_1 z_2 = R_0^2$

$R_0 = \sqrt{z_1 z_2}$

$= \sqrt{\frac{1}{j\omega C} \times j\omega L}$

$= \sqrt{\frac{L}{C}}$

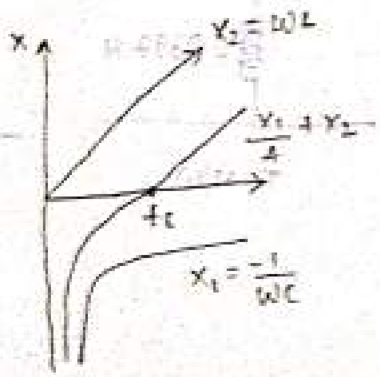
2. Cut off frequency:

$z_1 = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$x_1 = \frac{-1}{\omega C} \Rightarrow x_1 \propto \frac{-1}{\omega C}$

$z_2 = j\omega L$

$x_2 = j\omega L \Rightarrow x_2 \propto \omega L$



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from graph,

at f_c,

$$\frac{X_1}{4} + X_2 = 0$$

$$\frac{-1}{4\omega C} + \omega L = 0$$

$$\frac{+1}{4\omega C} = \omega L$$

$$4\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{4LC}$$

$$\omega = \frac{1}{2\sqrt{LC}}$$

$$2\pi f_c = \frac{1}{2\sqrt{LC}}$$

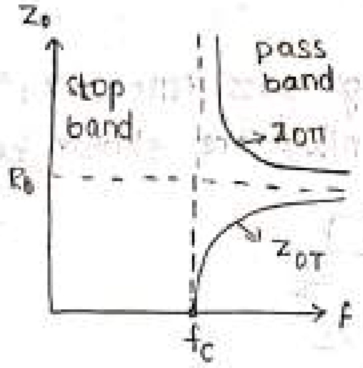
$$\boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

3. Z_{0T} & $Z_{0\pi}$

$$Z_{0T} = \sqrt{\left(\frac{Z_1}{2}\right)^2 + Z_1 Z_2}$$

$$= R_0\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{R_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$



For Z_{0T}:

put $f = f_c$, $Z_{0T} = \sqrt{1-1} = 0$

$f = \infty$, $Z_{0T} = R_0$

4 Design Equation:

We know, $R_0 = \sqrt{\frac{L}{C}}$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

case (i): $\frac{R_0}{f_c} = \sqrt{\frac{L}{C}} \times 4\pi\sqrt{LC} = 4\pi L \Rightarrow \boxed{L = \frac{R_0}{4\pi f_c}}$

case (ii): $R_0 \cdot f_c = \sqrt{\frac{L}{C}} \times \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4\pi C} \Rightarrow \boxed{C = \frac{1}{4\pi R_0 f_c}}$

1. Design constant K high pass filter if design impedance is 500Ω and cut off frequency is 1000 Hz.

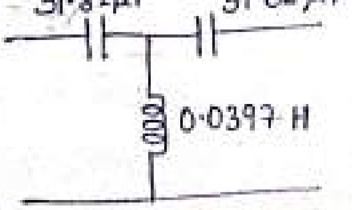
Sol: Given,

$R_0 = 500\Omega$

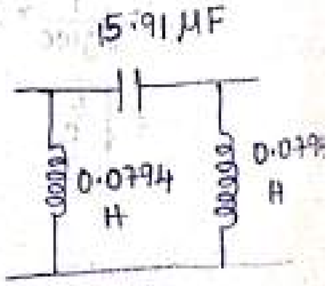
$f_c = 1000$ Hz

$$L = \frac{R_0}{4\pi f_c} = 0.0397 \text{ H}$$

$$C = \frac{1}{4\pi R_0 f_c} = 15.91\,\mu F$$

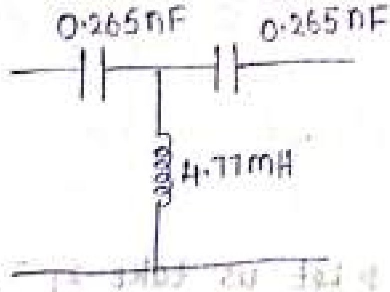


T N/W



π N/W

2. Design a T section constant K high pass filter having $f_c = 10\text{KHz}$ and $R_0 = 600\Omega$. Find its characteristic impedance (Z_{0T}) and phase constant ($Z_{0\pi}$) at 25 KHz



Sol: Given,

$f_c = 10\text{KHz}$

$R_0 = 600\Omega$

$$L = \frac{R_0}{4\pi f_c} = 0.0047\ \text{H} \ \text{or}\ 4.77\ \text{mH}$$

$$C = \frac{1}{4\pi R_0 f_c} = 0.132\text{nF}$$

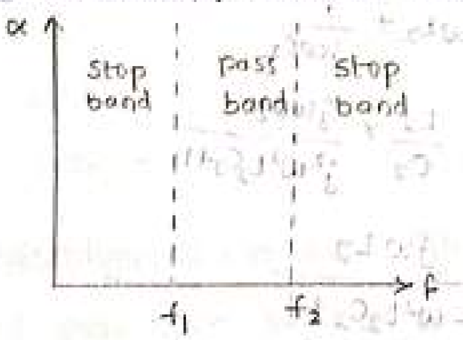
$$Z_{0T} = R_0\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 600\sqrt{1-\left(\frac{10\times10^3}{25\times10^3}\right)^2} = 549.9$$

$$Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 654.65$$

$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right) = 47.15^\circ$$

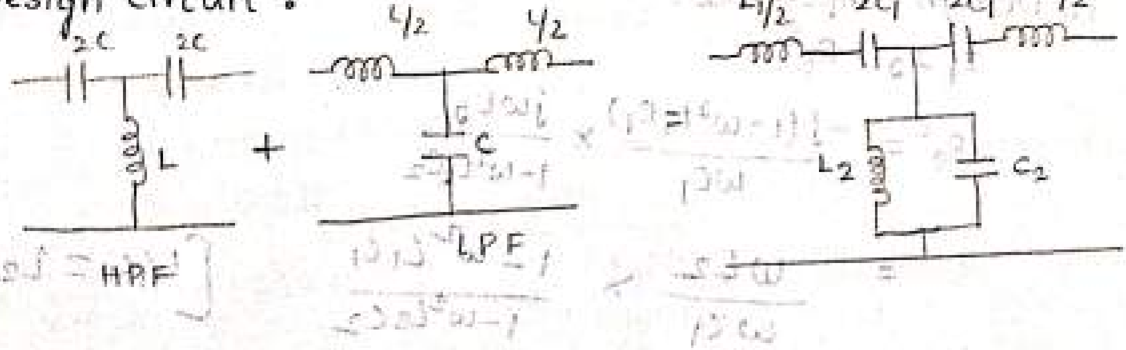
Constant K Band pass Filters:

→ Band pass Filters passes certain range of frequencies i.e., f_1 to f_2 and attenuates below & above frequency band.

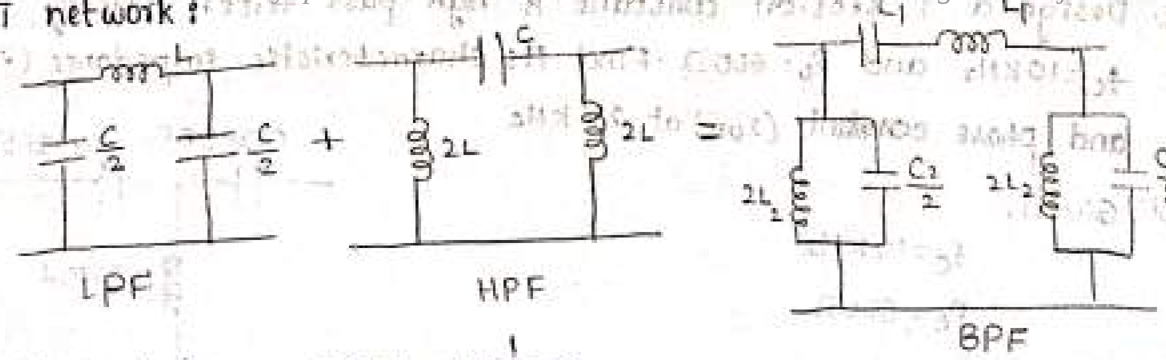


= HPF + LPF

Design circuit:



π network :



Let us take $z_1 = j\omega L_1 + \frac{1}{j\omega C_1}$

$z_2 = j\omega L_2$ parallel with $\frac{1}{j\omega C_2}$.

$$z_1 = j\omega L_1 - \frac{j}{\omega C_1}$$

$$= \frac{j\omega L_1 \times \omega C_1 - j}{\omega C_1}$$

$$= \frac{j\omega^2 L_1 C_1 - j}{\omega C_1}$$

$$= \frac{-j(1-\omega^2 L_1 C_1)}{\omega C_1}$$

$$z_2 = \frac{j\omega L_2 \times \frac{1}{j\omega C_2}}{j\omega L_2 + \frac{1}{j\omega C_2}}$$

$$= \frac{L_2}{C_2} \times \frac{j\omega C_2}{j^2\omega^2 L_2 C_2 + 1}$$

$$= \frac{j\omega L_2}{-\omega^2 L_2 C_2 + 1}$$

(i) Design Impedance :

$$z_1 z_2 = R_0^2$$

$$R_0^2 = \frac{-j(1-\omega^2 L_1 C_1)}{\omega C_1} \times \frac{j\omega L_2}{1-\omega^2 L_2 C_2}$$

$$= \frac{\omega L_2}{\omega C_1} \times \frac{1-\omega^2 L_1 C_1}{1-\omega^2 L_2 C_2} \qquad [L_1 C_1 = L_2 C_2]$$

$$\boxed{R_0^2 = \frac{L_2}{C_1}} \qquad \boxed{R_0 = \sqrt{\frac{L_2}{C_1}}}$$

(ii) Cut - off frequency :

At f_c , $\frac{z_1}{4} + z_2 = 0 \longrightarrow (1)$

where : z_1 = HPF + LPF

$$= \frac{1}{j\omega C_1} + j\omega L_1$$

z_2 = LPF + HPF

$$= j\omega L_2 + \frac{1}{j\omega C_2}$$

By solving z_1 & z_2 and substitute in eqn (1),

we get,

lower cut off frequency $f_1 = \frac{R_0 + \sqrt{R_0^2 + \frac{L_1}{C_1}}}{2\pi L_1}$

Higher cut off frequency $f_2 = -\left(\frac{R_0 + \sqrt{R_0^2 + \frac{L_1}{C_1}}}{2\pi L_1}\right)$

Resonant frequency $f_0 = \sqrt{f_1 f_2}$.

(iii) Design Equations :

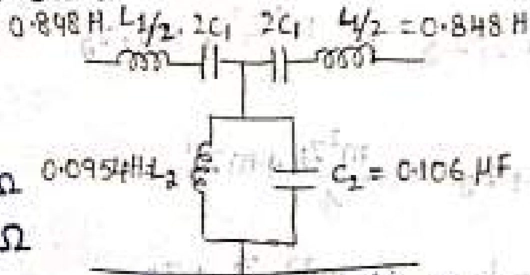
$$Z_{0T}(BPF) = Z_{0T}(HPF) + Z_{0T}(LPF)$$

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} \qquad C_1 = \frac{f_2 - f_1}{4\pi R_0 f_1 f_2}$$

$$L_2 = \frac{R_0(f_2 - f_1)}{4\pi f_1 f_2} \qquad C_2 = \frac{1}{\pi R_0 (f_2 - f_1)}$$

1. Design a prototype band pass filter if design Impedance $R_0 = 4K\Omega$ and pass band between 1.25 KΩ and 2 KΩ

Sol: Given that



$10^{-6} \times 10^{-3}$

$R_0 = 4 K\Omega$

$f_1 = 1250\ \Omega$

$f_2 = 2000\ \Omega$

$2C_1 = 11.92$ nF

$L_1 = \frac{R_0}{\pi(f_2 - f_1)} = 1.697$ H $\qquad L_2 = \frac{R_0(f_2 - f_1)}{4\pi f_1 f_2} = 0.0954$ H

$C_2 = \frac{1}{\pi R_0 (f_2 - f_1)} = 0.1061\ \mu F \qquad C_1 = \frac{f_2 - f_1}{4\pi R_0 f_1 f_2} = 5.96$ nF

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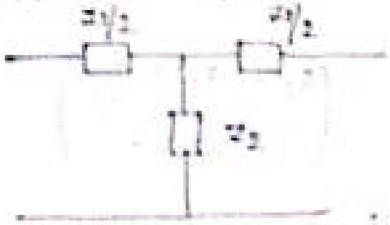
Disadvantages of prototype filters:

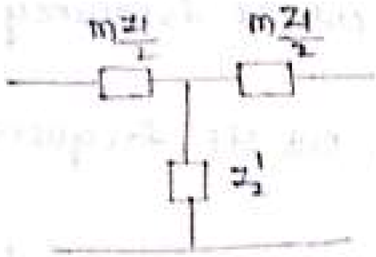
1. In case of Ideal, signal passes only in pass band and attenuates the signal in stop band. But in case of prototype filters, some of the signal passes through stop band also.
2. Ideally, for any line or in case of pass band characteristic impedance Z_{0T} & $Z_{0\pi}$ are always constant. But in case of prototype filters it varies from starting to cutoff frequency varies with frequencies.

→ In order to avoid two above difficulties, m-derived filters are introduced.

Derivations of equations for T and π sections in m-derived filters:



normal T-N/W



m-derived T-N/W

Characteristic impedances,

$$Z_{0T} = \sqrt{\left(\frac{Z_1}{2}\right)^2 + Z_1 Z_2} \quad , \quad Z_{0T(m\text{-}de)} = \sqrt{\left(\frac{mZ_1}{2}\right)^2 + mZ_1 Z_2'}$$

Inorder to maintain same characteristic impedance,

$$Z_{0T(n)} = Z_{0T(m\text{-}de)}$$

$$(Z_{0T_n})^2 = (Z_{0T_m})^2$$

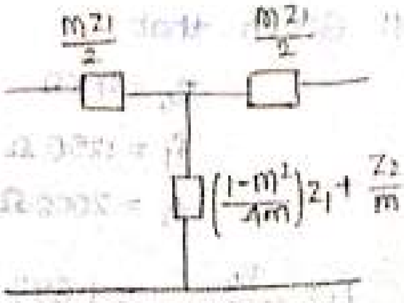
$$\left(\frac{Z_1}{2}\right)^2 + Z_1 Z_2 = \left(\frac{mZ_1}{2}\right)^2 + mZ_1 Z_2'$$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + mZ_1 Z_2'$$

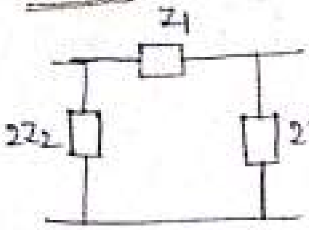
$$\frac{Z_1}{4} + Z_2 = \frac{m^2 Z_1}{4} + mZ_2'$$

$$mZ_2' = (1-m^2)\frac{Z_1}{4} + Z_2$$

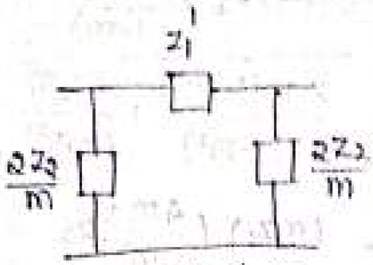
$$\boxed{Z_2' = \left(\frac{1-m^2}{4m}\right) Z_1 + \frac{Z_2}{m}}$$



π - N/W



normal - π



m-derived π

Characteristic Impedance :

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{\left(\frac{Z_1}{2}\right)^2 + Z_1 Z_2}} \quad , \quad Z_{0\pi(m)} = \frac{Z_1' \frac{Z_2}{m}}{\sqrt{\left(\frac{Z_1'}{2}\right)^2 + Z_1' \frac{Z_2}{m}}}$$

$$= \frac{\sqrt{Z_1 Z_2}\sqrt{Z_1 Z_2}}{\sqrt{Z_1 Z_2}\left(\sqrt{1+\frac{Z_1}{4Z_2}}\right)} \qquad = \frac{Z_1' \frac{Z_2}{m}}{\sqrt{Z_1' \frac{Z_2}{m}}\left(\sqrt{1+\frac{Z_1' m}{4Z_2}}\right)}$$

$$= \frac{\sqrt{Z_1 Z_2}}{\sqrt{1+\frac{Z_1}{4Z_2}}} \qquad = \frac{\sqrt{Z_1'^2 \frac{Z_2}{m}}}{\sqrt{1+\frac{Z_1' m}{4Z_2}}}$$

Remove square root on both sides

$$\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}} = \frac{Z_1' \frac{Z_2}{m}}{1+\frac{Z_1' m}{4Z_2}}$$

$$Z_1 Z_2 + \frac{Z_1 Z_1' m}{4} = Z_1' \frac{Z_2}{m} + \frac{Z_1 Z_1'}{4m}$$

$$Z_1'\left(\frac{Z_1}{4m} + \frac{Z_2}{m} - \frac{Z_1 m}{4}\right) = Z_1 Z_2$$

$$Z_1'\left[\frac{Z_1}{4}\left(\frac{1}{m} - m\right) + \frac{Z_2}{m}\right] = Z_1 Z_2$$

$$Z_1' = \frac{Z_1 Z_2}{Z_1\left(\frac{1-m^2}{4m}\right) + \frac{Z_2}{m}}$$

$$= \frac{Z_1 Z_2 (4m)}{Z_1(1-m^2) + 4Z_2}$$

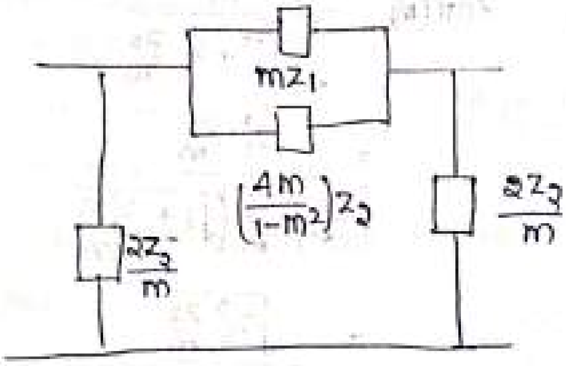
$$= \frac{(mZ_1)(4Z_2)}{Z_1(1-m^2)+4Z_2}$$

$$= \frac{(mZ_1)(4Z_2)}{Z_1(1-m^2)+4Z_2} \times \frac{\frac{m}{1-m^2}}{\frac{m}{1-m^2}}$$

Scanned with CamScanner

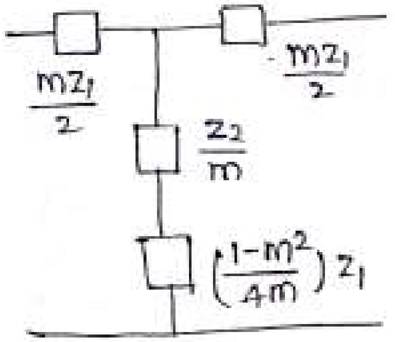
$$= \frac{(mZ_1) Z_2 \times \frac{4m}{1-m^2}}{\frac{Z_1 m(1-m^2)}{(1-m^2)} + \frac{4Z_2 m}{(1-m^2)}}$$

$$= \frac{(mZ_1)\left(\frac{4m}{1-m^2}\right)Z_2}{mZ_1 + \left(\frac{4m}{1-m^2}\right)Z_2} \Rightarrow \frac{ab}{a+b}$$

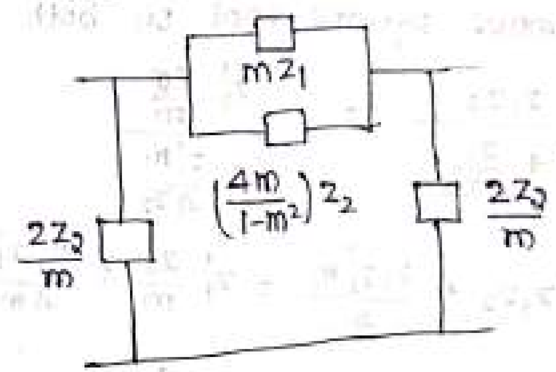


M-derived low pass filters:

Normal T n/w

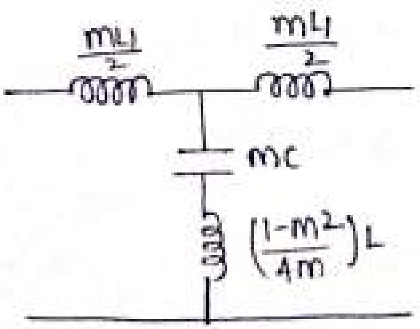


Normal π n/w

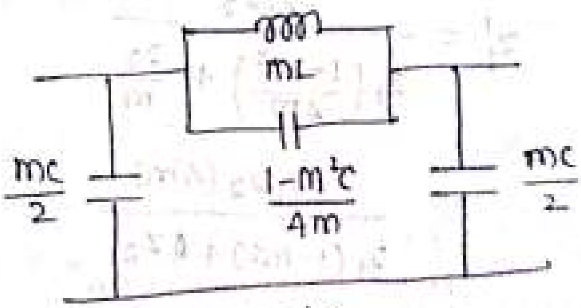


In case of LPF, we know $Z_1 = j\omega L$, $Z_2 = \frac{1}{j\omega C}$, then

$$\frac{mZ_1}{2} = j\omega \frac{mL}{2} \quad , \quad \frac{Z_2}{m} = \frac{1}{j\omega(mC)}$$



T N/W



π N/W

Consider shunt arm of T section resonates on a frequency of infinite attenuation that means f_α which is selected just above cut off frequency f_c. Then the frequency of resonance is given by

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$$f_\alpha = \frac{1}{2\pi\sqrt{(mC)\times\left(\frac{1-m^2}{4m}\right)L}}$$

$$= \frac{1}{2\pi\sqrt{LC\left(\frac{1-m^2}{4}\right)}}$$

$$= \frac{1}{\pi\sqrt{LC(1-m^2)}} \longrightarrow (1)$$

We know that in case of normal LPF

$$f_c = \frac{1}{\pi\sqrt{LC}} \longrightarrow (2)$$

Substitute eqn(2) in eqn(1)

$$f_c = \frac{\frac{1}{\pi\sqrt{LC}}}{\sqrt{1-m^2}} \Rightarrow f_\alpha = \frac{f_c}{\sqrt{1-m^2}}$$

$$\sqrt{1-m^2} = \frac{f_c}{f_\alpha}$$

$$(\sqrt{1-m^2})^2 = \left(\frac{f_c}{f_\alpha}\right)^2$$

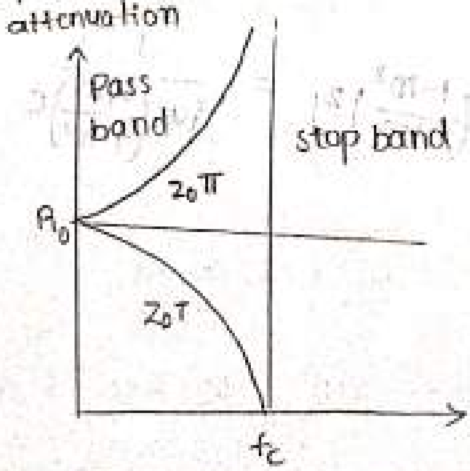
$$1-m^2 = \left(\frac{f_c}{f_\alpha}\right)^2$$

$$m^2 = 1-\left(\frac{f_c}{f_\alpha}\right)^2$$

$$m = \sqrt{1-\left(\frac{f_c}{f_\alpha}\right)^2}$$

In this case the design equations and variation Z_0 vs f are same in case of normal LPF.

Z_0 vs f :



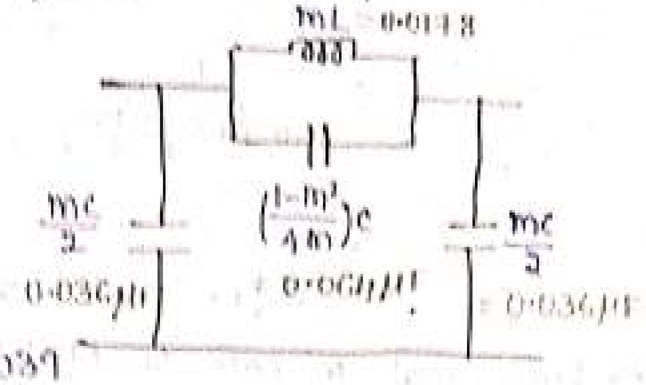
Design Equations:

$$L = \frac{R_0}{\pi f_c}$$

$$C = \frac{1}{R_0 \pi f_c}$$

1. Design m derived low pass filter to work into load of 500Ω and cut off frequency is 4KHz and peak attenuation at 4.5 KHz.

Ans: Given data,



$R_0 = 500\Omega$

$f_c = 4000$ Hz

$f_\infty = 4500$ Hz

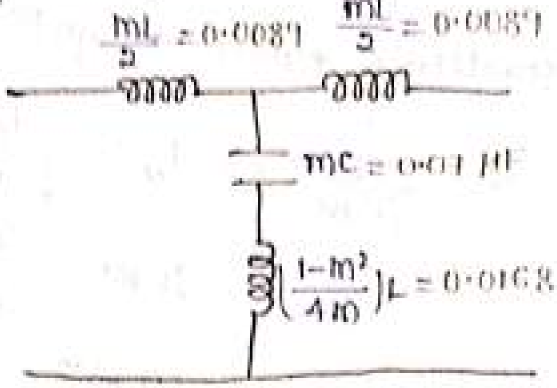
$$L = \frac{R_0}{\pi f_c} = \frac{500}{\pi \times 4000} = 0.039$$

$$C = \frac{1}{\pi R_0 f_c} = \frac{1}{\pi \times 500 \times 4000} = 0.159\ \mu F$$

$$m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$$

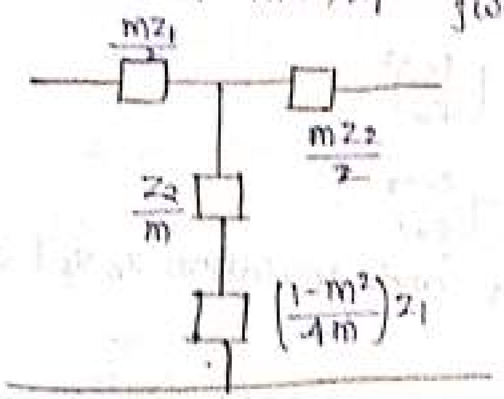
$$= \sqrt{1 - \left(\frac{4000}{4500}\right)^2}$$

$$= 0.458$$

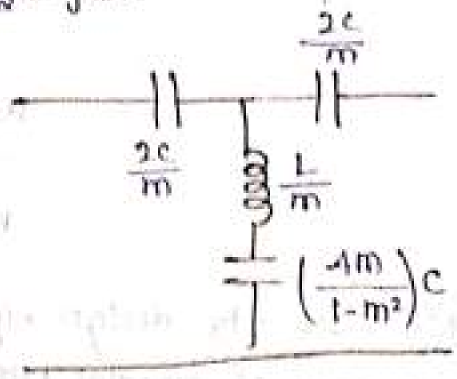


M-derived High Pass Filter:

In case of HPF, $Z_1 = \frac{1}{j\omega C}$, $Z_2 = j\omega L$



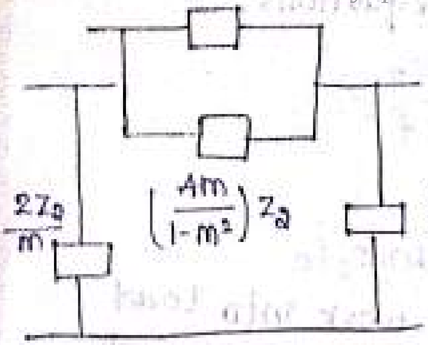
Normal T N/W

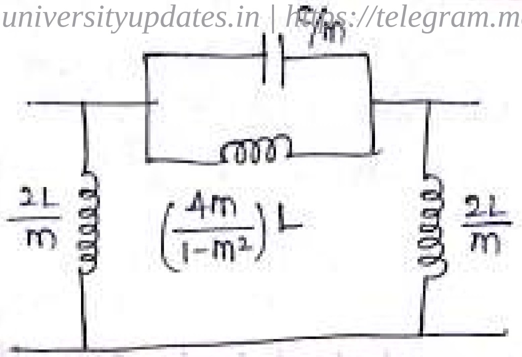


$$\frac{mZ_2}{2} = \frac{m}{2} \times \frac{1}{j\omega C} \qquad \left(\frac{1-m^2}{4m}\right)Z_1 = \frac{1}{j\omega\left(\frac{4m}{1-m^2}\right)C}$$

$$= \frac{1}{j\omega\left(\frac{2C}{m}\right)}$$

$$\frac{Z_2}{m} = j\omega\left(\frac{L}{m}\right)$$





$$mZ_1 = m\frac{1}{j\omega C} = \frac{1}{j\omega\left(\frac{C}{m}\right)} \qquad \left(\frac{4m}{1-m^2}\right)Z_2 = \left(\frac{4m}{1-m^2}\right)j\omega L$$

$$\frac{2Z_2}{m} = \frac{2}{m}j\omega L = j\omega\left(\frac{2L}{m}\right) \qquad = j\omega\left(\frac{4m}{1-m^2}\right)L$$

Consider shunt arm of T section resonates at a frequency of infinite attenuation that means f_α which is selected just above cut off frequency f_c then the frequency of resonance is given by

$$f_\alpha = \frac{1}{2\pi\sqrt{\left(\frac{L}{m}\right)\times\left(\frac{4m}{1-m^2}\right)C}}$$

$$f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}} \longrightarrow (1)$$

But in case of HPF

$$f_c = \frac{1}{4\pi\sqrt{LC}} \longrightarrow (2)$$

substitute eqn(2) in eqn(1)

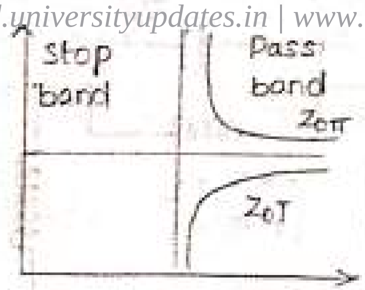
$$f_\alpha = \sqrt{1-m^2}\times f_c$$

$$\left(\frac{f_\alpha}{f_c}\right)^2 = 1-m^2$$

$$m^2 = 1-\left(\frac{f_\alpha}{f_c}\right)^2$$

$$m = \sqrt{1-\left(\frac{f_\alpha}{f_c}\right)^2}$$

Z_0 Vs f and eqns are same in case of HPF.



Design Equations:

$$L = \frac{R_0}{4\pi f c},$$

$$C = \frac{1}{4\pi R_0 f c}$$

1. Design m-derived high pass filter to work into load of 600Ω with cut off frequency of 1000/π Hz and peak attenuation frequency at 300 Hz.

Sol: Given that,

$$R_0 = 600\,\Omega$$

$$f_c = \frac{1000}{\pi} = 318.3\text{ Hz}$$

$$f_\alpha = 300\text{ Hz}$$

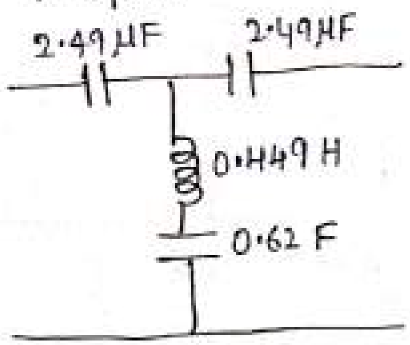
$$L = \frac{R_0}{4\pi f_c} = \frac{600}{4\pi \times 318.3} = 0.15\text{ H}$$

$$C = \frac{1}{4\pi R_0 f_c} = \frac{1}{4\pi \times 600 \times 318.3} = 0.147\ \mu F$$

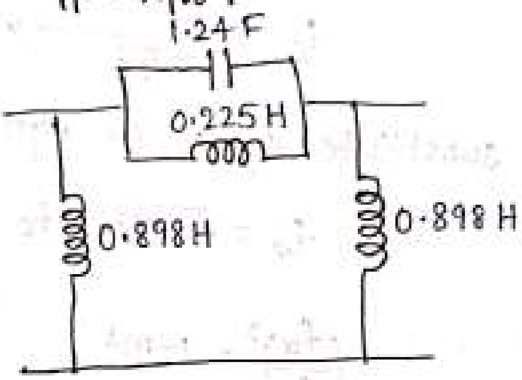
$$m = \sqrt{1 - \left(\frac{f_\alpha}{f_c}\right)^2}$$

$$= 0.334$$

T N/w:



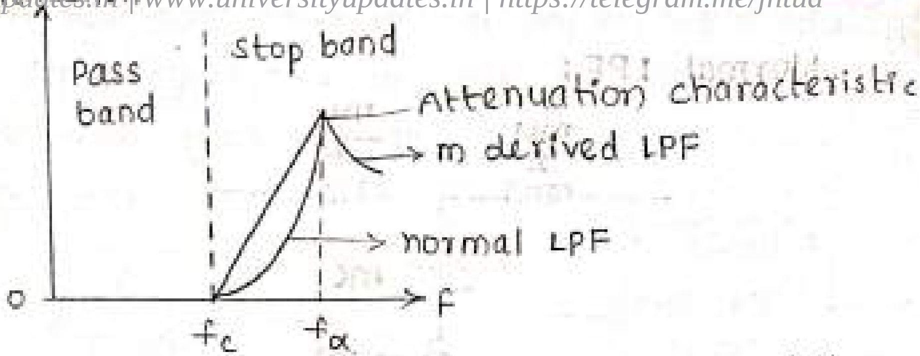
π- N/w:



Composite Filter:

→ Attenuation of prototype filter in stop band increases slowly from zero while in the m-derived filter, attenuation reaches very high value at a frequency (f_α).

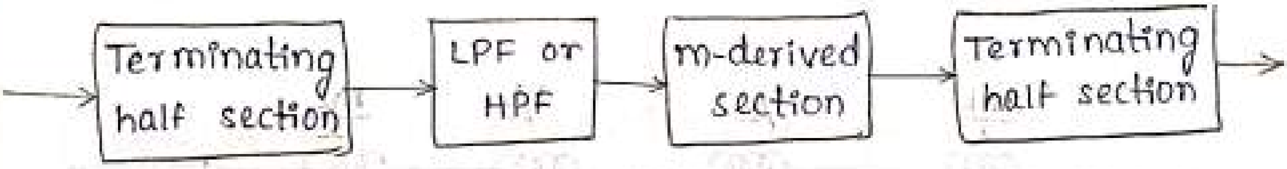
→ However, the attenuation decreases after f_α in the LPS and below f_α in High pass section.

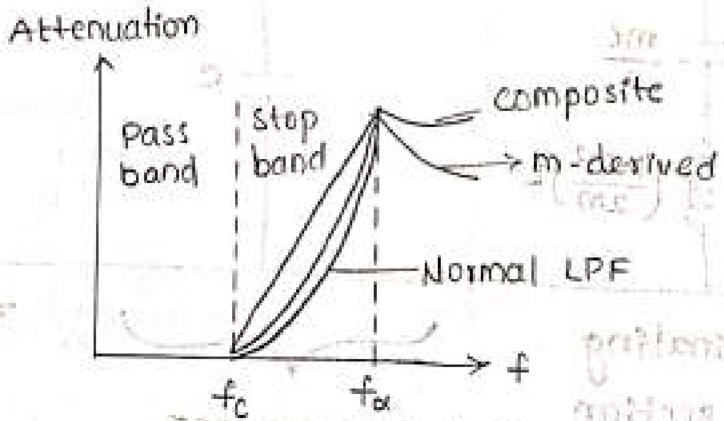


→ In order to avoid these difficulties, a special type filter called composite filter is introduced.

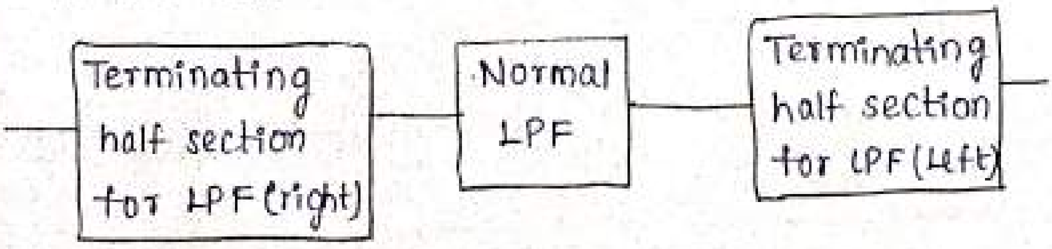
→ Composite filter consisting of

1. One or more constant k sections are prototype section to produce specific cut off frequency.
2. One or more m-derived sections in order to produce infinite attenuation which is closed to f_c.
3. Two no. of terminating half sections to provide almost constant input and output impedances.

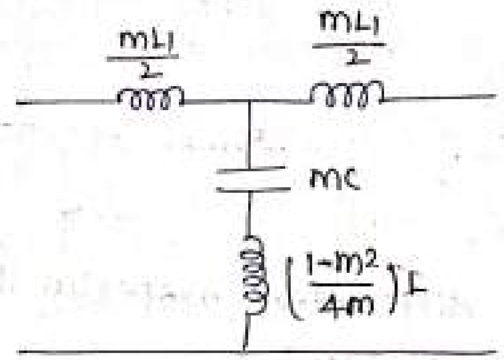


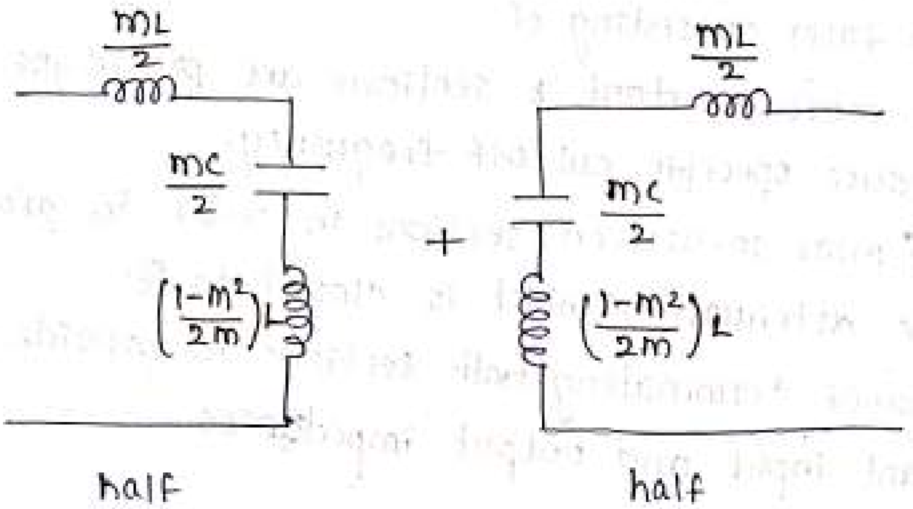


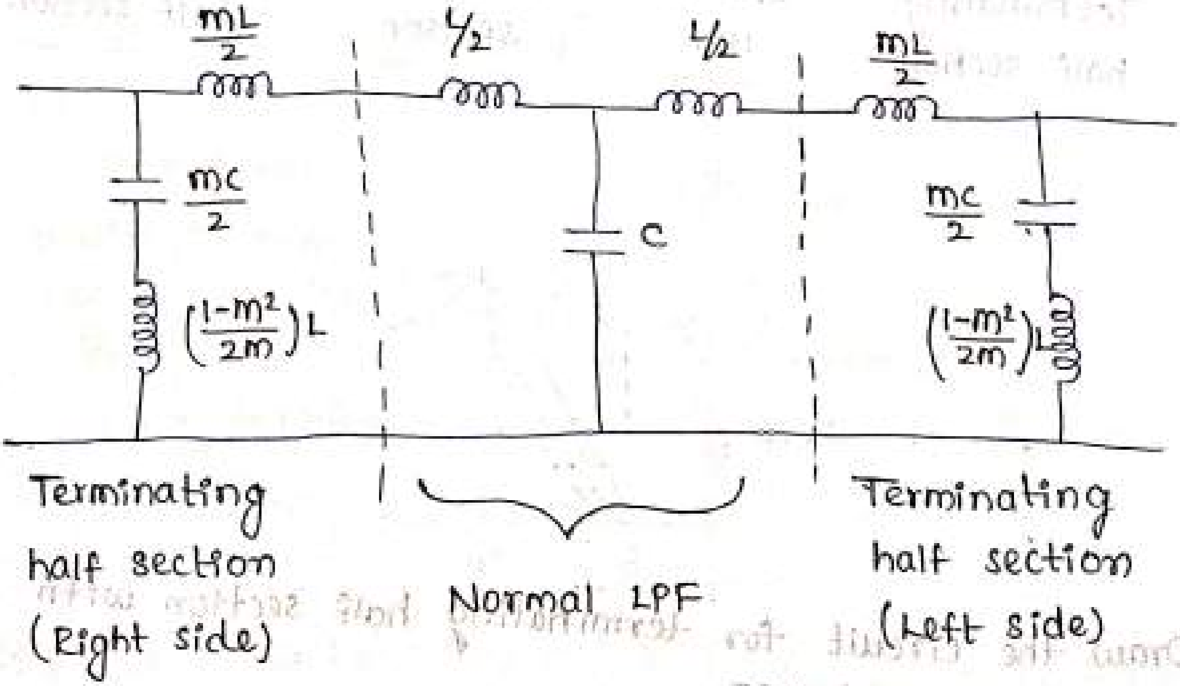
1. Draw the circuit for terminating half section with normal LPF and HPF.



Normal LPF :

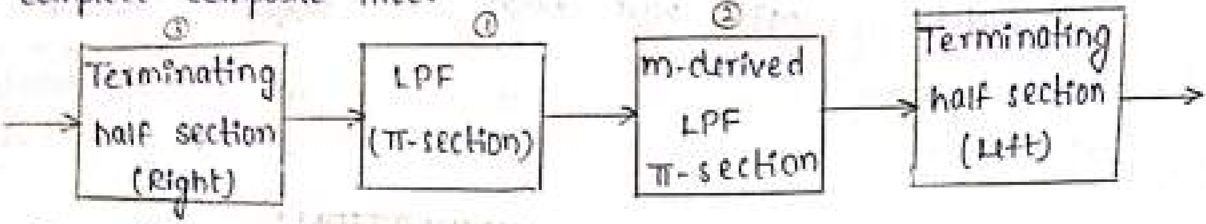

mL1/2
mL1/2
mC
(1-m²/4m) L


mL/2
mC/2
(1-m²/2m) L
+
mL/2
mC/2
(1-m²/2m) L
half
half


mL/2
mC/2
(1-m²/2m) L
L/2
L/2
C
mL/2
mC/2
(1-m²/2m) L
Terminating
half section
(Right side)
Normal LPF
Terminating
half section
(Left side)

2. Design a complete low pass filter π section to work into 600Ω if cut-off frequency of 2 kHz and very high attenuation of 2.2 KHz also terminate filter properly.

Sol: Complete composite filter.



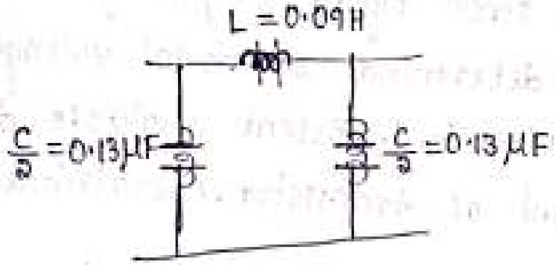
Given that,

$$R_0 = 600\Omega \; , \; f_c = 2000\text{ Hz} \; \text{ and } \; f_\alpha = 2200\text{ Hz}$$

(A) Normal LPF (π-section):

$$L = \frac{R_0}{\pi f_c} = \frac{600}{\pi \times 2000} = 0.09\text{ H}$$

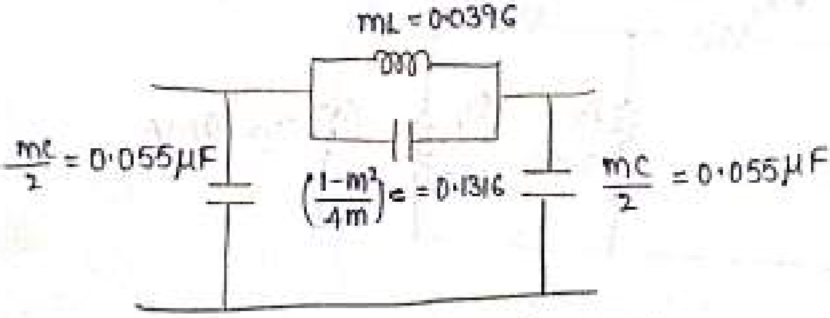
$$C = \frac{1}{R_0 \pi f_c} = \frac{1}{600 \times \pi \times 2000} = 0.265\ \mu F$$



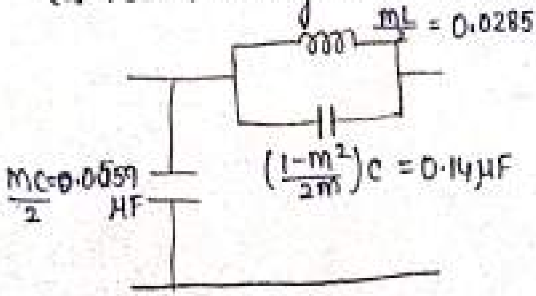
(B) M-derived low pass filter (π-section):

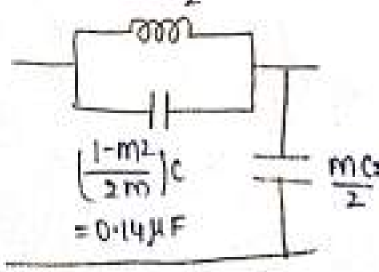
$$m = \sqrt{1-\left(\frac{f_c}{f_\alpha}\right)^2}$$

$$= 0.416$$

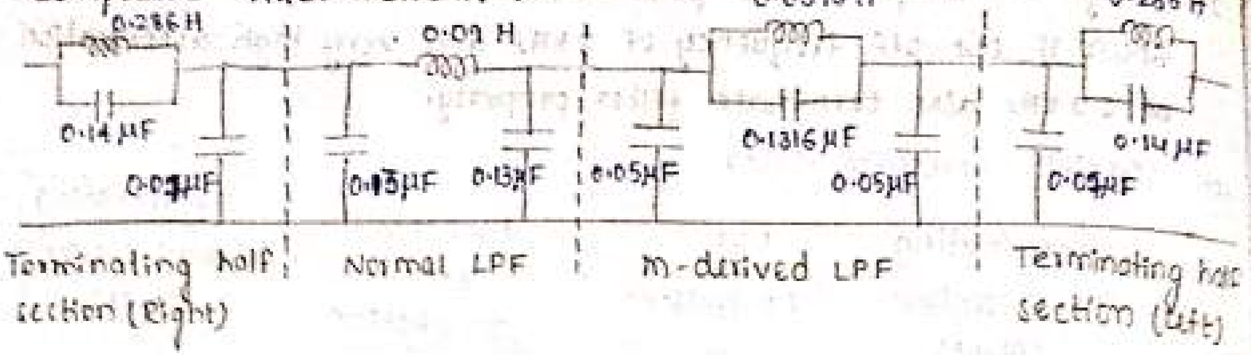


(C) Terminating half section (m = 0.6): $\frac{mL}{2} = 0.0285\text{ H}$





Composite filter circuit :

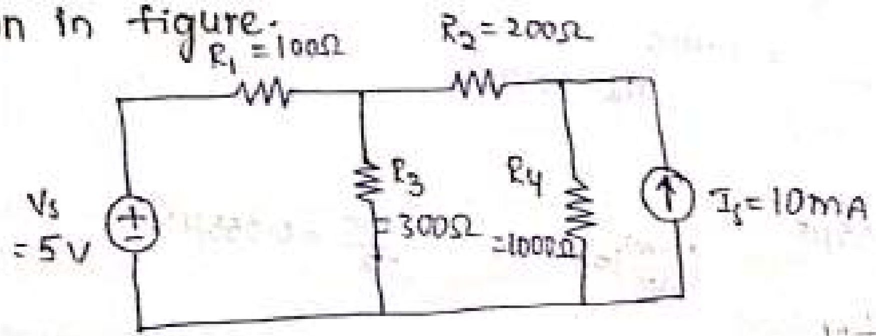


CIRCUIT SIMULATION:

→ PSPICE is a personal simulated program with integrated circuit emphasis.

→ PSPICE is a general purpose software that includes simulation of different circuits and can perform various analysis of electrical and electronic circuits including time domain response, small signal frequency response, total power dissipation, determinal of nodal voltages and mesh analysis in the circuit, transient analysis, determination of operating point of transistor, operational amplifiers etc.

1. Write the PSPICE program and (write) also find the current flowing through resistances R_2 & R_3 as shown in figure.



Sol: