A Course Module

on

FUNDAMENTALS OF ELECTRICAL CIRCUITS (20A02101T)

Prepared by

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SREE RAMA ENGINEERING COLLEGE

Approved by AICTE, New Delhi – Affiliated to JNTUA, Ananthapuramu
Accredited by NAAC with ‘A’ Grade
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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

B.Tech (EEE)– I Sem

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(20A02101T) FUNDAMENTALS OF ELECTRICAL CIRCUITS

Course Objectives:

To make the student learn about

- Basic characteristics of R, L, C parameters, their Voltage and Current Relations and Various combinations of these parameters.
- The Single Phase AC circuits and concepts of real power, reactive power, complex power, phase angle and phase difference
- Series and parallel resonances, bandwidth, current locus diagrams
- Network theorems and their applications
- Network Topology and concepts like Tree, Cut-set , Tie-set, Loop, Co-Tree

Unit- 1

Introduction to Electrical & Magnetic Circuits

Electrical Circuits: Circuit Concept – Types of elements - Source Transformation-Voltage - Current Relationship for Passive Elements. Kirchhoff's Laws – Network Reduction Techniques- Series, Parallel, Series Parallel, Star-to-Delta or Delta-to-Star Transformation. Examples

Magnetic Circuits: Faraday's Laws of Electromagnetic Induction-Concept of Self and Mutual Inductance-Dot Convention-Coefficient of Coupling-Composite Magnetic Circuit-Analysis of Series and Parallel Magnetic Circuits, MMF Calculations.

Learning Outcomes:

At the end of this unit, the student will be able to

- To know about Kirchhoff's Laws in solving series, parallel, non-series-parallel configurations in DC networks
- To know about voltage source to current source and vice-versa transformation in their representation
- To understand Faraday's laws
- To distinguish analogy between electric and magnetic circuits
- To understand analysis of series and parallel magnetic circuits

Unit- 2

Network Topology

Definitions – Graph – Tree, Basic Cutset and Basic Tieset Matrices for Planar Networks – Loop and Nodal Methods of Analysis of Networks & Independent Voltage and Current Sources – Duality & Dual Networks. Nodal Analysis, Mesh Analysis.

Learning Outcomes:

At the end of this unit, the student will be able to

- To understand basic graph theory definitions which are required for solving electrical circuits
- To understand about loop current method

- To understand about nodal analysis methods
- To understand about principle of duality and dual networks
- To identify the solution methodology in solving electrical circuits based on the topology

Unit- 3

Single Phase A.C Circuits

R.M.S, Average Values and Form Factor for Different Periodic Wave Forms – Sinusoidal Alternating Quantities – Phase and Phase Difference – Complex and Polar Forms of Representations, j-Notation, Steady State Analysis of R, L and C (In Series, Parallel and Series Parallel Combinations) with Sinusoidal Excitation-Resonance - Phasor diagrams - Concept of Power Factor- Concept of Reactance, Impedance, Susceptance and Admittance-Apparent Power, Active and Reactive Power, Examples.

Learning Outcomes:

At the end of this unit, the student will be able to

- To understand fundamental definitions of 1-ϕ AC circuits
- To distinguish between scalar, vector and phasor quantities
- To understand voltage, current and power relationships in 1-ϕ AC circuits with basic elements R, L, and C.
- To understand the basic definitions of complex immittances and complex power
- To solve 1-ϕ AC circuits with series and parallel combinations of electrical circuit elements R, L and C.

Unit- 4

Network Theorems

Superposition, Reciprocity, Thevenin's, Norton's, Maximum Power Transfer, Millmann's, Tellegen's, and Compensation Theorems for D.C and Sinusoidal Excitations.

Learning Outcomes:

At the end of this unit, the student will be able to

- To know that electrical circuits are 'heart' of electrical engineering subjects and network theorems are main part of it.
- To distinguish between various theorems and inter-relationship between various theorems
- To know about applications of certain theorems to DC circuit analysis
- To know about applications of certain theorems to AC network analysis
- To know about applications of certain theorems to both DC and AC network analysis

Unit- 5

Three Phase A.C. Circuits

Introduction - Analysis of Balanced Three Phase Circuits – Phase Sequence- Star and Delta Connection - Relation between Line and Phase Voltages and Currents in Balanced Systems - Measurement of Active and Reactive Power in Balanced and Unbalanced Three Phase Systems. Analysis of Three Phase Unbalanced Circuits - Loop Method - Star Delta Transformation Technique – for balanced and unbalanced circuits - Measurement of Active and reactive Power – Advantages of Three Phase System.

Learning Outcomes:

At the end of this unit, the student will be able to

- To know about advantages of 3-ϕ circuits over 1-ϕ circuits
- To distinguish between balanced and unbalanced circuits
- To know about phasor relationships of voltage, current, power in star and delta connected balanced and unbalanced loads
- To know about measurement of active, reactive powers in balanced circuits
- To understand about analysis of unbalanced circuits and power calculations

Text Books:

1. Fundamentals of Electric Circuits Charles K. Alexander and Matthew. N. O. Sadiku, Mc Graw Hill, 5th Edition, 2013.
2. Engineering circuit analysis William Hayt and Jack E. Kemmerly, Mc Graw Hill Company, 7th Edition, 2006.

Reference Books:

1. Circuit Theory Analysis & Synthesis A. Chakrabarti, Dhanpat Rai & Sons, 7th Revised Edition, 2018.
2. Network Analysis M.E Van Valkenberg, Prentice Hall (India), 3rd Edition, 1999.
3. Electrical Engineering Fundamentals V. Del Toro, Prentice Hall International, 2nd Edition, 2019.
4. Electric Circuits- Schaum’s Series, Mc Graw Hill, 5th Edition, 2010.
5. Electrical Circuit Theory and Technology John Bird, Routledge, Taylor & Francis, 5th Edition, 2014.

Course Outcomes:

After completing the course, the student should be able to do the following

- Given a network, find the equivalent impedance by using network reduction techniques and determine the current through any element and voltage across and power through any element.
- Given a circuit and the excitation, determine the real power, reactive power, power factor etc,.
- Apply the network theorems suitably
- Determine the Dual of the Network, develop the Cut Set and Tie-set Matrices for a given Circuit. Also understand various basic definitions and concepts.

UNIT- 1
INTRODUCTION TO ELECTRICAL & MAGNETIC CIRCUITS

BASIC DEFINITIONS:

Voltage(V) : The potential difference between force applied to two oppositely charged particles to bring them as near as possible is called as potential difference .(in electrical terminology it s voltage).

$V = W / Q$ (v)

$\upsilon = dw / dq$ (v)

Units- volts ,

Current(I) : The flow of electrons develops the current.

$I = Q/ t$ (A)

$i = dq / dt$ (A)

A = Ampere, units of current.

Power(P): It is defined as product of voltage and power in electrical circuits orRate of change of energy.

$P = dw/dt = dw/ dq . dq/dt = \upsilon .i$ (W)W = watts, units of power.

Energy : It is the capacity to do work or it is defined as power consumed overGiven interval of time.(w)

$W = \int P\, dt.$(J)

J = Joules, units of energy1J = 1 watt-sec.

Q 1. Types of Network Elements

We can classify the Network elements into various types based on some parameters.

Active Elements and Passive Elements

Linear Elements and Non-linear Elements

Bilateral Elements and Unilateral Elements

Lumped and Distributed Elements

Active Elements and Passive Elements

Active Elements deliver power to other elements, which are present in an electric circuit.. Examples: Voltage sources and current sources.

Passive Elements can’t deliver power (energy) to other elements, however they can absorb power. Examples: Resistors, Inductors, and capacitors.

Linear Elements and Non-Linear Elements

Linear Elements are the elements that show a linear relationship between voltage and current. Examples: Resistors, Inductors, and capacitors.

Non-Linear Elements are those that do not show a linear relation between voltage and current. Examples: Voltage sources and current sources.

Bilateral Elements and Unilateral Elements

Bilateral Elements are the elements that allow the current in both directions. Examples: Resistors, Inductors and capacitors.

Unilateral Elements are those that allow the current in only one direction.

Examples: Diode , SCR

Lumped- Distributed Element:

• Physical dimensions of circuit are such that voltage across and current through conductors connecting elements does not vary. Example :Resistor

Distributed Circuits:

• Current varies along conductors and elements;

• Voltage across points along conductor or within element varies

Example: Long transmission lines

Q 2. Explain Basic Parameters of Electrical Circuits
or
Write the properties of RLC elements
or
Explain V-I Relation ship in Passive elements
or
Derive the Energy storied in Inductor and Capacitor

Resistor: resistor is the element which restricts flow of electrons and thisProperty of opposing electrons is called as resistance.

OHM's Law : ohm's law states that current flowing through circuit isDirectly proportional to potential difference applied. (at constant temperature)

I α V, at constant T.

I.R = V.

R = V / I, hence resistance can be calculated as ratio of Volatge to current in any element or circuit.

= OHMS Ω, units of resistance.

R 

Inductor: An length of wire twisted forms the basic inductor.(L). when Alternating current is allowed through such a element it inducesVoltage in it.

L 

e, ϕ

Where, e = emf induced ; ϕ = flux developed in it for current i.

Hence, e = L di/dt (v).

L= is the inductance of the coil(H)

H = henry units of inductance.

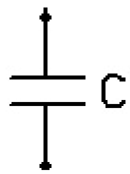
Energy stored in inductor,

W = $\int$ e.i dt = $\int$ L di/dt. i dt. = (1/2) L i^2

Properties:

- Inductor doesn’t allow sudden changes in current.
- If DC supply is provided to inductor it acts as short circuit.
- stores energy in the form of magnetic field.

Capacitor : two parallel plates oppositely charged separated by an dielectric medium constitutes an capacitor.



v, i

when some voltage v is applied , i is thecurrent flowing through capacitor, given as

i = c dv/dt

c = capacitance of the capacitor.(F)

Farad is the unit of capacitance,

Voltage across capacitor is given as, $v = (1/c) \int i\, dt + v(0+)$.

Energy stored in capacitor is , $w = \int v.i\, dt = \int c\, dv/dt.\, i\, dt. = (1/2)\, cv^2$

Properties:

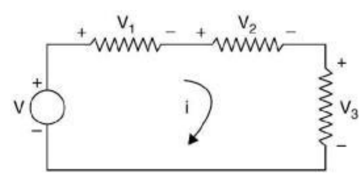
- Capacitor doesn’t allow sudden changes in voltage.
- If DC supply is provided to capacitor it acts as open circuit.
- Stores energy in the form of electric field.

Q3. Explain Kirchoff’s Laws

Or

Explain Voltage Division & Current Division Rules

Kirchoff’s voltage law(KVL): States that algebraic sum of voltages in an loop is equal to zero.



$\sum v = o.$

V = V1 + V2 + V3

Where, V1 = i.R1

V2 = i.R2

V3 = i.R3,

Voltage division rule:

V1 = i.R1

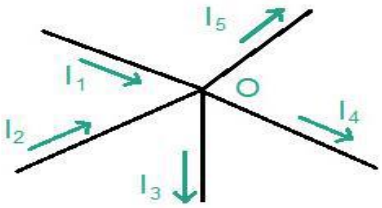
We know that , i = V / (R1 + R2 + R3) = Vs / Rt

Substituting i in V1, V1 = Vs / Rt * R1.

Similarly, V2 = Vs / Rt * R2.

V3 = Vs / Rt * R3.

Kirchoff's current law(KCL): States that sum of the current entering junction is equal to sumof the leaving the junction



Applying KCL:-

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

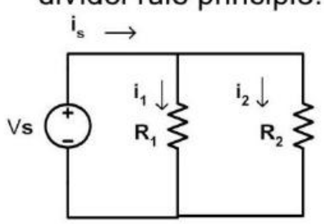
$$I_1 + I_2 = I_3 + I_4 + I_5$$

According to KCL:-

$$\boxed{\sum I = 0}$$

Current Division Rule :

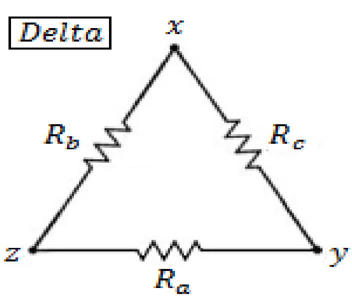
Whenever current has to be divided among resistors in parallel, use current divider rule principle.

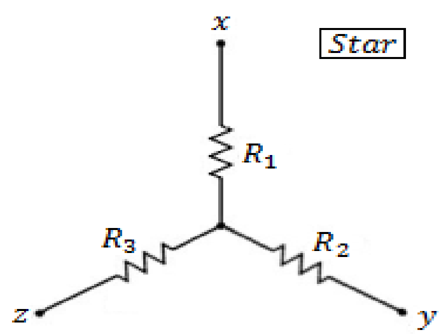


$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s$$

Q4. **Explain Star-to-Delta or Delta-to-Star Transformation**





$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_2 = \frac{R_aR_c}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Above figure shows elements connected in star and other elements connected in delta.

In star connection,

$R_{xy} = (R_1 + R_2)$

$R_{yz} = (R_2 + R_3)$

$R_{zx} = (R_3 + R_1)$

In delta connection,

$R_{xy} = R_c(R_a+R_b) / (R_a+R_b)$

$R_{yz} = R_a(R_c+R_b) / (R_c+R_b)$

$R_{zx} = R_b(R_a+R_c) / (R_a+R_c)$

Now for transformation resistance between the terminals should be equal

$(R_1 + R_2)= R_c(R_a+R_b) / (R_a+R_b)$ --------------------------- 1

$(R_2 + R_3)= R_a(R_c+R_b) / (R_c+R_b))$--------------------------- 2

$(R_3 + R_1) = R_b(R_a+R_c) / (R_a+R_c)$-----------------------------3

Delta to star transformation:

By solving (1+3-2) we can find R_1 in terms of Ra,Rb and Rc.

$R_1 = R_bR_c / (R_a+R_b+R_c)$

Similarly, for R_2 solve (1+2-3) and R_3 solve (2+3-1)

$R_2 = R_aR_c / (R_a+R_b+R_c)$

$R_3 = R_bR_a / (R_a+R_b+R_c)$

Star to Delta Transformation:

$R_a =(R_1 R_2+R_2 R_3+R_3 R_1)/ R_1$

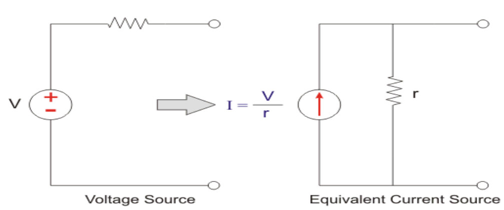
$R_b =(R_1 R_2+R_2 R_3+R_3 R_1)/ R_2$

$R_c =(R_1 R_2+R_2 R_3+R_3 R_1)/ R_3$

Q5. Explain Source Transformation Technique

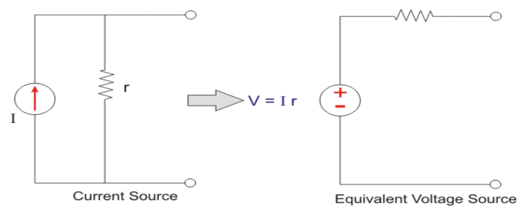
Voltage Source to Current Source Conversion

Assume a voltage source with terminal voltage V and the internal resistance r. This resistance is in series. The current supplied by the source is equal to: $I = \frac{V}{R}$
when the source of the terminals are shorted. This current is supplied by the equivalent current source and the same resistance r will be connected across the source. The voltage source to current source conversion is shown in the following figure.



Current Source to Voltage Source Conversion

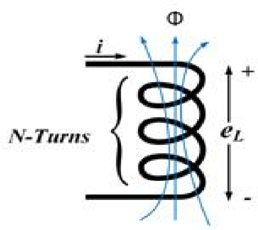
Similarly, assume a current source with the value I and internal resistance r. Now according to the Ohm's law, the voltage across the source can be calculated as V=I*R
Hence, voltage appearing, across the source, when terminals are open, is V.



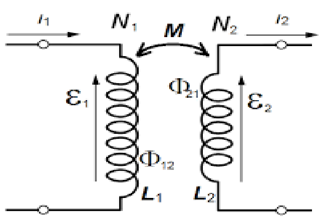
Q6.Explain Faraday's Laws of Electromagnetic Induction

Faraday's First law:
it states that
Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced.
Faraday's second law
it states that
The induced emf in a coil is equal to the rate of change of flux linkage

$$emf = -N\frac{d\emptyset}{dt}$$



Q7. Concept of Self and Mutual Inductance



Self Inductance

Self-induction means the coils induce the emf themselves. There is a change in the magnetic flux through that coil and because of this, the current will be induced in the coil by itself. So once the current get induced, the current tries to oppose the flux.

$Induced\ emf = -N\frac{d\emptyset}{dt}$

$Voltage\ across\ inductance = L\frac{di}{dt}$

From above equations

$$L = N\frac{d\emptyset}{di} \quad \text{Self Inductance}$$

Mutual Inductance

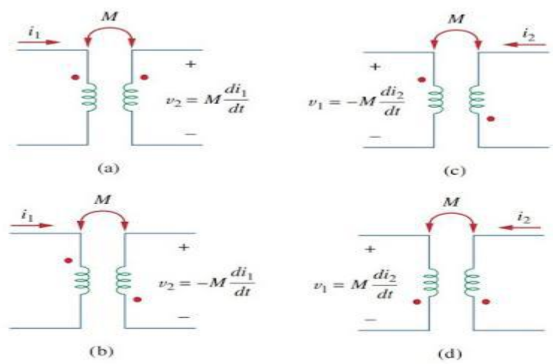
Here, there are two coils placed near each other. The first coil will make turns and carry the current which results in the magnetic field. As both the coils nearly close to each other, the magnetic field through one coil will all pass through the other coil. So one coil causes the change in magnetic flux because of which current is induced in the other coil.

$$Mutuallly\ Induced\ emf\ = -M\frac{di}{dt}$$

Q8. Importance of dot convention

Dot convention is a technique, which gives the details about voltage polarity at the dotted terminal. This information is useful, while writing KVL equations.

- If the current enters at the dotted terminal of one coil (or inductor), then it induces a voltage at another coil (or inductor), which is having positive polarity at the dotted terminal.
- If the current leaves from the dotted terminal of one coil (or inductor), then it induces a voltage at another coil (or inductor), which is having negative polarity at the dotted terminal.



Q9. Importance of dot convention

The amount of coupling between the inductively coupled coils is expressed in terms of the coefficient of coupling, which is defined as

Coefficient of Coupling $k = \frac{M}{\sqrt{L_1 L_2}}$

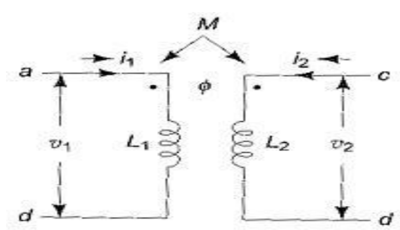
where

M = mutual inductance between the coils

L_1 = self inductance of the first coil, and

L_2 = self inductance of the second coil

Coefficient of coupling is always less than unity, and has a maximum value of 1 (or 100%). This case, for which K = 1, is called perfect coupling, when the entire flux of one coil links the other.



For a pair of mutually coupled circuits shown in Fig. let us assume initially that i_1, i_2 are zero at t = 0.

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$

Initial energy in the coupled circuit at t = 0 is also zero. The net energy input to the system shown in Fig. at time t is given by

$$W(t) = \int_0^t [v_1(t)\, i_1(t) + v_2(t)\, i_2(t)]\, dt$$

Substituting the values of $\upsilon_1(t)$ and $\upsilon_2(t)$ in the above equation yields

$$W(t) = \int_0^t \left[L_1 i_1(t) \frac{di_1(t)}{dt} + L_2 i_2(t) \frac{di_2(t)}{dt} + M(i_1(t)) \frac{di_2(t)}{dt} + i_2(t) \frac{di_1(t)}{dt} \right] dt$$

From which we get

$$W(t) = \frac{1}{2} L_1[i_1(t)]^2 + \frac{1}{2} L_2[i_2(t)]^2 + M[i_1(t) i_2(t)]$$

If one current enters a dot-marked terminal while the other leaves a dot marked terminal, the above equation becomes

$$W(t) = \frac{1}{2} L_1[i_1(t)]^2 + \frac{1}{2} L_2[i_2(t)]^2 - M[i_1(t)\, i_2(t)]$$

According to the definition of passivity, the net electrical energy input to the system is non-negative. W(t) represents the energy stored within a passive network, it cannot be negative.

$$W(t) = \frac{1}{2}\left(\sqrt{L_1}\, i_1 - \frac{M}{\sqrt{L_1}} i_2\right)^2 + \frac{1}{2}\left(L_2 - \frac{M^2}{L_1}\right) i_2^2$$

The first term in the parenthesis of the right side of the above equation is positive for all values of i_1 and i_2, and, thus, the last term cannot be negative; hence

$$L_2 - \frac{M^2}{L_1} \geq 0$$

$$\frac{L_1 L_2 - M^2}{L_1} \geq 0$$

$$L_1 L_2 - M^2 \geq 0$$

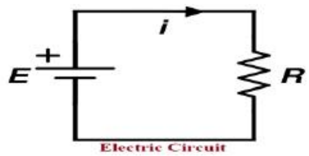
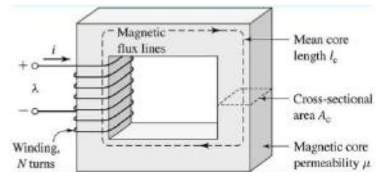
$$\sqrt{L_1 L_2} \geq M$$

$$M \leq \sqrt{L_1 L_2}$$

Obviously the maximum value of the mutual inductance is $\sqrt{L_1 L_2}$. Thus, we define the coefficient of coupling for the coupled circuit as

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

Q10. Distinguish analogy between electric and magnetic circuits

Electric Circuit	Magnetic Circuit
	
Path traced by the current is known as electric current.	Path traced by the magnetic flux is called as magnetic circuit.
EMF is the driving force in the electric circuit. The unit is Volts.	MMF is the driving force in the magnetic circuit. The unit is ampere turns.
There is a current I in the electric circuit which is measured in amperes.	There is flux φ in the magnetic circuit which is measured in the weber.
The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
Resistance (R) oppose the flow of the current. The unit is Ohm	Reluctance (S) is opposed by magnetic path to the flux. The Unit is ampere turn/weber.
$R = \rho.\ l/a$. Directly proportional to l. Inversely proportional to a. Depends on nature of material.	$S = l/\ (\mu_0\mu_r a)$. Directly proportional to l. Inversely proportional to $\mu = \mu_0\mu_r$. Inversely proportional to a
The current I = EMF/ Resistance	The Flux = MMF/ Reluctance
The current density	The flux density
Kirchhoff current law and voltage law is applicable to the electric circuit.	Kirchhoff mmf law and flux law is applicable to the magnetic flux.

1.Q Two coils have self inductances of L_1 and L_2 respectively when they are connected in series, the total inductance in series aiding and total inductance in series opposing are 25mH and 5mH respectively. Coefficient of coupling between coils is 0.6. Find out mutual inductance between coils, L_1 and L_2.

Sol: Let L_1, L_2 are self inductances of coils

we know that,

when two coils are connected in series aiding is

$$L_{eq} = L_1 + L_2 + 2M$$

Two coils are connected in series opposing is

$$L_{eq} = L_1 + L_2 - 2M$$

Given that,

$$25mH = L_1 + L_2 + 2M \quad -(1)$$

$$5mH = L_1 + L_2 - 2M \quad -(2)$$

Solving (1) and (2), we get,

$$\cancel{L_1} + \cancel{L_2} + 2M - 25 = 0$$

$$\cancel{L_1} + \cancel{L_2} - 2M - 5 = 0$$

$$- \quad - \quad + \quad +$$

$$4m - 20 = 0$$

$$4m = 20$$

$$m = \frac{20}{4}$$

$$\boxed{M = 5mH}$$

∴ coefficient of coupling is 0.6.

WKT: $K = \dfrac{M}{\sqrt{L_1 L_2}}$

$$\Rightarrow 0.6 = \frac{5}{\sqrt{L_1 L_2}}$$

$\rightarrow \sqrt{L_1 L_2} = \frac{5}{0.6}$

Squaring on both sides

$L_1 L_2 = \frac{25}{(0.6)^2}$

$L_1 L_2 = \frac{25}{0.36}$

$L_1 L_2 = 69.44$

2.Q. A mild steel ring of circumference 50 cm and cross sectional area of 5 cm 2 has a coil of 250 turns wound around it. calculate : (a) the reluctance. (b) the current required to produce a flux of 700 mWb. The relative permeability $\mu_r = 380$.

Sol: Given, circumference of a steel ring $= 50$ cm

$= 50 \times 10^{-2}$ m . m.f

$= H \times l = N \times I$

$A = 5 \times 2 \times 10^{-4} \times 10^{-3} m^2$

$N = 250$

$B = \phi/A \Rightarrow$ flux density

$H = \frac{NI}{l}$ $\quad (\because A = \pi r^2,\ l = 2\pi r)$

$\mu = \mu_0 \mu_r$

$\frac{B}{H} = \mu_0 \mu_r$

i) Reluctance $(S) = \frac{l}{\mu_0 \mu_r A}$

$= \frac{50 \times 10^{-2}}{4\pi \times 10^{-7} \times 380 \times 10^{-3}} = 1047071/H.$

ii) $\phi = 700\times10^{-3}$ wb

$\Rightarrow B = \phi/A = \dfrac{700\times10^{-3}}{10^{-3}}$

$B = 700$

$\Rightarrow \dfrac{B}{H} = \mu_0\mu_r$

$H = \dfrac{B}{\mu_0\mu_r} = \dfrac{700}{4\pi\times10^{-7}\times380}$

$H = 14659\times10^{2}$

$H = \dfrac{NI}{l}$

$I = \dfrac{Hl}{N}$

$I = \dfrac{14659\times10^{2}\times50\times10^{-2}}{250}$

$I = 2931.8$ A.

3Q. An iron ring of mean length 40 cm has an air gap of 2mm and a winding of 300 turns. If the permeability of the iron core is 300, when a current of 1A flows through the coil, find the flux density.

Sol: Given, $l = 40$ cm

Air gap $= 2$mm $\Rightarrow 2\times10^{-2}$cm

$N = 300$

$\mu_r = 300$

Current $= 1$A.

$B = ?$

we know that, $H = \dfrac{NI}{l} = \dfrac{300\times1}{40\times2\times10^{-2}} = \dfrac{3000}{8}$

$\therefore \dfrac{B}{H} = \mu_0\mu_r$

$$B = \mu_0 \mu_r \times H$$

$$B = 4\pi \times 10^{-7} \times 300 \times \frac{3000}{8}$$

$$B = \underline{0.14T}$$

Q.4 Two coils connected in series have a resistance of 18Ω and when connected in parallel have a resistance of 4Ω. Find the value of resistances.

Sol: Given, $\cancel{\frac{R_1 R_2}{18} = 4}$

2. $\cancel{R_1 R_2 = 72}$

$\cancel{R_2 = \frac{72}{R_1}}$

Let R_1, R_2 be resistances connected in series

$R_1 + R_2 = 18$ —(1) , $\frac{R_1 R_2}{R_1 + R_2} = 4$

$$\frac{R_1 R_2}{18} = 4$$

$$R_1 R_2 = 72$$

$$R_2 = \frac{72}{R_1}$$

from (1), $R_1 + \frac{72}{R_1} = 18$

$$R_1^2 + 72 = 18R_1$$

$$R_1^2 - 18R_1 + 72 = 0$$

$$R_1^2 - 6R_1 - 12R_1 + 72 = 0$$

$$R_1(R_1 - 6) - 12(R_1 - 6) = 0$$

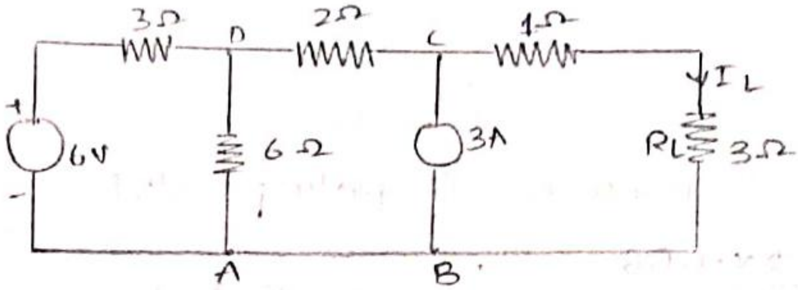
$$(R_1 - 6)(R_1 - 12) = 0$$

$\therefore R_1 = 6, \underline{R_2 = 12}$

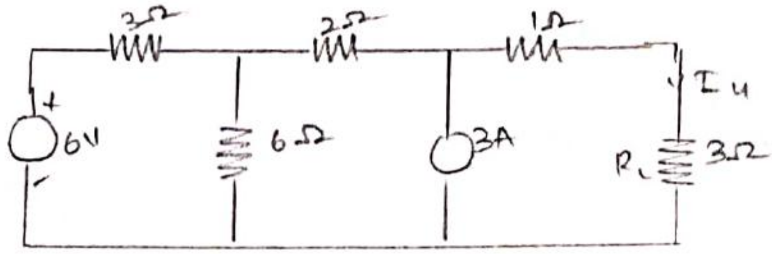
(or)

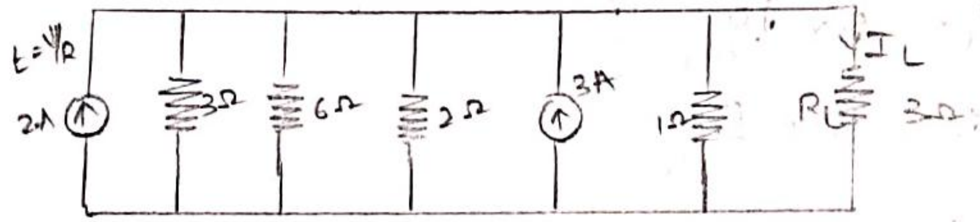
$R_1 = 12, R_2 = 6.$

Q.5 Using source transformation technique, determine the load current in the circuit shown in figure

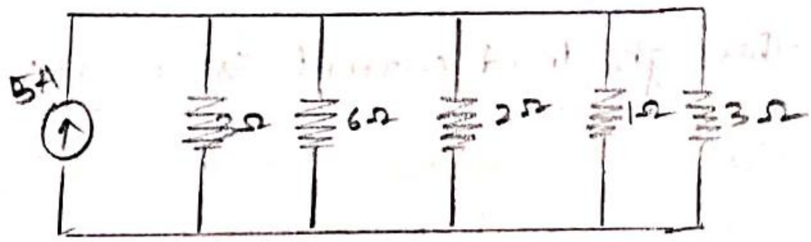


Sol: Given circuit is





Here, 3, 6, 2, 1, are parallel.

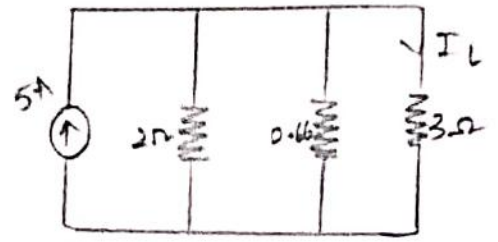


Here, 3, 6 are in parallel.,

$$\frac{3 \times 6}{3+6} = 2\Omega.$$

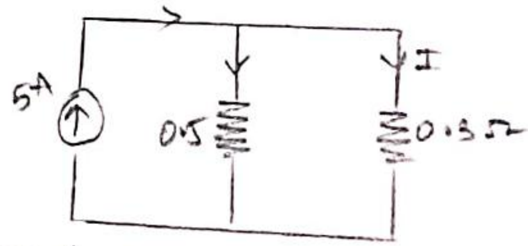
2, 1, are in parallel.

$$\frac{2 \times 1}{2+1} = \frac{2}{3} = 0.66\,\Omega$$



Here, 2, 0.66 are in ~~pallel~~ parallel.

$$\frac{2 \times 0.66}{2+0.66} = \frac{1.32}{2.66} = 0.5\,\Omega$$



Current passing through 3Ω resistor,

$$i_L = 5 \times \frac{0.5}{3+0.5}$$

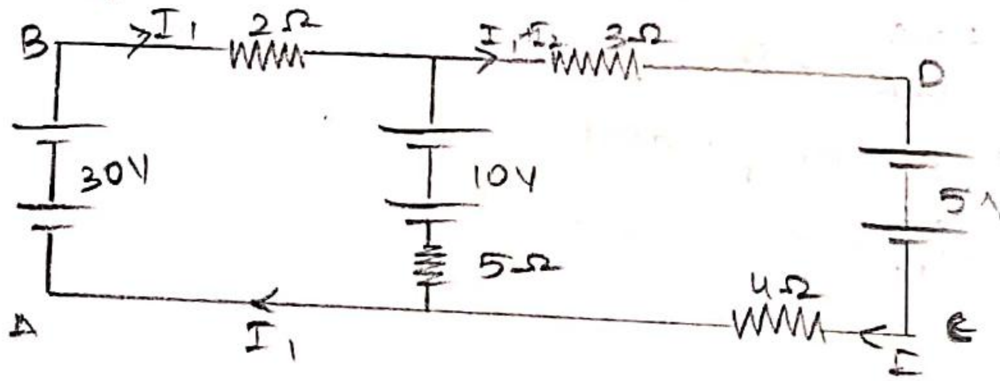
$$i_L = 5 \times \frac{0.5}{3.5}$$

$$i_L = 5 \times 0.14$$

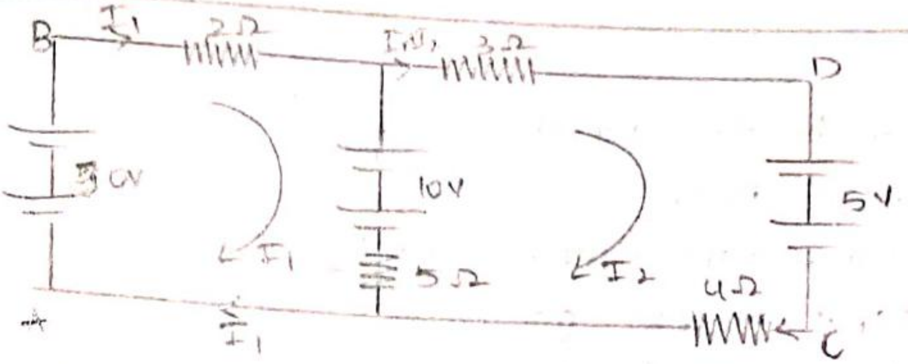
$$i_L = \underline{\underline{0.71\ A}}.$$

∴ Current passing through load current is $\underline{\underline{0.71A}}$.

Q.6. For the circuit shown in figure below, find the current flowing in all branches by applying kirchhoff's laws.



Sol.



Let i_1, i_2 are the currents.

By Applying KVL to loop ①,

$$2i_1 + 10V + 5(i_1 - i_2) - 30 = 0$$

$$7i_1 - 5i_2 = 20V \quad - ①$$

By Applying KVL to loop ②,

$$3i_2 + 5V + 4i_2 + 5(i_2 - i_1) - 10 = 0$$

$$3i_2 - 5V + 4i_2 + 5i_2 - 5I_1 = 0$$

$$-5i_1 + 12i_2 = 5V \quad - ②$$

By sloving ① and ②, we get,

$$i_1 = 4.5A, \quad i_2 = 2.2A.$$

Current passing through branches,

$$I_{2\Omega} = I_1 = 4.5A.$$

$$I_{5\Omega} = I_1 - I_2 = 2.3A.$$

$$I_{5\Omega} = I_2 - I_1 = -2.3A.$$

$$I_{3\Omega} = I_2 = 2.2A.$$

$$I_{4\Omega} = I_2 = 2.2A.$$

Q.7. When two identical coupled coils are connected in series, the inductances of the combination is found to be 80 mH. When the connections to one of the coils are reversed, a similar measurement indicates 20 mH. Find the coupling coefficient between the coils.

Sol: Let L_1, L_2 be the inductance of coils

$$L_1 + L_2 + 2M = 80$$

Similar conductances means $L_1 = L_2$

$$2L_1 + 2M = 80 \quad -(1)$$

Connected in opposite direction

$$2L_1 - 2M = 20 \quad -(2)$$

By Sloving (1) and (2), we get,

$$4L_1 = 100,$$

$$L_1 = 25 \text{ mH}$$

Subctitute L_1 in (1), we get,

$$2(25) + 2m = 80$$

$$50 + 2m = 80$$

$$50 + 2m = 80$$

$$2m = 30$$

$$\boxed{m = 15 \text{ mH}}$$

Q.8. An iron ring of 30 cm in different and $10 cm^2$ cross-section is wound with 300 turns of wire. For a flux density of $1 wb m^2$ and a permeability of 600. Find the exciting current and inductances when there is a 1mm air gap.

Sol. Given, $d = 30 cm$

$A = 10 cm^2 = 10 \times (10^{-2})^2 m$

$N = 300$

$B = 1 wb/m^2$

$\mu = 600$

$I = ?$

Now, $\frac{B}{H} = \mu_0 \mu_r$

$H = \frac{B}{\mu_0 \mu_r} = \frac{1}{4\pi \times 10^{-7} \times 300}$

$H = 2652.5$

Now, $H = \frac{NI}{l}$

$I = \frac{l \times H}{N} = \frac{2\pi r \times H}{N}$

$= \frac{2 \times 3.14 \times 15 \times 10^{-2} \times 2652.5}{300}$

$= \frac{2498.18}{300}$

$I = 8.3A.$

$L = \frac{\phi}{I}$

$B = \frac{\phi}{A} \Rightarrow \phi = BA$

$\phi = 1 \times 10^{-3} m$

$L = \frac{10^{-3}}{8.3} = 1 \times 10^{-4}$

$L = 0.1 mH.$

Q.a) An iron ring of mean length 40 cm has an air gap of 2mm and a winding of 300 turns. If the permeability of the iron core is 200, when a current of 1A flows through the coil, find the flux density.

Sol: Given, $l = 40\,cm = 40 \times 10^{-2}\,m$

Air gap $= 2mm = 2 \times 10^{-3}\,m$

$N = 300$

$\mu = 300$

$I = 1A$

$B = ?$

$\frac{B}{H} = \mu_0 \mu_r$

$H = \frac{NI}{l}$

$= \frac{300 \times 1}{40 \times 10^{-2}}$

$= \frac{3 \times 10^{4}}{4}$

$H = 750$

$B = H \times \mu_0 \mu_r$

$B = 750 \times 4\pi \times 10^{-7} \times 300$

$B = 0.28T$.

~~Now, $N_1 = 750$, $N_2 = 1200$~~

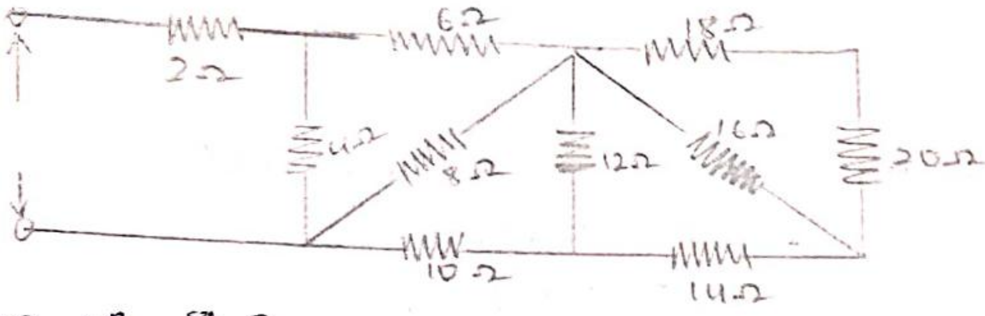
~~$I = 7.5\,A$.~~

~~leaking flux $= 0.25\,mWb$~~

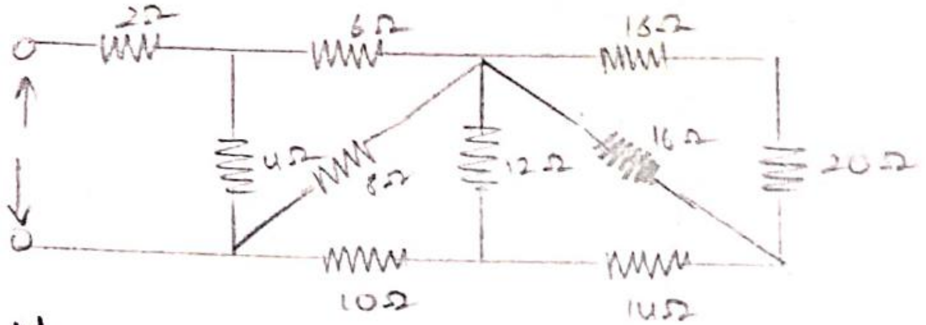
mutual flux $= 0.75\,mWb$.

3.10 Determine the equivalent resistance of the circuit shown below.

Sol:

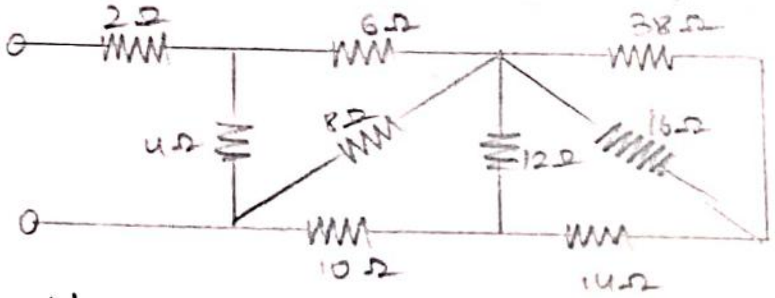


Sol: Given circuit is



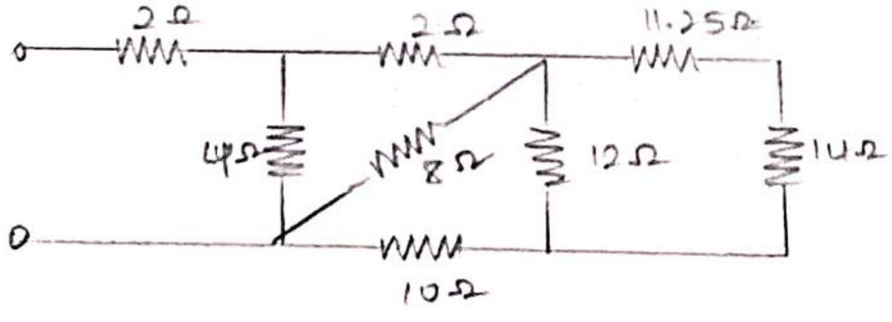
Here, 18, 20 are in series

then, $18+20 = \underline{38\Omega}$



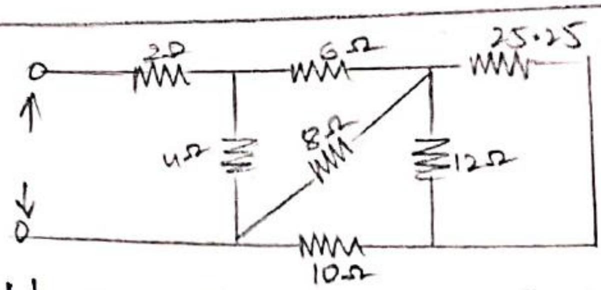
Here, 16, 38 are in parallel.

$$\frac{16\times 38}{16+38} = \underline{11.25\Omega}$$



Here, 11.25, 14 are in series

$$11.25+14 = \underline{\underline{25.25}}.$$

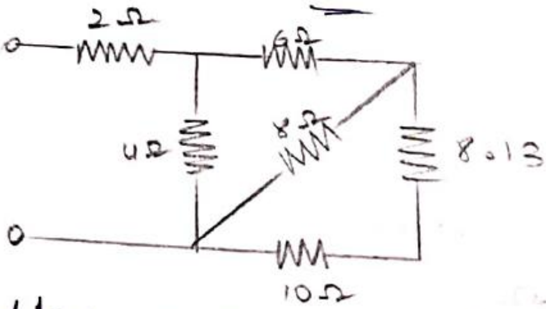


Here, 25.25, 12 are in parallel

$$= \frac{25.25 \times 12}{25.25 + 12}$$

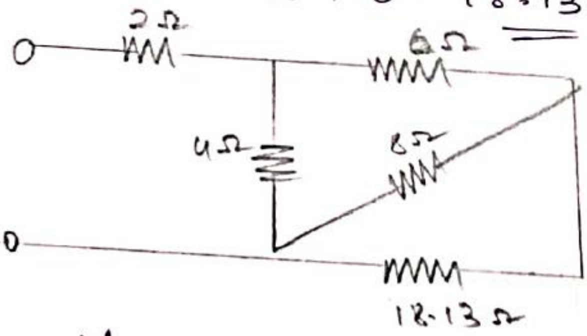
$$= \frac{302.4}{37.2}$$

$$= 8.13\,\Omega$$



Here, 8.13, 10 are in series

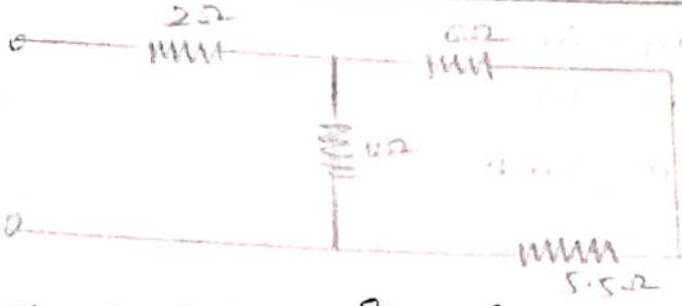
$$8.13 + 10 = 18.13\,\Omega$$



Here, 8, 18.13 are in parallel.

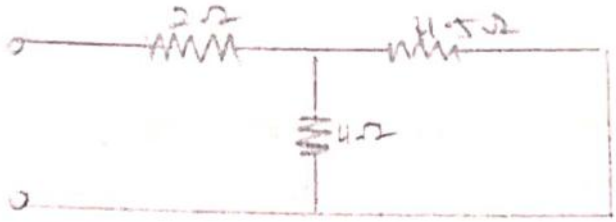
$$\frac{8 \times 18.13}{8 + 18.13} = \frac{145.04}{26.13}$$

$$= 5.5\,\Omega.$$



Here, 6, 5.5 are in series

$6+5.5 = \underline{11.5\,\Omega}$



Here 11.5, 4 are in parallel

$$\frac{11.5\times4}{11.5+4} = \frac{46}{15.6}$$

$$= \underline{\underline{2.96\,\Omega}}$$

These are in series

$2+2.96$

$\underline{\underline{Req}} = \underline{\underline{4.96\,\Omega}}$

Q.11. Two coils connected in series have self inductance of 40mH and 10mH respectively. The total inductance of the circuit is found to be 60mH. Determine i) The mutual inductance between the two coils. ii) The coeffient of coupling

Sol: Given, $L_1 = 40\,mH$.

i) $L_2 = 10\,mH$.

$L_1 + L_2 + 2M = L_{eq}$

$L_{eq} = 60\,mH$.

$$110 + 110 + 2m = 60$$
$$2m = 10$$
$$m = 5\,mH$$

ii) $k = \frac{M}{\sqrt{L_1 L_2}}$

$$= \frac{5}{\sqrt{4 \times 10}} = \frac{5}{\sqrt{40}} = \frac{5}{6.32}$$

$$k = 0.79.$$

12. Two coils having 750 and 1200 turns are wound on a common non-magnetic core. The leakage flux and mutual flux due to a current of 7.5A in a coil are 0.25 m.wb and 0.75 m wb, respectively. Calculate i) self inductance of the coil, ii) mutual inductance, iii) coefficient of coupling

Sol:

Given, $N_1 = 750$

$N_2 = 1200$

$I = 7.5A$

Leakage flux $= 0.25\,m\,wb$

mutual flux $= 0.75\,m\,wb$

i) $L = N \times \phi / i$

$$= 750 \times \frac{0.25}{7.5}$$

$$= \frac{\cancel{75} \times 0.25 \times 100}{\cancel{75}}$$

$$\Rightarrow L = 25\,H$$

ii) $m = N \times \phi / I$

$= 1200 \times \frac{0.25}{7.5}$

$= \frac{12 \times 25}{75} \times 10$

$M = 120 H.$

iii) $K = \frac{M}{\sqrt{L_1 L_2}}$

$K = \frac{120}{0.25 \times 25}$

$= \frac{120}{\sqrt{6.25}} \Rightarrow \frac{120}{2.5}$

$= \frac{1200}{25}$

$K = 48$

Q.13. In a coupled circuit $L_2 = 4L_1$ and coupling coefficient $K = 0.6$. When L_1 and L_2 are connected in series opposing the equivalent inductances is 44.2 mH. Find L_1, L_2 and M.

Sol: Given, $L_2 = 4L_1$

$K = 0.6$

$K = \frac{M}{\sqrt{L_1 L_2}}$

$L_1 + L_2 - 2M = 44.2$

$5L_1 - 2 \times 0.6 \sqrt{L_1 L_2} = 44.2$

$5L_1 - 1.2 \times 2L_1 = 44.2$

$L_1 [5 - 2.4] = 44.2$

$$L_1 = \frac{44.2}{2.6}$$

$L_1 = 17$

$L_2 = 4L_1$

$L_2 = 68$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$M = 0.6\sqrt{17 \times 68}$

$M = 0.6 \times 34$

$M = \underline{\underline{20.4}}.$

UNIT- II

1) Explain mesh analysis by consdering with 3 loop circuit.

Sol Consider a circuit with 2 voltage sources having 3 loops as shown in figure.

By KVL to loop-1 : Let $i_1, i_2 \& i_3$ are loop currents

$$i_1 R_1 + R_2(i_1 - i_2) - V_1 = 0$$

$$(R_1+R_2)i_1 - R_2 i_2 = V_1 \longrightarrow (1)$$

Applying KVL to loop-2

$$i_2 R_3 + R_4(i_2 - i_3) + R_2(i_2 - i_1) = 0$$

$$-R_2 i_1 + (R_2+R_3+R_4) i_2 - R_4 i_3 = 0 \longrightarrow (2)$$

Applying KVL to loop-3

$$i_3 R_5 + V_2 + R_4(i_3 - i_2) = 0$$

$$-R_4 i_2 + (R_4 + R_5) i_3 = -V_2 \longrightarrow (3)$$

By solving eq^n (1), (2) & (3), we get loop current $i_1, i_2 \& i_3$

The branch current are

$$I_{R_1} = i_1$$

$$I_{R_2} = i_1 - i_2$$

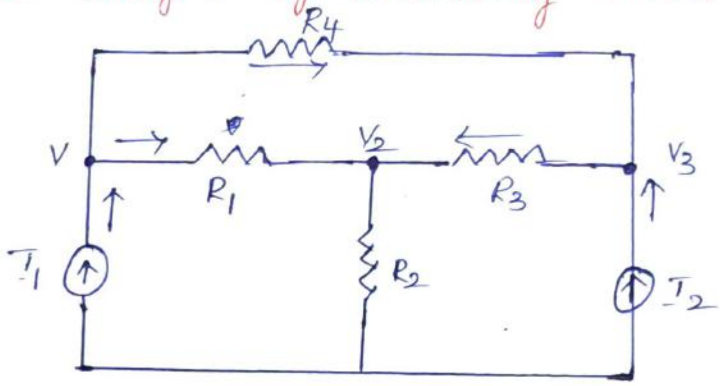
$$I_{R_3} = i_2$$

$$I_{R_4} = i_2 - i_3$$

$$I_{R_5} = i_3$$

2. Explain nodal analysis by considering 3 node circuits.

Sol:



consider a circuit having 3 nodes, with 2 current sources and resistors as shown in fig.

Let node voltages are V_1, V_2 & V_3

By applying KCL at node 1:

$$I_1 = \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_4}$$

$$V_1\left[\frac{1}{R_1} + \frac{1}{R_4}\right] - \frac{1}{R_1} V_2 - \frac{1}{R_4} V_3 = I_1 \longrightarrow (1)$$

By applying KCL at node 2:

$$\frac{V_1 - V_2}{R_1} + \frac{V_3 - V_2}{R_3} = \frac{V_2}{R_2}$$

$$\frac{1}{R_1} V_1 - \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2}\right) V_2 + \frac{1}{R_3} V_3 = 0 \longrightarrow (2)$$

By applying KCL at node 3:

$$I_2 + \frac{V_1 - V_3}{R_4} = \frac{V_3 - V_2}{R_3}$$

$$\frac{1}{R_4} V_1 + \frac{1}{R_3} V_2 - \left(\frac{1}{R_4} + \frac{1}{R_3}\right) V_3 = -I_2 \longrightarrow (3)$$

By solving eq^n (1), (2) & (3), we get node voltage V_1, V_2 & V_3

Branch currents are given by

$$I_{R_1} = \frac{V_1 - V_2}{R_2} \qquad I_{R_3} = \frac{V_3 - V_2}{R_3}$$

$$I_{R_2} = \frac{V_2}{R_2} \qquad I_4 = \frac{V_1 - V_3}{R_4}$$

3) Define i) Graph ii) Tree iii) Tie sets iv) cut-set v) chrode(or) Twings vi) link vii) Co-tree

sol:

i) Graph: It can be obtain by replacing active and passive elements in a circuit with short circuit (or) lineseg ment.

ii) Tree : Tree is a part of graph which contain all the nodes and they shouldn't be any closed loop.

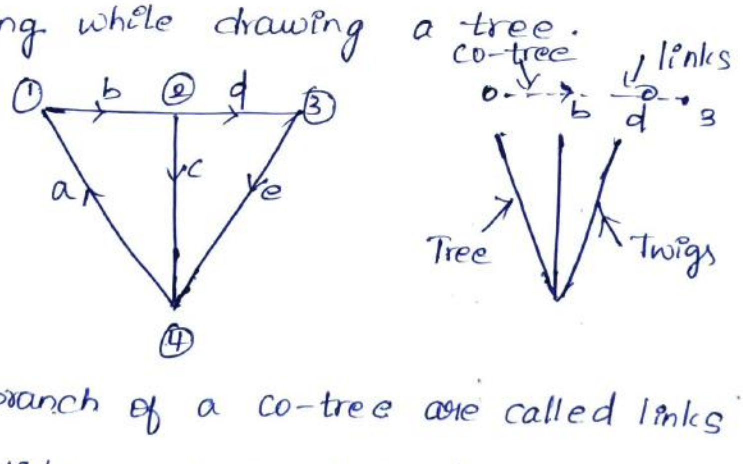
* A graph having no. of trees.

* The branches of a tree is called "Twings"

* If no. of nodes in graph is "n" then no. of twigs = n-1

iii) Tie-sets : The tree of a given graph, by connecting 1 link it forms a closed loop. It is called fundamental loop (or) Tie-set.

vii) co-tree: Co-Tree of a graph is obtained with branches eliminating while drawing a tree.



* The branch of a co-tree are called links

* No. of links equal to b-(n-1)

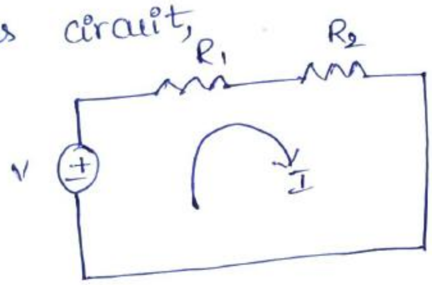
No. of links = b-(n-1)

iii) cut-set : Is a minimum set off branches of a connected graph such that the removal of this branches causes graph to be cut into exactly 2 parts.

4) What is duality? Write down the procedure to obtain dual network by taking any one example.

Duality : Two electric circuits are said to dual are mesh eq^n of one circuit is analogos to nodel eq^n of another circuits.

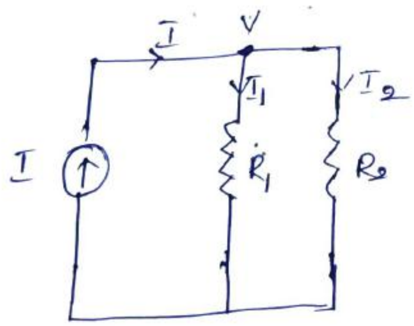
Consider a series circuit,



By applying KVL,

$$V = IR_1 + IR_2 \longrightarrow (1)$$

consider a parallel circuit,



By KCL,

$$I = I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\boxed{\therefore G = \frac{1}{R}}$$

$$I = VG_1 + VG_2 \longrightarrow (2)$$

From eq^n(1) & (2), it is clear that KVL of first circuit is similiar to KCL of 2nd circuit.

∴ The above circuits are called Dual circuits.

for example:

Volt - current

Resistance(R) - Conductance(C)

Mesh - Node

Some dual elements:

1) Voltage (V) ←→ Current (I)
2) Resistor (R) ← → Conductance (G)
3) Inductor (I) ← → Capacitor (C)
4) KVL ← → KCL
5) V(t) ← → I(t)
6) Mesh ← → nodal
7) Series ← → parallel
8) Vsinwt ← → Icoswt
9) Open circuit ← → short circuit
10) Thevenin ← → Norton
11) Link ← → twig
12) Cut set ← → tie set
13) Tree ← → co-tree
14) Switch in series (getting closed) ← → switching in parallel (

Procedure to Obtain a Dual Network:

These rules illustrated below are only for **planar** or flat networks which do not have any of their branches crossing other branches.

1) Place a dot in every loop of the network whose dual is obtained and a dot outside the network. Each dot is numbered according to the loop in which it is placed. The outside dot is called the reference node and give number as 0.

2) Connect two dots by a line through each branch. The dots are the nodes of the dual network between two nodes; the element to be connected is the dual of the element crossed by the line.

3) When sources are included, then the line joining the dots should intersect the sources also; between these two nodes the dual of the source is included.

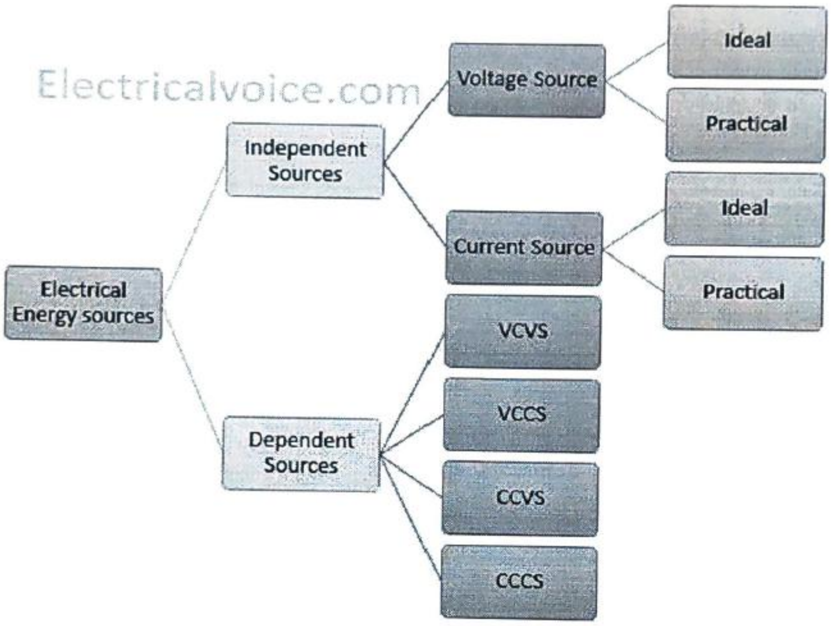
4) The polarity of the source is decided by the fallowing rule. A voltage or current source which drives a current in clockwise in j^{th} loop, then place a positive polarity at j^{th} the dual network. Negative if it is opposite

Classification of Electrical Energy Sources

Electrical energy sources are majorly classified into two classes i.e. Independent sources and Dependent sources.

The **independent sources** are further divided into two types namely **voltage source** and the **current source**. There are four types of the **dependent sources**. They are as follows

1. Voltage Controlled Voltage Source (VCVS)
2. Voltage Controlled Current Source (VCCS)
3. Current Controlled Voltage Source (CCVS)
4. Current Controlled Current Source (CCCS)



Independent Sources

Ideal Voltage Source

Voltage source is an active circuit element which delivers energy to the electrical circuit with a specified terminal voltage. If the voltage of such a voltage source is independent of the current due to the absence of internal resistance it is said to an **ideal voltage source**.

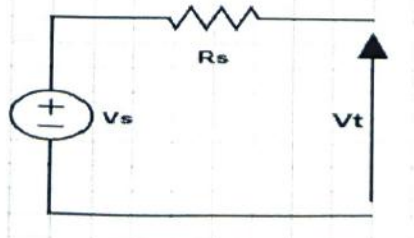
In ideal voltage source, the load voltage is independent of load current.

Practical Voltage Source

Practically, all the voltage sources have some internal resistance in contrast to its ideal case. A **practical voltage source** is modelled as, an ideal voltage source in series with its internal resistance indicated by a resistor.

Due to the presence of the internal resistance, the voltage delivered by a practical voltage source is no more constant as in the ideal case, but it changes as the current changes and is dependent on the current it delivers. The voltage will drop as the current delivered by it increases.

Circuit of a practical voltage source



The fig. 1 show the practical voltage source in which the voltage delivered from the source is indicated by V_s and its internal resistance with a resistor R_s. V_t is the actual terminal voltage across the source. Hence, the terminal voltage can be obtained by applying the Kirchhoff's voltage law (KVL) as below,

If I is the current in the loop. Then the loop equation is,

$-V_s + IR_s + V_t = 0$

$\therefore V_t = V_s - IR_s$

Thus, as the value of current, I increase the terminal voltage decreases.

V-I characteristics

The fig. 2 shows the V-I characteristics of a practical voltage source along with its comparison with the characteristic of an ideal source. One can observe the drop in the voltage is due to the internal resistance.

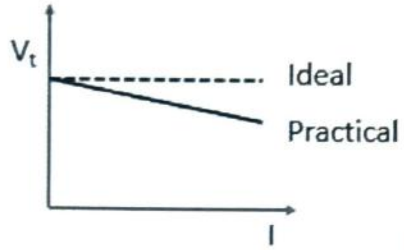


Fig. 2 Characteristic of a practical voltage source

Practically, the current sources don't have infinite resistance across them but have some finite and high resistance. Due to the finite resistance, the practical current source shows some dependency on the voltage across it. So the current delivered by a practical current is not a constant as in case of the ideal one.

Ideal Current Source

Current source is a circuit element which delivers energy with a specified current through it. If such a source maintains constant current for any voltage then it is called as an **ideal current source**. An ideal current source has infinite resistance (or impedance) across it.

In ideal current source, the load current is independent of load voltage.

Practical Current Source

A **practical current source** is modelled as an ideal current source in parallel with its internal resistance indicated by a resistor.

Circuit of a practical current source

The Fig. 1 shows the practical current source in which the current delivered from the source is indicated by I_s and its internal resistance with a resistor R_s. It is the current delivered to the external circuit.

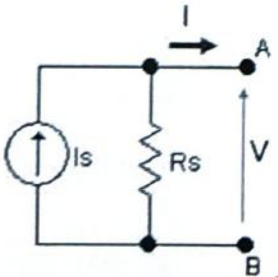


Fig. 1 Practical current source

The current to delivered to the external circuit It is given by,

$I = I_s - (V/R_s)$

V-I characteristics

The Fig. 2 shows the V-I characteristics of a practical current source along with its comparison with the characteristic of an ideal current source. The reduction of the current is due to the internal resistance R_s.

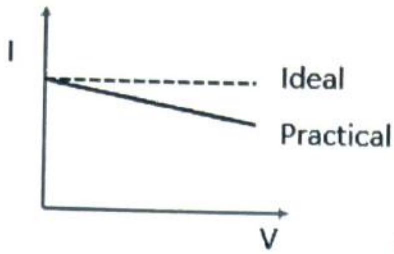


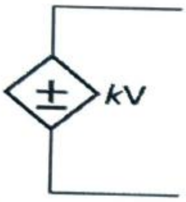
Fig. 2 Characteristics of a practical current source

Dependent Sources

The voltage or current delivered by these sources are dependent on other circuit parameters and hence they are called as **dependent sources**. These are classified into four types as given under along with their circuit diagrams.

Voltage Controlled Voltage Source (VCVS)

In the voltage controlled voltage source, the voltage of the source depends on the voltage across the another branch element or terminal. It is denoted by the following symbol. Here k is the controlled variable.

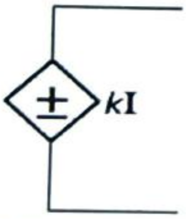


Voltage Controlled Current Source (VCCS)

In the voltage controlled current source, the current of the source depends on the voltage across the another branch element or terminal. It is denoted by the following symbol.

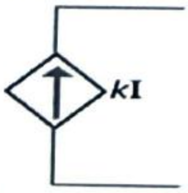
Current Controlled Voltage Source (CCVS)

In the current controlled voltage source, the voltage source is dependent on the current of the another branch. It is denoted by the following symbol.



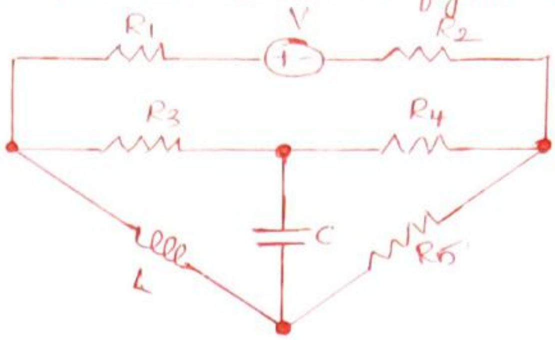
Current Controlled Current Source (CCCS)

In the Current Controlled Current Source, the current source is dependent on the current of the another branch. It is denoted by the following symbol.

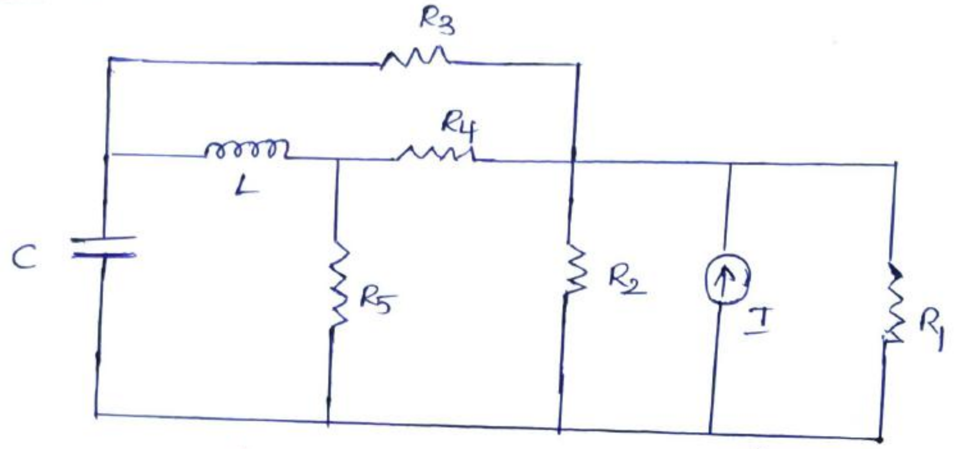


Note: Dependent sources are linear with respect to their controlled variables.

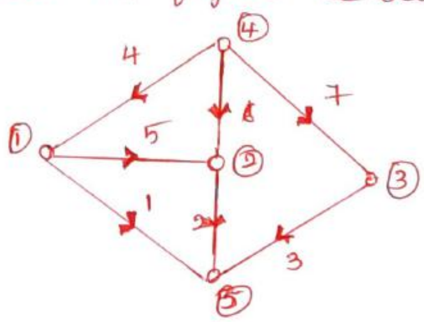
6. Draw the dual of the network shown in figure below.



solution:

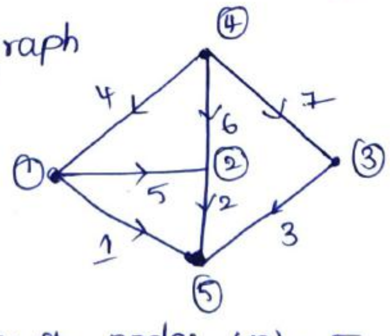


7. Obtain the fundamental loop and ~~functional~~ fundamental cut-set matrices for the graph shown in figure below.



solution:

Given graph



No. of nodes $(n) = 5$

no. of branches $(b) = 7$

no. of twigs $= (n-1) = (5-1) = 4$

no. of links $= (b-(n-1))$

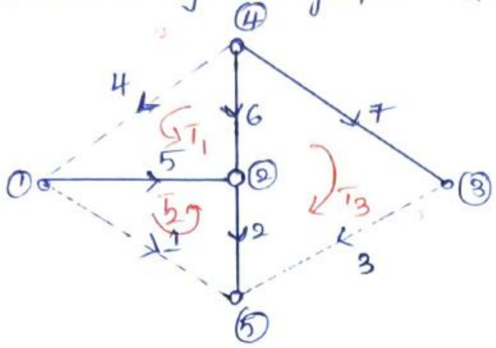
$= 7-4 = 3$

No. of tie sets = no. of links $= 3$

No. of Tie sets = 3

Tie set:

By representation given graph in terms of twigs and links



$T_1 : \{4, 5, 6\}$

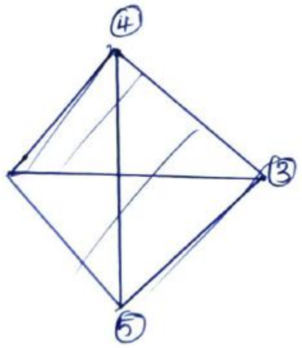
$T_2 : \{1, 2, 5\}$

$T_3 : \{3, 2, 6, 7\}$

Tie set matrix:

$$\begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

cut-set:

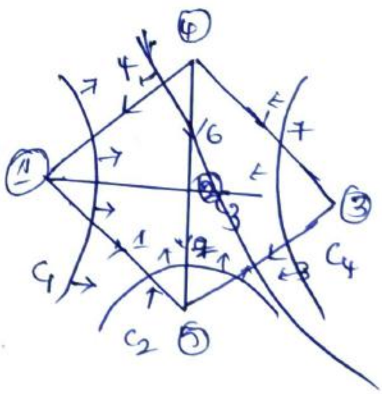


$C_1 : \{1, 4, 5\}$

$C_2 : \{1, 2, 3\}$

$C_3 : \{3, 4, 6\}$

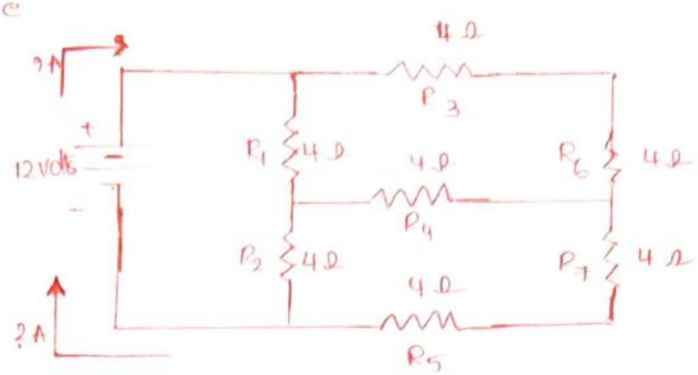
$C_4 : \{3, 7\}$



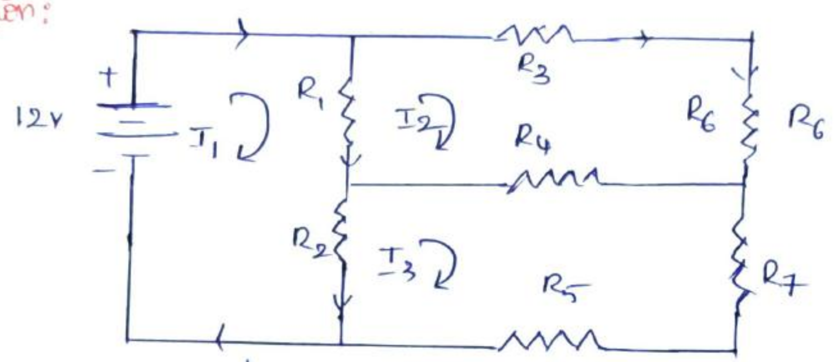
cut-set matrix:

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

8. For the circuit show below, determine the current supplied by the 12V d.c source



solution:



By loop analysis

Apply KVL to loop-1

$$R_1(I_1-I_2)+R_2(I_1-I_3)-12=0$$

$$R_1I_1 - R_1I_2+R_2I_1-R_2I_3=12$$

$$I_1(R_1+R_2)-I_2R_1-I_3R_2=12$$

$$8I_1-4I_2-4I_3=12 \longrightarrow (1)$$

Applying KVL to loop-2

$$R_3I_2+R_6I_2+R_4(I_2-I_3)+R_1(I_2-I_1)=0$$

$$R_3I_2+R_6I_2+R_4I_2-R_4I_3+R_1I_2-R_1I_1=0$$

$$I_1[-R_1]+I_2[R_1+R_3+R_4+R_6]+I_3[-R_4]=0$$

$$-4I_1+16I_2-4I_3=0 \longrightarrow (2)$$

Applying KVL to loop-3

$$R_4(I_3-I_2)+R_7(I_3)+R_5I_3+R_2[I_3-I_1]=0$$

$$R_4I_3-R_4I_2+R_7I_3+R_5I_3+R_2I_3-R_2I_1=0$$

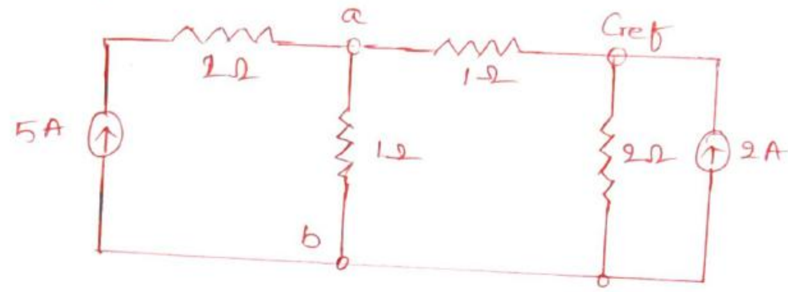
$I_1[-R_2] + I_2[-R_4] + I_3[R_4 + R_7 + R_5 + R_2] = 0$

$-4I_1 - 4I_2 + 16I_3 = 0$

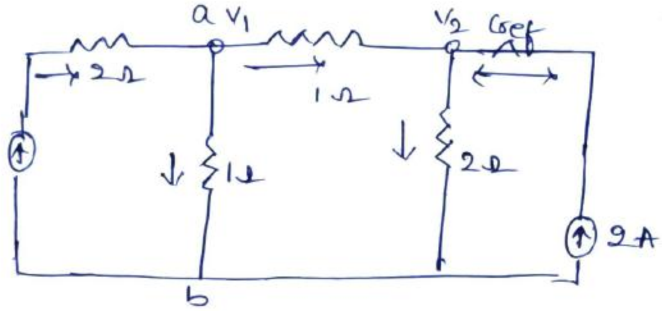
$I_1 = 2.25$, $I_2 = 0.75$, $I_3 = 0.75$

$\therefore$ Correct supplied by 12V dc source $I_1 = 2.25$

9. Using node analysis, find the current flowing through the resistors.



solution:



Let V_1, V_2 are node voltages

By applying KCL to node (1)

$$5 = \frac{V_1}{1} + \frac{V_1 - V_2}{L}$$

$$5 = 2V_1 - V_2$$

$$2V_1 - V_2 = 5 \longrightarrow (1)$$

KCL at node (2)

$$2 + \frac{V_1 - V_2}{1} = \frac{V_2}{2}$$

$$2 + 2V_1 - 2V_2 = V_2$$

$$2V_1 - 3V_2 = -4 \longrightarrow (2)$$

$$V_1 = 4.75 , \quad V_2 = 4.5$$

current passing resistors

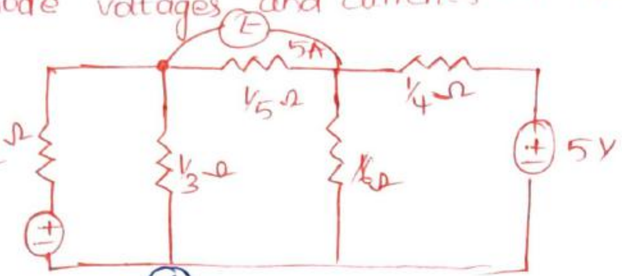
$I_2 = 5A$

$I_1 = \frac{V_1}{1} = 4.75$

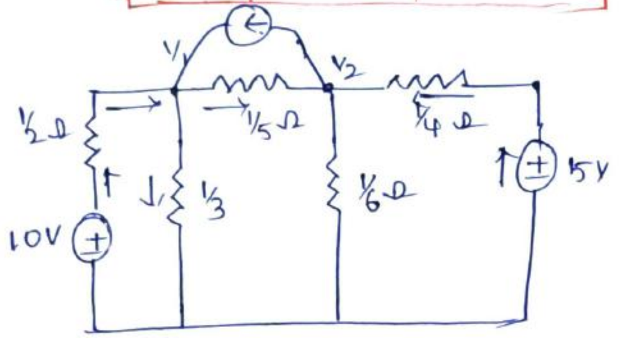
$I_1 = \frac{V_1 - V_2}{1} = 0.25$

$I_2 = \frac{V_2}{2} = \frac{4.5}{2} = 2.25$

10. Using node voltage analysis for the circuit shown in figure below. Find all the node voltages and currents in all the branches.



sol:



Let V_1, V_2 are node voltages

KCL to node (1)

$$5 + \frac{10 - V_1}{1/2} = \frac{V_1 - V_2}{1/8} + \frac{V_1}{1/3}$$

$$[2(10 - V_1) + 8(V_1 - V_2)] = 3V_1$$

$$20 - 2V_1 + 8V_1 - 8V_2 - 3V_1 = 0$$

$$3V_1 - 8V_2 = -20 \longrightarrow (1)$$

$$\frac{5 - V_2}{1/4} + \frac{V_1 - V_2}{1/8} = \frac{V_2}{1/6} + 5$$

$$5 + \frac{10 - V_1}{1/2} = \frac{V_1 - V_2}{1/8} + \frac{V_1}{1/3}$$

$$5+20-2V_1 = 8V_1 - 8V_2 + 3V_1$$

$$11V_1 - 8V_2 - 25 + 2V_1 = 0$$

$$13V_1 - 8V_2 = 25 \longrightarrow (2)$$

$$\frac{5-V_2}{1/4} + \frac{V_1-V_2}{1/8} = \frac{V_2}{1/6} + 5$$

$$4(5-V_2) + 8(V_1-V_2) = 6V_2 + 5$$

$$20 - 4V_2 + 8V_1 - 8V_2 = 6V_2 + 5$$

$$8V_1 - 16V_2 + 15 = 0$$

$$8V_1 - 16V_2 = -15 \longrightarrow (2)$$

Node voltages,

$$V_1 = 3.6$$

$$V_2 = 2.74$$

Currents:

$$I_{1/2} = \frac{10-V_1}{1/2} = 2(10-3.6)$$

$$\boxed{I_{1/2} = 12.8}$$

$$\boxed{I_{1/3} = 3V_1 = 10.8}$$

$$I_{1/8} = \frac{V_1-V_2}{1/8} = 8(V_1-V_2)$$

$$\boxed{I_{1/8} = 6.88}$$

$$I_{1/6} = \frac{V_2}{1/6} = 6V_2$$

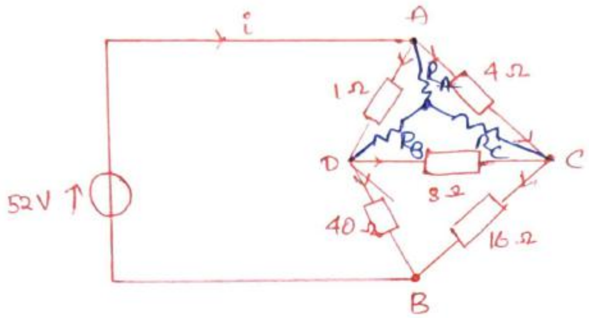
$$\boxed{I_{1/6} = 16.44}$$

$$I_{1/4} = \frac{5-V_2}{1/4}$$

$$I_{1/4} = 4(5-V_2)$$

$$\boxed{I_{1/4} = 9.04}$$

11. for a bridge network shown in figure below by using suitable delta-star transformation, determine : (i) The value of the single equivalent resistance that replaces the network between terminals A & B.
(ii) The current supplied by the 52 V source.

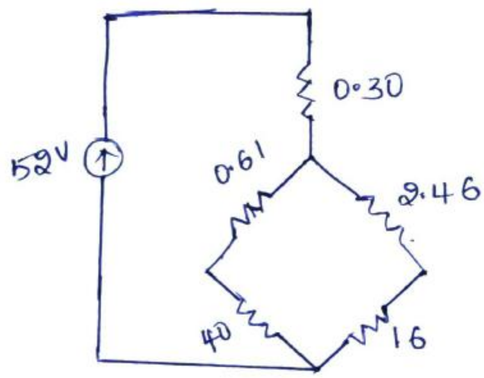


sol:

$$R_A = \frac{4\times1}{4+1+8} = \frac{4}{13} = 0.30$$

$$R_C = \frac{4\times8}{4+1+8} = 2.46$$

$$R_D = \frac{8\times1}{4+1+8} = 0.61$$

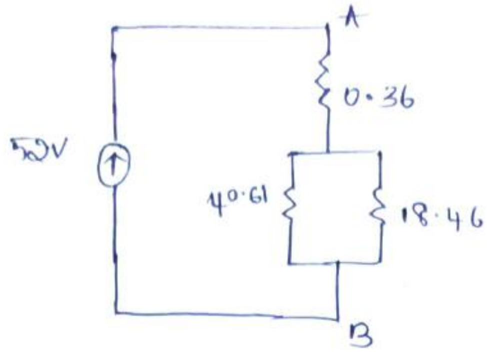


Here 40 Ω and 0.61 Ω in series

16 Ω and 2.46 Ω in series

$40 + 0.61 = 40.61\ \Omega$

$16 + 2.46 = 18.46\ \Omega$

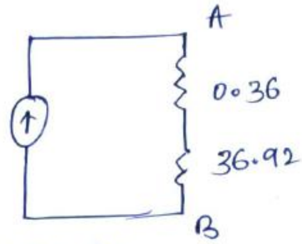


Here 40.61 & 18.46 are in parallel

$$= \frac{40.61 \times 18.46}{40.61 + 18.46}$$

$$= 36.92$$

Here 0.36 & 36.92 are in series



$$= 0.36 + 36.92$$

$$= 37.28\,\Omega$$

i) Resistance b/w terminals A and B = 37.28 Ω

ii) The current supplied by 52 V

$$I = \frac{52}{37.28}$$

$$I = 1.39$$

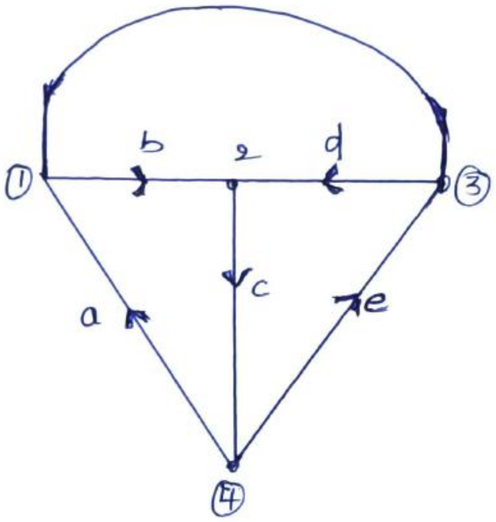
15. The reduced incidence matrix of a graph is given below. Draw the graph corresponding to it.

$$\begin{bmatrix} -1 & +1 & 0 & 0 & 0 & -1 \\ 0 & -1 & -1 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 & -1 & +1 \end{bmatrix}$$

Sol:

Nodes

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & +1 & 0 & 0 & 0 & -1 \\ 0 & -1 & -1 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 & -1 & +1 \\ +1 & 0 & 0 & -1 & +1 & 0 \end{bmatrix} \end{array}$$



UNIT - III

SINGLE PHASE A.C CIRCUITS

Answer the following questions :-

1. Define,

i) Average Value :-

Average value is a periodic function f(t) with time period T is given by $f_{avg} = \frac{1}{T}\int_0^T f(t)\,dt$.

ii) RMS value :-

The root mean square value of a periodic function of f(t) having time period T is given by if RMS.

$$f_{RMS} = \sqrt{\frac{1}{T}\int_0^T f(t)^2\,dt}$$

iii) Form factor :-

It is defined as ratio of RMS value and average value.

i.e. $F.F = \frac{f_{rms}}{f_{avg}}$

iv) Peak Factor (or) Crest factor :-

It is defined as ratio of maximum peak value and RMS value.

i.e. P.F or $C.F = \frac{f_{max}}{f_{rms}}$

v) Active Power (or) True Power :-

The power which is actually consumed or utilised in A.C circuit it is representing with P. Units = Kilowatt

Active Power $(P) = V_{rms} I_{rms} \cos\phi$

Where $\cos\phi$ = Power fact.

vi) Reactive Power :-

The portion of power due to stored energy which returns to stores in each cycle. It is representing with Q. Units = KVAR

Where K = Kilo, V = Volt, A = Amphere, R = Resistance

Reactive Power $(Q) = V_{rms} I_{rms} \sin\phi$

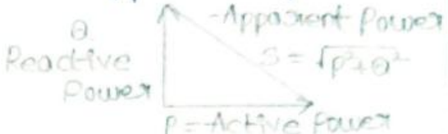
vii) Apparent Power :-

The product of RMS value of voltage and current is called Apparent power. It is the total amount of power drawn by the circuit from source.

It is representing with S. Units = KVA
Where K = Kilo, V = volt, A = Amphere
Apparent Power (S) = $V_{rms} \, I_{rms}$

viii) Power Triangle :-

The relation between active and reactive and apparent powers are given by $S^2 = P^2 + Q^2 \Rightarrow S = \sqrt{P^2+Q^2}$



ix) Impedance (Z) :-

It is defined as opposition offered by circuit two alternating current. It is representing with Z. Units = (Ω) ohm
It is combination of resistance and reactance
i.e $Z = R + j(X_L - X_C)$
Where, Z = impedance, R = Resistance, $X_L - X_C$ = Reactants.

x) Admitance (Y) :-

It is defined as inverse of impedance. It is representing with Y. Units = mho (℧). $\Rightarrow Y = \frac{1}{Z}$

xi) Reactance (X) :-

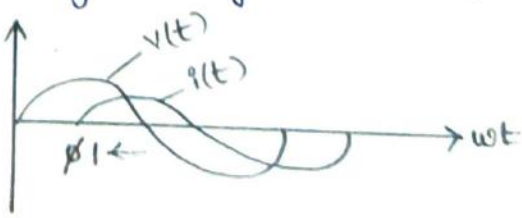
It is defined as opposition offered by inductor and capacitor to alternating current. It is representing with X. Units = ohm (Ω)
Inductive Reactance $X_L = 2\pi f_L$.
capacitive Reactance $X_C = \frac{1}{2\pi} f_C$
Where f = frequency of A.C supply = 50 H.

xii) Susceptance (S) :-

It is defined as inverse of reactance. It is representing with S. Units = mho (℧) or Seimen. $\Rightarrow S = \frac{1}{X}$.

xiii) Power Factor :-

It is defined as cosine angle between voltage and current. Let phase angle between voltage and current in an A.C circuit is ø. The power factor of the circuit is given by $P.F = \cos\phi$
We know that active power,
$P = V_{rms} \, I_{rms} \cos\phi$
and
Apparent Power $S = V_{rms} \cdot I_{rms}$



Power factor $\cos\phi = \frac{P}{S} = \frac{\text{Active Power}}{\text{Apparent power}}$.

In this power factor phase angle is also calculated from impedance triangle. $\cos\phi = \frac{R}{Z}$ or $\phi = \tan^{-1}\left(\frac{X}{R}\right)$

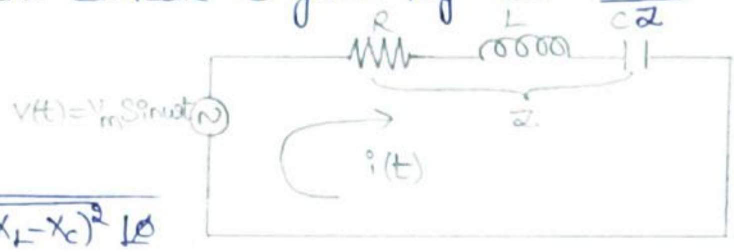
xiv) Analysis of Series RLC Circuit with AC Excitation:-

Consider a series RLC circuit excited with A.C voltage $v(t) = V_m \sin \omega t$ as shown in figure.

Current passing through the circuit is given by $i(t) = \frac{v(t)}{Z}$

$$Z = R + jX_L - jX_C$$

$$Z = R + j(X_L - X_C)$$



$$i(t) = \frac{V_m \sin \omega t}{R + j(X_L - X_C)} = \frac{V_m \angle 0^\circ}{\sqrt{R^2 + (X_L - X_C)^2} \angle \phi}$$

$$i(t) = \frac{V_m \angle 0}{\sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$

If $X_L > X_C$ current lagging with respect to voltage.

If $X_L < X_C$ current leading with respect to voltage.

If $X_L = X_C$ current and voltage are inphase.

(2) Show that the power through pure capacitor when excited with $v(t) = V_m \sin \omega t$ is zero?

(or)

Obtain voltage current relationship in pure capacitor with A.C excitation

(or)

Show that power factor in pure capacitor is zero?

(or)

Show that phase angle between voltage and current in capacitor excited with AC voltage is 90°?

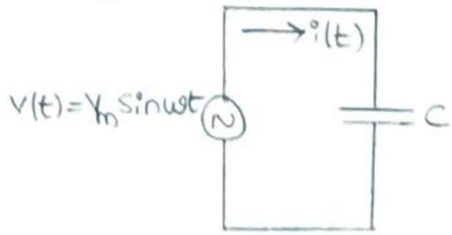
(A) Consider a capacitor C ~~connect~~ excited across AC voltage $v(t) = V_m \sin \omega t$ as shown in figure.

The current passing through the circuit is given by

$$V = \frac{1}{C}\int i\,dt$$

$$v(t) = \frac{1}{C}\int i(t)\,dt$$

$$i(t) = C \cdot \frac{d(v(t))}{dt}$$



$i(t) = C \cdot \frac{d}{dt}(V_m \sin \omega t)$

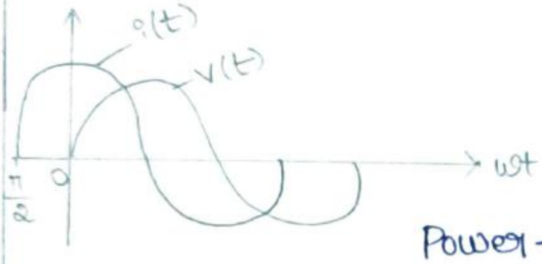
$i(t) = V_m C \omega \cos \omega t$

$i(t) = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\frac{\pi}{2} + \omega t\right)$

Impedance is given by $Z = \frac{V(t)}{i(t)} = \frac{1}{\omega C} = X_C$

The vector representation of impedance is given by $Z = -J X_C$

Where $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$



From this wave-forms it is clear that voltage and currents are out of phase lead by 90° with respect to voltage.

Power-factor $P.F = \cos\phi = \cos\frac{\pi}{2} = 0$.

Active power $P = V_{rms} \, i_{rms} \cos\phi = 0$.

③ Show that the power through pure inductor when excited with $V(t) = V_m \sin \omega t$ is zero?

(or)

Obtain voltage current relationship in pure inductor with Ac excitation?

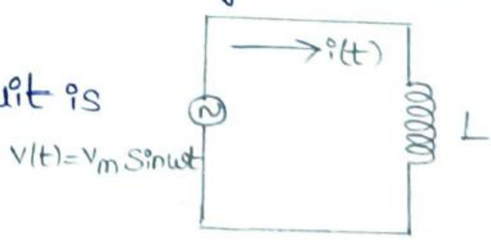
(or)

Show that power-factor in pure inductor is zero?

(or)

Show that phase angle between voltage and current in inductor excited with Ac voltage is 90°?

Ⓐ Consider a pure inductor L excited with Ac voltage $V(t) = V_m \sin \omega t$ as shown in figure.



The current passing through the circuit is given by $V = L \cdot \frac{di}{dt}$

$V(t) = L \cdot \frac{di(t)}{dt}$

$i(t) = \int \frac{V(t)}{L} dt$

$i(t) = \frac{1}{L} \int V_m \sin \omega t \cdot dt$

$i(t) = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$

$$i(t) = \frac{V_m}{\omega L}\left(-\sin\left(\frac{\pi}{2} - \omega t\right)\right)$$

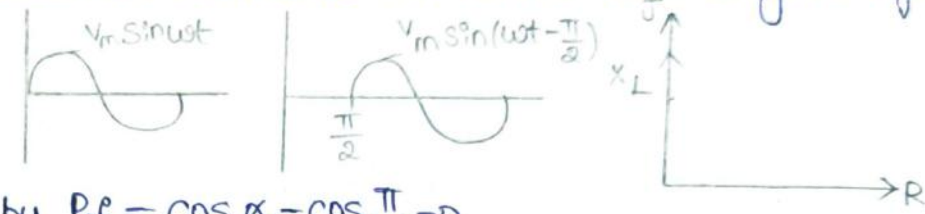
$$i(t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

We know that $i(t) = \frac{v(t)}{Z}$

Therefore impedance (Z) = Inductive reactance $= \omega L$

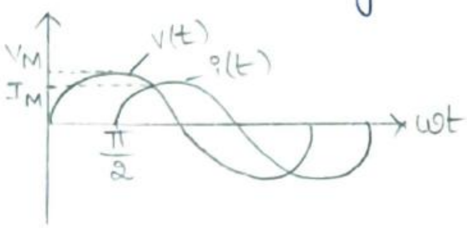
The presentation of inductive reactance in vector form is given by

$$Z = j\omega L$$
$$Z = jX_L$$
$$X_L = 2\pi f L$$



Power factor is given by $P.f = \cos\phi = \cos\frac{\pi}{2} = 0$

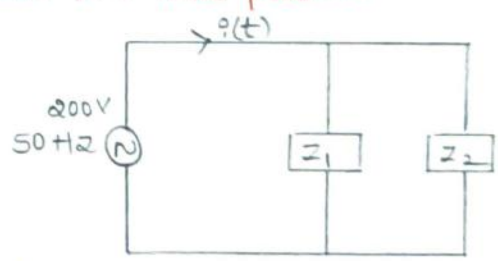
Power consumed by inductor is given by $P = V_{rms} I_{rms} \cos\phi = 0$



From this wave forms it is clear that in pure inductor voltage and current are out of phase, current is lagging by 90° with respect to voltage.

(4) Two impedances $(15 - j10)\Omega$ and $(10 + j15)\Omega$ are connected in parallel. The supply voltage is 200V, 50Hz. calculate: (i) Admittance, (ii) Conductance (iii) Susceptance, (iv) Total current, (v) Total power?

(A)
$$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{(15 - j10)(10 + j15)}{(15 - j10) + (10 + j15)}$$



$$Z_{eq} = \frac{(15 - j10)(10 + j15)}{25 + j5} = 12.5 + j2.5$$

i) Admittance: $Y = \frac{1}{Z} = \frac{1}{12.5 + j2.5} = 0.07 - j0.01 \; \mho$

ii) Conductance: $G = \frac{1}{R} = \frac{1}{12.5} = 0.08 \; \mho$

iii) Susceptance: $S = \frac{1}{X} = \frac{1}{j2.5} = 0.4 \; \mho$ or Seimen

iv) Total current: $i(t) = \frac{V(t)}{Z} = \frac{200}{12.5 + j2.5} = 15.3j - j3.07 = 15.6 \angle{-11.3}$

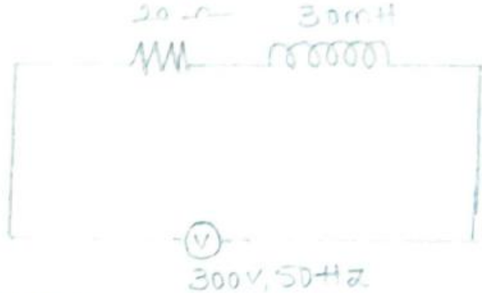
v) Total Power: Active power$(P) = V_{rms} I_{rms} \cos\phi$

$$= (200)(15.6)\cos(11.3) = 3.05 \text{ kW}$$

Reactive power$(Q) = V_{rms} I_{rms} \sin\phi = (200)(15.6)\sin(11.3) = 611.3$ VAR.

Apparent power$(S) = V_{rms} \cdot I_{rms} = (200)(15.6) = 3120$ VA

⑤ A 20 Ω resistor and a 30 mH inductor are connected in series across a 300 V, 50 Hz a.c supply. Find: (i) Impedance of the circuit (ii) Voltage across the resistor (iii) voltage across the inductor (iv) Apparent power (v) Active power (vi) Reactive power?



Ⓐ (i) Impedance of the circuit : $Z = R + jX_L$

$$Z = 20 + j\,2\pi(50)(30 \times 10^{-3} m)$$

$$Z = 20 + j\,9.42$$

(ii) Voltage across the resistor : $V_R = iR$

$$I = \frac{V}{Z} = \frac{300}{20 + j9.42} = 12.27 - j5.7 = 13.5 \angle -25.2 = 13.5 \times 20 = 270 \text{ Volts}$$

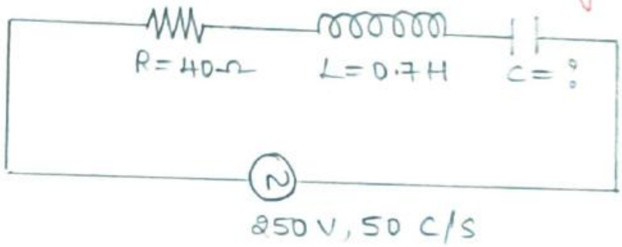
(iii) Voltage across the inductor : $V_L = Ix = 13.5 \times 9.4 = 126.9$ Volts.

(iv) Apparent power : $V_{rms} \cdot I_{rms} = (300)(13.5) = 4050$ VA.

(v) Active power : $V_{rms} \cdot I_{rms} \cos\phi = 300(13.5)\cos(25.2) = 3604$ KW.

(vi) Reactive power : $V_{rms} \cdot I_{rms} \cdot \sin\phi = 300(13.5)\sin(25.2) = 1724$ VAR

⑥ A coil of resistance 40 Ω and inductance 0.75 H forms part of a series circuit for which the resonant frequency is 55 c/s. If the supply is 250 V, 50 c/s. Find (i) Line current, (ii) Power factor, (iii) Voltage across the coil?



Ⓐ Series Resonance Circuit :

$R = 40\,\Omega,\ L = 0.7\,H,\ C = ?,\ f_r = 55$ c/s

We know that resonance frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$55 = \frac{1}{2\pi\sqrt{0.7 \times C}} = 11\,\mu f$$

$$C = 11\,\mu f$$

(i) Line current : $i(t) = \frac{V(t)}{Z}$

$$Z = R + j(X_L - X_C)$$

$$X_L = 2\pi f_L = 2\pi(50)(0.7) = 219.$$

(ii) $X_C = \frac{1}{2\pi f_C} = \frac{1}{2\pi \times 11 \times 10^{-6} \times 50} = 289$

$$Z = 40 + j(219 - 289) = 40 - j70$$

$$i(t) = \frac{250}{40 - j70} = 3 \angle 60$$

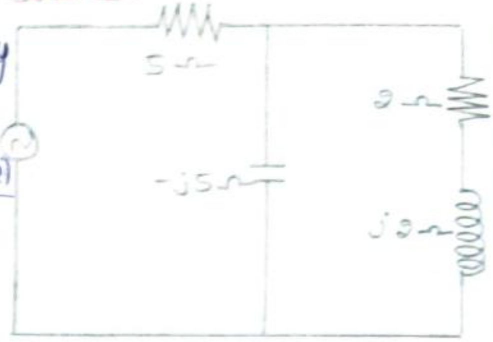
(ii) Power factor : $\cos\phi = \cos 60 = 0.5$

(iii) Voltage across the coil : $V_{coil} = Z \cdot i$

$$= |Z| \cdot |i| = |40 - j70|\,|3\angle 60| = \sqrt{40^2 + 70^2} \times 3 = 24\text{V}$$

(7) Determine the value of the voltage source and power factor in the given network if it delivers a power of 100 W to the circuit shown below. Find also the reactive power drawn from the source:

(A) The total impedance of the circuit is given by



$$z = 5 + (-j5) \,//\, (2 + j2)$$

$$z = 5 + \frac{(-j5)(2+j2)}{-j5+2+j2} = \frac{10-15j+(-j5)(2+j2)}{2-j3}$$

$$z = 5 + 3.8 + j0.7 = 8.8 + j0.7$$

Given $P = V_{rms}\, i_{rms} \cos\phi = 100$ watts

$$P = I^2 R = 100$$

$$I^2 = \frac{100}{R} = \frac{100}{8.8} = 11.36$$

$$I = \sqrt{11.36} = 3.37$$

$$P = V_{rms} \cdot I_{rms} \cos\phi$$

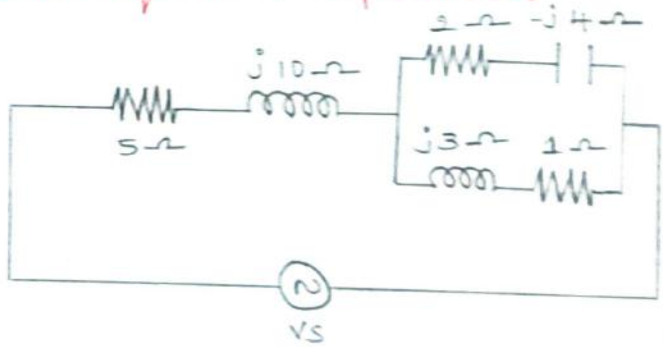
$$100 = V_{rms} \cdot (3.37) \cos\left(\tan^{-1}\left(\frac{0.7}{8.8}\right)\right)$$

$$V_{rms} = \frac{100}{3.37 \times 0.99} = 30$$

Reactive power $Q = V_{rms} \cdot I_{rms} \sin\phi = 30 \times 3.37 \times 0.07 = \cancel{7.077} = 8$ VAR

(8) In the circuit shown below, determine the equivalent impedance:

(A) $I = z_3 + z_1 \,//\, z_2$



Where $z_1 = 2-j4$, $z_2 = 1-j3$,

$z_3 = 5+j10$

$$z = (5+j10) + \frac{(2-j4)(1-j3)}{(2-j4)+(1-j3)}$$

$$z = 5+j10 + \frac{2-j4-6j-12}{3-j7}$$

$$z = \frac{75-10j}{3-j7} = 5.6 + j8.2$$

(9) Series circuit the voltage, current is given by $v(t) = 282.8 \sin 314t$, $i(t) = 14.14 \sin(314t - 60°)$. Calculate value of the circuit elements and calculate active, reactive and apparent power:

(A) We know that $\frac{v(t)}{i(t)}$

$$z = \frac{v(t)}{i(t)} = \frac{282.8 \sin(314t)}{14.14 \sin(314t-60)} = \frac{282.8 \angle 0°}{14.14 \angle -60} = 10 + j17.3 = R + jX_L$$

$R = 10$

$X_L = 17.3 = \omega L$

$2\pi f L = \omega L$

$L = \frac{17.3}{314} = 0.05\ H$ (or) 55 mH.

i) Active Power : $V_{rms} \cdot I_{rms} \cos\phi$

$$= \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos(60^\circ) = \frac{282.8}{\sqrt{2}} \cdot \frac{14.14}{\sqrt{2}} \cdot \frac{1}{2} = 999$$

ii) Reactive power : $V_{rms} \cdot I_{rms} \sin\phi$

$$= \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{14.14}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = 1729$$

iii) Apparent power : $V_{rms} \cdot I_{rms}$

$$= \left(\frac{282.8 \times 14.14}{2}\right) = 1999$$

(10) Two impedances in electrical network are given by $Z_1 = 4.7\angle 35^\circ$, $Z_2 = 7.36\angle 48^\circ$. Determine in the polar form the total impedance Z_T. Given that $Z_T = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$?

(A) $$Z_T = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{4.7\angle 35^\circ \times 7.36\angle 48^\circ}{4.7\angle 35^\circ + 7.36\angle 48^\circ} = 9.849$$

(11) For an R-L series circuit, the current $i = 14.14 \sin(157t)$, $R = 5\,\Omega$, $L = 30$ mH. determine (i) Inductive Reactance (ii) Impedance, (iii) Total voltage, (iv) Power factor, (v) Real power and (vi) Reactive power?

(A) Given $i = 14.14 \sin(157t)$, $R = 5\,\Omega$, $L = 30$ mA.

i) Inductive Reactance $\Rightarrow X_L = \omega L$

$$X_L = 157 \times 30 = 4.71\,\Omega$$

ii) Impedance $(Z) = R + jX_L = 5 + j(4.71) = 5 + j(4.7)$

iii) Total voltage $V(t) = i(t) \times Z$

$$V(t) = 14.14\angle 0 \times (5 + j4.7) = 97.03\angle 43^\circ$$

$$V(t) = 97.03 \sin(157t + 43^\circ)$$

iv) Power factor (PF) $= \cos\phi = \cos(43^\circ) = 0.73$

v) Real factor $P = V_{rms} \cdot I_{rms} \cos\phi$

$$= \frac{V(t)}{\sqrt{2}} \cdot \frac{I(t)}{\sqrt{2}} \cdot \cos(43)$$

$$= \frac{97.03}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \times \cos(43^\circ)$$

$$= 501.7$$

vi) Reactive power $= V_{rms} \cdot I_{rms} \cdot \sin\phi = \frac{V(t)}{\sqrt{2}} \cdot \frac{I(t)}{\sqrt{2}} \cdot \sin(43^\circ)$

$$= \frac{97.03}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \times \sin(43^\circ)$$

$$= 467.8$$

(12) Calculate the reactance of a 2 µF capacitor at : i> 50 Hz ii> 2.5 kHz
iii> 1.5 MHz?

(A) We know that $X_C = \frac{1}{2\pi f_C}$, f = 50

(i) $X_C = \frac{1}{2\pi \times 50 \times 2\mu} = 1591$

(ii) $X_C = \frac{1}{2\pi \times 2500 \times 2\mu} = 31.8$

(iii) $X_C = \frac{1}{2\pi \times 1500 \times 2\mu} = 0.05$

(13) A coil has an inductance of 20 mH and a resistance of 5 Ω. It is connected across a supply voltage v = 50 sin 314t. Obtain a similar expression for the supply current?

(A) We know that $i(t) = \frac{v(t)}{Z}$

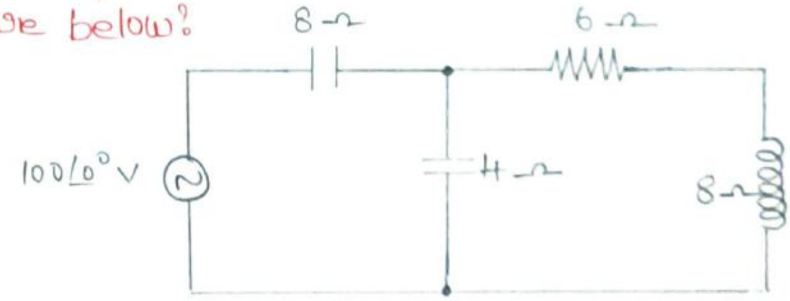
$Z = R + jX_L$ $\qquad X_L = \omega L$

$Z = 5 + j6.28$ $\qquad = 314 \times 20 \times 10^{-3} = 6.28$

$i(t) = \frac{50 \angle 0^\circ}{5 + j6.2} = 6.2 \angle -51^\circ$

$i(t) = 6.2 \sin(314t - 51^\circ)$.

(14) Determine the total current of power factor and power consumed by the circuit shown in figure below?



(A) $Z = \frac{V(t)}{i(t)}$

$Z = -j8 + (6+j8) \parallel (-j4)$

$Z = -j8 + \frac{(6+j8)(-j4)}{(6+j8)+(-j4)}$

$Z = 1.8 - j13.2$

$I(t) = \frac{V(t)}{Z} = \frac{100 \angle 0^\circ}{1.8 - j13.2} = 7.5 \angle 82^\circ$

i> Power factor : $\cos\phi = \cos(82^\circ) = 0.13$

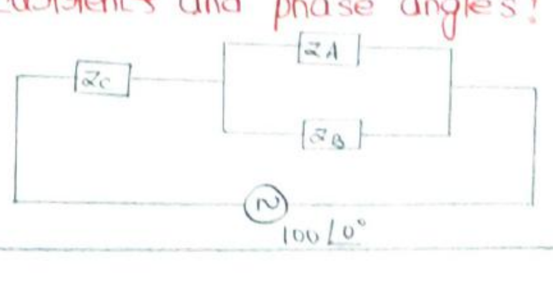
ii> Power consumed by the circuit : $P = I^2 R = (7.5)^2 (1.8) = 101.25$

(15) In a series parallel circuit the two parallel branches A and B are in series with C. The impedances are $Z_a = 10 + j8$, $Z_b = 9 - j6$ and $Z_c = 3 + j2$ Ω voltage across Z_c is $(100 + j0)$V. Find the currents and phase angles!

(A) Given $Z_a = 10 + j8$, $Z_b = 9 - j6$, $Z_c = 3 + j2$

Here Z_a & Z_b are in parallel then $\frac{Z_a \cdot Z_b}{Z_a + Z_b}$

$\rightarrow \frac{(10+j8)(9-j6)}{(10+j8)+(9-j6)} = 7.249 - 0.131j$



then z_c is in series $\Rightarrow 7.249 - 0.131j + 3 + j2 = 10.249 + 1.869i = z$

Current: $i(t) = \frac{V(t)}{z} = \frac{100}{10.249 + 1.869i} = 9.4430 - 1.72i$

In polar form $r\angle\theta$, $\Rightarrow 9.598 \angle -10.334$.

Phase Angle: -10.334.

(16) A resistor R is connected in series with a capacitor c and the combination is connected across a 100V, 50 Hz supply. The voltage drop across the resistor is 60V, the power dissipated in the resistor is 108W Find R and c?

(A) Given $z = R - jx_c$, Voltage $V = 100\angle 0°$

Voltage drop across resistor is 108.

$V_R = i_R = 60°$

$P = I^2 R = 108°$

$i = \frac{108}{60} = 1.8$

$R = \frac{60}{1.8} = 33.3\Omega$

$i = \frac{V(t)}{z}$

$1.8 = \frac{100}{33 - jx_c} = \frac{100}{\sqrt{(33)^2 + x_c^2}} \Rightarrow C = 44$ $\Rightarrow f = 50$ $c = 44$

$x_c = \frac{1}{2\pi f_c} = \frac{1}{2\pi \times 50 \times 44} = 7.2 \times 10^{-5}$

$c = 72\ \mu F$.

(17) A coil of power factor 0.6 is in series with a 100 μF capacitor. When connected to a 50 Hz supply, the pontential drop across the coil is equal to the potential drop across the capacitor. Find the resistance and inductance of the coil?

(A) $Coil = R + jx_L$

$x_c = \frac{1}{2\pi f_c} = \frac{1}{2\pi \times 50 \times (100)\mu} = 31.8$

$x_c = 31.8$

Given voltage drop across coil = voltage drop across capacitor

$i_z = ix_c$

$z = x_c = 31.8$

$R = 0.6z$

$= 0.6 \times 31.8 = 19\Omega$.

$z = \sqrt{R^2 + x_L^2}$

$x_L = \sqrt{z^2 - R^2}$

$$X_L = \sqrt{(31.8)^2 - (19)^2} = \sqrt{650.24} = 25.4$$

$$2\pi f L = \sqrt{650.24} = 25.4$$

$$2\pi \times 50 \times L = 25.4$$

$$L = \frac{25.4}{2\pi \times 50} = 0.08$$

(18) A coil of resistance 5 Ω and inductance 120 mH in series with a 100 μF capacitor is connected 100, 300V, 50 Hz supply calculate:

(i) The current flowing in the circuit

(ii) The phase difference between the supply voltage and current.

(iii) The voltage across the capacitor.

(iv) Real and Reactive power.

(A) Given, resistance $(R) = 5\,\Omega$

Inductance $(L) = 120\,mH$

Capacitance $(C) = 100\,\mu F$

Frequency $(F) = 50\,Hz$

i) Current $i(t) = \frac{V(t)}{Z}$

$$Z = R + j(X_L - X_C)$$
$$= 5 + j(X_L - X_C)$$
$$= 5 + j(5.84)$$

$$X_L = 2\pi f_L$$

$$X_L = 2 \times 3.14 \times 50 \times 120 \times 10^{-3} = 37.68$$

$$X_C = \frac{1}{2\pi f_C} = \frac{10^4}{2 \times 3.14 \times 50} = 31.84$$

$$i(t) = \frac{300}{5 + j(5.84)} = 25.37 - 29.6j = 39\underline{|-49.43}$$

ii) Phase difference between the supply voltage and current $= -49$

iii) $V(t) = i(t) \times Z$

$$= |Z| = 39 \times 7.08 = 299.5$$

voltage across capacitor $V(t) = i(t) \times X_C$

$$= 39 \times (31.84) = 1241.76$$

iv) Real power $P = V_{rms} \cdot I_{rms} \cdot \cos\phi$

$$= \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) \times \cos(-49) = 211 \times 27.5 \times 0.65 = 3771 \text{ watts}$$

Reactive power $Q = V_{rms} \cdot I_{rms} \cdot \sin\phi$

$$= 211 \times 27.5 \times 0.75 = 4351$$

(19) A coil having a resistance of 10 Ω and an inductance of 125 mH is connected in series with a 60 μF capacitor across a 120 V supply. At what frequency does resonance occur? Find the current flowing at the resonant frequency?

(A) Given,

Resistance $(R) = 10\,\Omega$

Inductance $(L) = 125\,mH$

Capacitor $(C) = 60\,\mu F$

Voltage $(V) = 120\,V$

Resonance Frequency condition $X_L = X_C$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$= \frac{1}{4\pi^2 LC}$$

$$= \frac{1}{4\times(3.14)^2 \times 125\times10^{3} \times 60\times10^{-6}} = \frac{1}{2.95\times10^{-4}}$$

$$= \frac{1}{2.95}\times10^{4} = 0.3389\times10^{4} = 58.2\,Hz$$

At $f = 58.2\,Hz$ resonance will occur current

$$I(t) = \frac{V(t)}{Z} = \frac{120}{10 - j(4.4)}$$

$$= 10.05 + 4.4j$$

$$= 10.9\angle 23.7$$

$$X_L = 2\pi f L$$
$$= 215.6$$

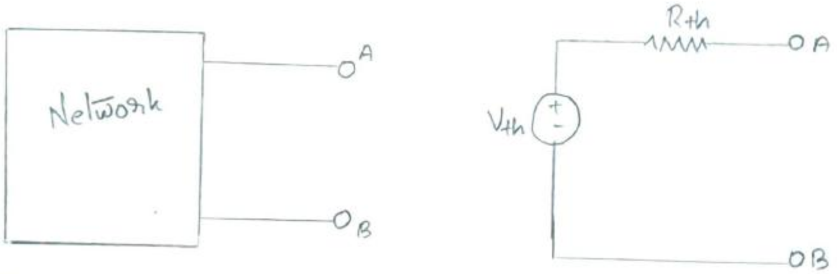
$$X_C = \frac{1}{2\pi f c} = \frac{1}{0.02}$$
$$= 50$$

UNIT- IV

Thevenin's Theorem:-

Statement:-

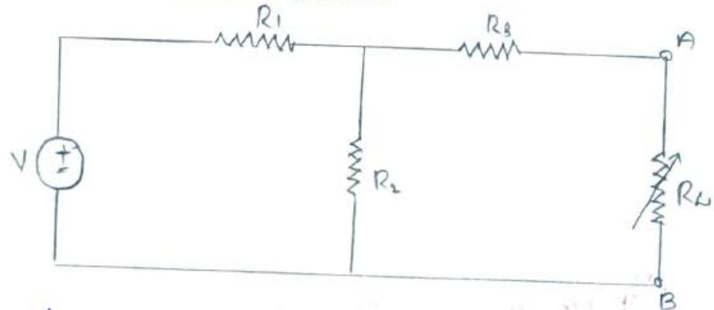
Thevenin's Theorem states that in a linear, bilateral two terminal network having no. of sources and resistance can be replaced with equivalent voltage source in series with equivalent resistance i.e.



where V_{th} = Open circuit voltage between ~~voltag~~ terminals AB.

R_{th} = Equivalent resistance between terminals AB.

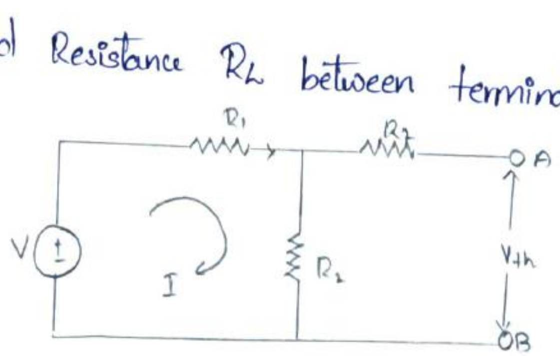
Consider a network shown below.



The thevenin's equivalent b/w terminals AB is obtained as follows.

Step1:- To find V_{th}

Remove load Resistance R_L between terminals, open circuit voltage is given by



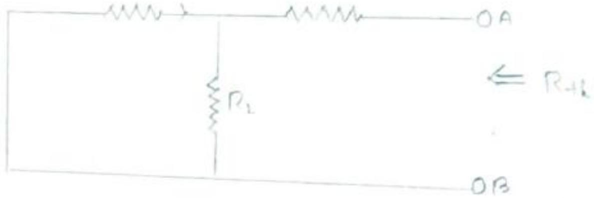
V_{th} = voltage across R_2

$= IR_2$

$$\boxed{V_{th} = \frac{V}{R_1+R_2} R_2}$$

Step2:- To find R_{th}

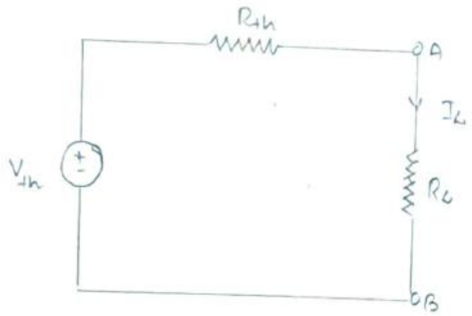
Replace voltage source with short circuit and current source with open circuit.



$$R_{th} = R_1 // R_2 + R_3$$

$$\boxed{R_{th} = \frac{R_1 R_2}{R_1 + R_2} + R_3}$$

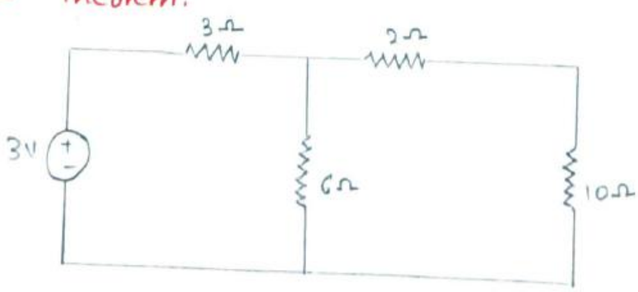
Step3:- Thevenins equivalent



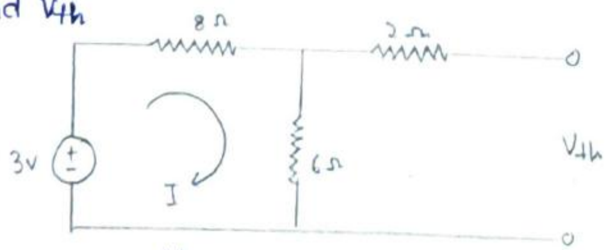
$$\boxed{I_L = \frac{V_{th}}{R_{th} + R_L}}$$

Example:-

Calculate current passing through 10Ω resistor in a circuit given below using thevenin's theorem.



Step1:- To find V_{th}



V_{th} = Voltage across 6Ω

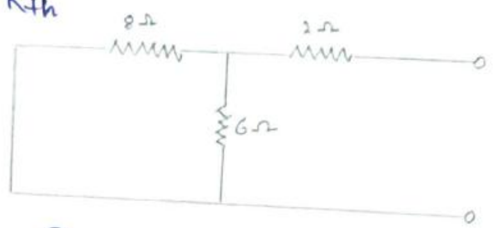
$$= I_6$$

$$= \frac{5}{8+6} \cdot 6$$

$$V_{th} = \frac{5}{14} \cdot 6$$

$$\boxed{V_{th} = 2.1V}$$

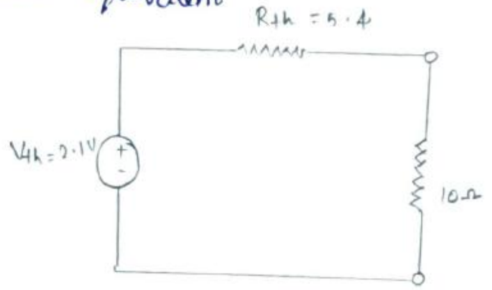
Step2:- To find R_{th}



$$R_{th} = 8 // 6 + 2 = \frac{(8)(6)}{8+6} + 2$$

$$R_{th} = 5.4 \Omega$$

Step3:- Thevenin's equivalent



Current passing through 10Ω resistor is given by

$$I_{10} = \frac{V_{th}}{R_{th}+10} = \frac{2.1}{5.4+10} = 0.13A$$

$$\boxed{I_{10} = 0.13A}$$

Norton's theorem :-

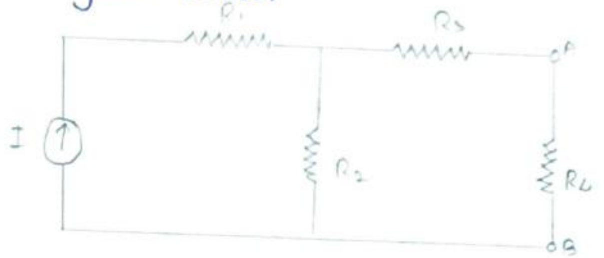
Statement :-

Norton's theorem states that in a linear, bilateral, two terminal network having no. of sources and resistors can be replaced with equivalent current source in parallel with equivalent resistance i.e.

Where I_N = short circuit current between A and B.

R_N = Equivalent resistance between A and B.

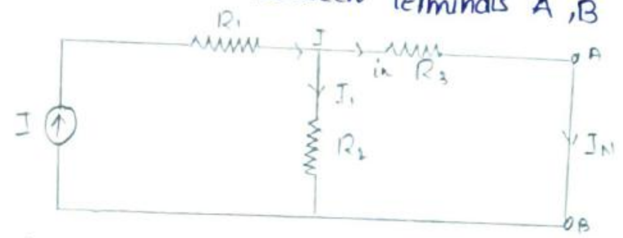
Consider a circuit given below.



Norton's equivalent circuit between A and B is obtained as follows.

Step1:- To find I_N

Make a short circuit between terminals A, B

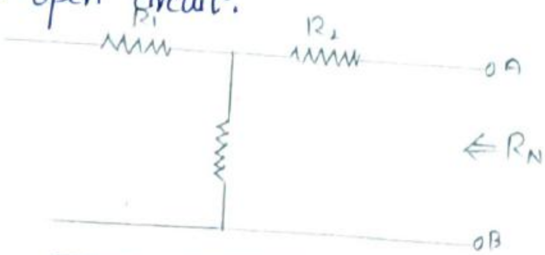


Using current division rule.

$$I_N = I * \frac{R_2}{R_2 + R_3}$$

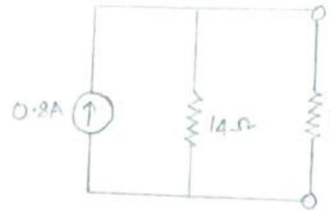
Step2:- To find R_N

Replace voltage source with short circuit, current source with open circuit.



$$\boxed{R_N = R_2 + R_3}$$

Step3:- Norton's equivalent



Current passing through 5Ω resistor is given by

$$I_5 = 0.8 * \frac{14}{14+5}$$

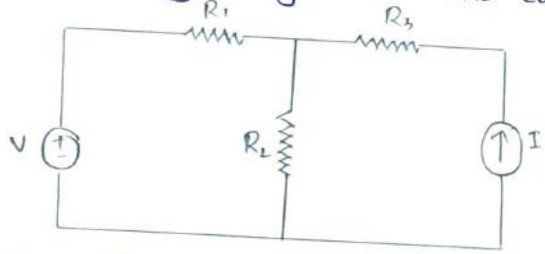
$$\boxed{I_5 = 0.58A}$$

Superposition theorem:-

Statement:- It states that in a linear bilateral network having no. of sources and resistances the response in any element is algebric sum of responses while single source acting alone.

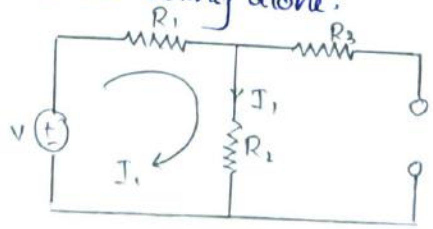
By calculating response by making single source active, voltage sources are replaced with short circuit and current sources are replaced with open circuit.

Consider a circuit having voltage source and current source shown below.



The current passing through resistance R_2 is obtained using super position theorem as follows.

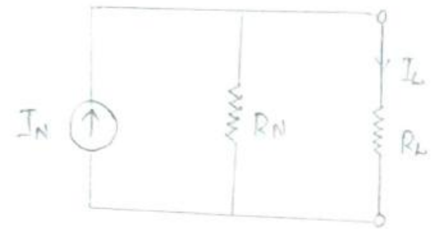
Case 1:- Voltage source acting alone.



$$\boxed{I_1 = \frac{V}{R_1+R_2}}$$

Case 2:- Current source acting alone

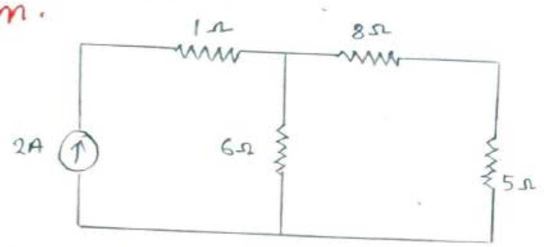
Step3:- Norton's equivalent



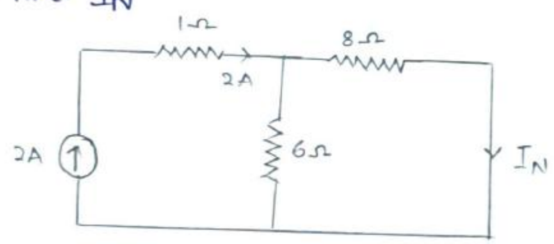
$$\boxed{\text{Load current } I_L = I_N * \frac{R_N}{R_N + R_L}}$$

Example:-

find current passing through 5Ω in the circuit given below using Norton's theorem.



Step1:- To find I_N



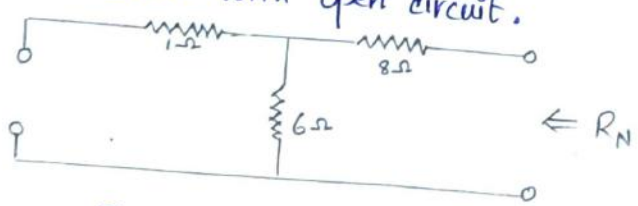
$$I_N = 2 * \frac{6}{6+8}$$

$$= 2 * \frac{6}{14}$$

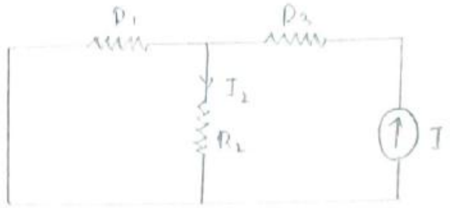
$$\boxed{I_N = 0.8}$$

Step2:- To find R_L

Replace current source with open circuit.



$$R_N = 6 + 8 = 14\Omega$$

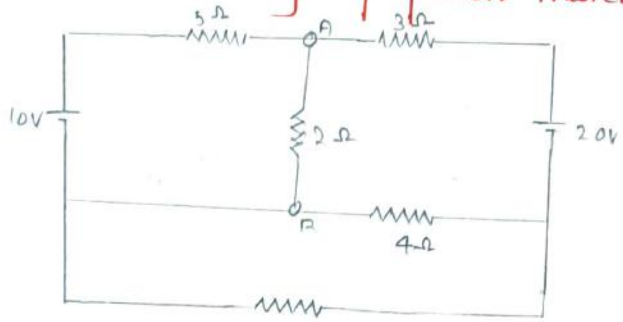


$$I_2 = I \times \frac{R_1}{R_1+R_2}$$

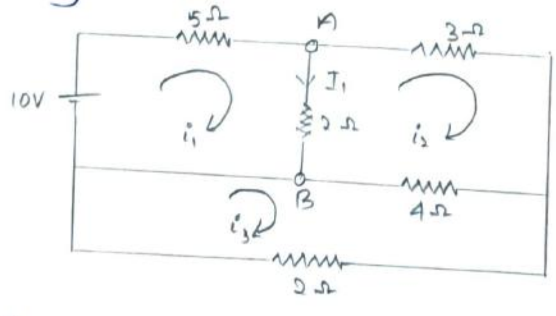
$\therefore$ Current passing through resistance R_2, according to superposition theorem

$$\boxed{I_{R_2} = I_1 + I_2}$$

Example:- Find the current passing through 2Ω resistor between A and B for the network shown below using superposition theorem.



Case1:- 10V acting alone



KVL to loop1:-

$$5i_1 + 2(i_1 - i_2) - 10 = 0$$

$$7i_1 - 2i_2 - 10 = 0 \quad \text{—— (1)}$$

KVL to loop2:-

$$3i_2 + 4(i_2 - i_3) + 2(i_2 - i_1) = 0$$

$$3i_2 + 4i_2 - 4i_3 + 2i_2 - 2i_1 = 0$$

$$-2i_1 + 9i_2 - 4i_3 = 0 \quad \text{—— (2)}$$

KVL to loop3:-

$$4(i_3 - i_2) + 2i_3 = 0$$

$$4i_3 - 4i_2 + 2i_3 = 0$$

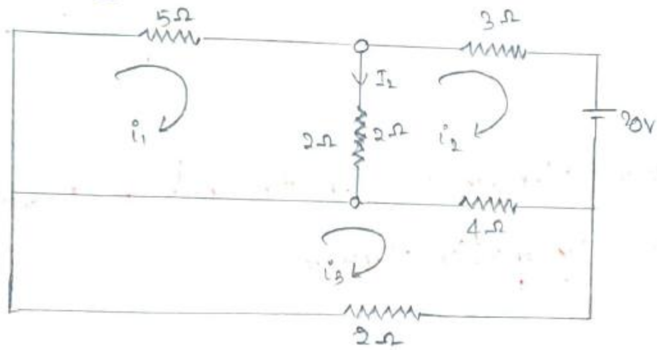
$$-4i_2 + 6i_3 = 0 \quad \text{——} \ (3)$$

$i_1 = 1.5$, $i_2 = 0.4$, $i_3 = 0.3$

$$I_1 = i_1 - i_2$$

$$I_1 = 1.1A$$

Case2:- 20V acting alone



KVL to loop1:-

$$5i_1 + 2(i_1 - i_2) = 0$$

$$7i_1 - 2i_2 = 0 \quad \text{——} \ (1)$$

KVL to loop2:-

$$3i_2 + 2(i_2 - i_1) + 4(i_2 - i_3) = -20$$

$$-2i_1 + 9i_2 - 4i_3 = -20 \quad \text{——} \ (2)$$

KVL to loop3:-

$$4(i_3 - i_2) + 2i_3 = 0$$

$$-4i_2 + 6i_3 = 0 \quad \text{——} \ (3)$$

$i_1 = -0.99$, $i_2 = -3.47$, $i_3 = -2.3$

$$I_2 = i_1 - i_2 = 2.48$$

According to superposition theorem, current across terminals A and B

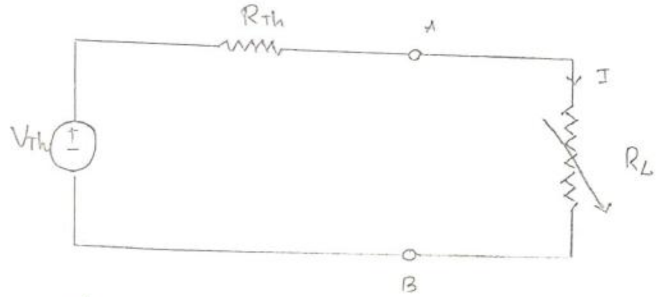
$$\boxed{I_{2\Omega} = I_1 + I_2}$$

$$= 1.1 + 2.4$$

$$\boxed{I_{2\Omega} = 3.5}$$

Maximum Power Transfer Theorem:-

It states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.



The amount of power dissipated across the load resistor is

$$P_L = I^2 R_L$$

substitute $I = \dfrac{V_{Th}}{R_{Th} + R_L}$ in the above equation

$$P_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$\Rightarrow P_L = V_{Th}^2 \left(\frac{R_L}{(R_{Th} + R_L)^2}\right) \quad \text{——— (1)}$$

Condition for Maximum power Transfer:

For maximum & minimum, first derivative will be zero. so, differentiate eq (1) with respect to R_L and make it equal to zero.

$$\frac{dP_L}{dR_L} = V_{Th}^2 \left(\frac{(R_{Th} + R_L)^2 \times 1 - R_L \times 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right) = 0$$

$$\Rightarrow (R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L) = 0$$

$$\Rightarrow (R_{Th} + R_L)(R_{Th} + R_L - 2R_L) = 0$$

$$\Rightarrow (R_{Th} - R_L) = 0$$

$$\Rightarrow R_{Th} = R_L \text{ or } R_L = R_{Th}$$

Therefore, the condition for maximum power dissipation across the load is $R_L = R_{Th}$. That means, if the value of load resistance is equal to the value of source resistance i.e., Thevenin's resistance, then the power dissipated across the load will be of maximum value.

The value of Maximum power Transfer

Substitute $R_L = R_{Th}$ & $P_L = P_{L,Max}$ in equation 1.

$$P_{L,Max} = V_{Th}^2 \left(\frac{R_{Th}}{(R_{Th} + R_{Th})^2} \right)$$

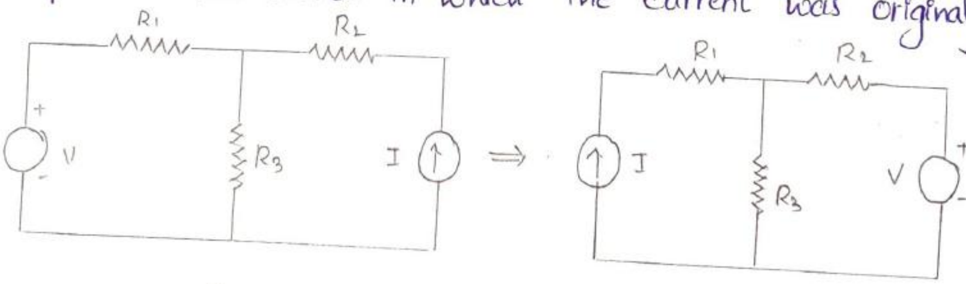
$$P_{L,Max} = V_{Th}^2 \left(\frac{R_{Th}}{4R_{Th}^2} \right)$$

$$\Rightarrow P_{L,Max} = \frac{V_{Th}^2}{4R_{Th}}$$

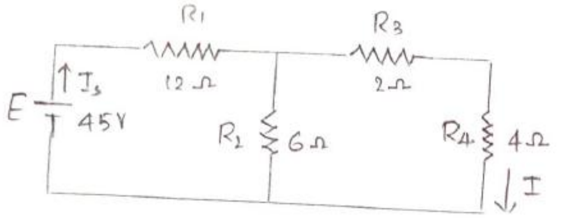
$$\Rightarrow P_{L,Max} = \frac{V_{Th}^2}{4R_L}, \text{ Since } R_L = R_{Th}$$

Reciprocity Theorem:-

Reciprocity theorem states that - In any branch of a network & circuit, the current due to a single source of voltage (V) in the network is equal to the current through that branch in which the source was originally placed when the source is again put in the branch in which the current was originally obtained.



Example:-



Solution:

$R_{TH} = [(2+4)//6] + 12 = 15\Omega$

$I_s = 45/15 = 3A$

By using current division rule

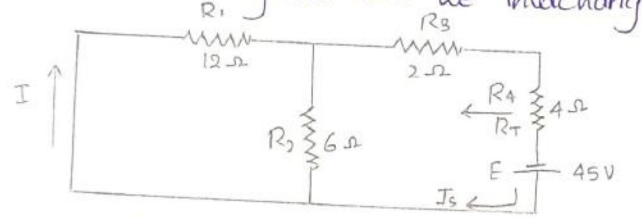
$I = 3 * 6/12 = 1.5 A$

$\boxed{I = 1.5A}$

Now we have to interchange E and I

Since I is pointed downwards in the given problem. So the positive terminal placed downwards when we interchange.

Since E positive terminal points upwards in the given problem, the current I also points upwards in above diagram when we interchange.



$R_T = (12 \| 6) + 2 + 4 = 10\,\Omega$

$I_s = 45/10 = 4.5\,A$

By using current division rule

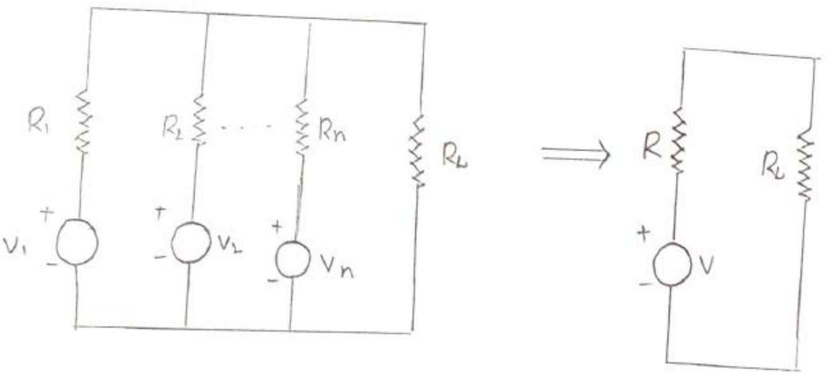
$I = 4.5 * 6/(12+6)$

$I = 1.5\,A$

Since in both cases I are equal reciprocity theorem is verified.

Millman's Theorem:-

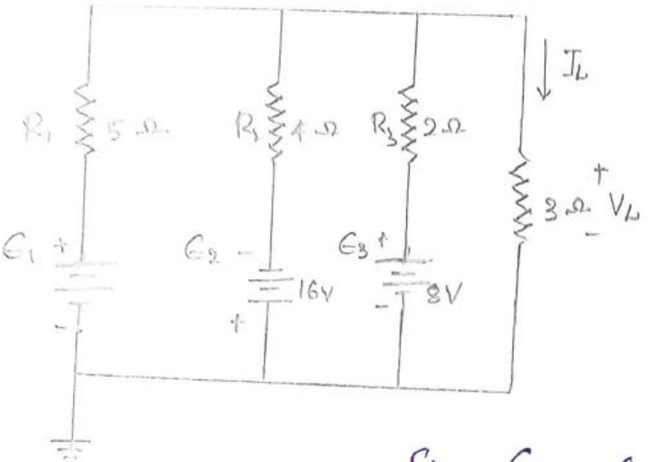
The Millman's Theorem states that - when a number of voltage sources ($V_1, V_2, V_3 \ldots V_n$) are in parallel having internal resistance ($R_1, R_2, R_3 \ldots R_n$) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R.



$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \pm V_n G_n}{G_1 + G_2 + \cdots + G_n} \text{ and}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots + G_n}$$

Example:- Using Millman's theorem, find the current through and voltage across the resistor R_L.



$$E_{eq} = \frac{+\frac{E_1}{R_1} - \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$E_{eq} = \frac{+\frac{10V}{5\Omega} - \frac{16V}{4\Omega} + \frac{8V}{2\Omega}}{\frac{1}{5\Omega} + \frac{1}{4\Omega} + \frac{1}{2\Omega}}$$

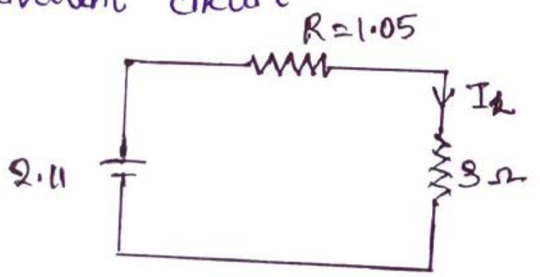
$$= \frac{2A - 4A + 4A}{0.2S + 0.25S + 0.5S}$$

$$= \frac{2A}{0.95S} = 2.11V$$

$$R_{eq} = \frac{1}{G_1 + G_2 + G_3}$$

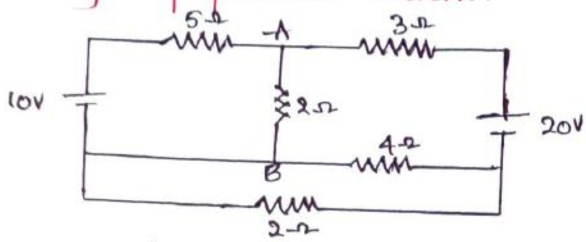
$$= \frac{1}{\frac{1}{5} + \frac{1}{4} + \frac{1}{2}} = 1.05$$

Equivalent circuit

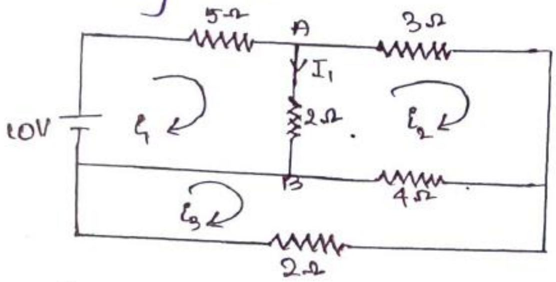


$$I_L = \frac{2.11}{1.05 + 3} = 2.009A$$

1. Find the current in the 2Ω resistor between A and B for the network shown below using superposition theorem.



Case 1: 10V acting alone.



Applying KVL to loop 1

$$i_1 5 + 2(i_1 - i_2) - 10V = 0$$

$$7i_1 - 2i_2 = 10V \text{ ——— } (1)$$

KVL to loop 2

$$3i_2 + 4(i_2 - i_3) + 2(i_2 - i_1) = 0$$

$$3i_2 + 4i_2 - 4i_3 + 2i_2 - 2i_1 = 0$$

$$-2i_1 + 9i_2 - 4i_3 = 0 \text{ ——— } (2)$$

KVL to loop 3

$$4(i_3 - i_2) + 2i_3 = 0$$

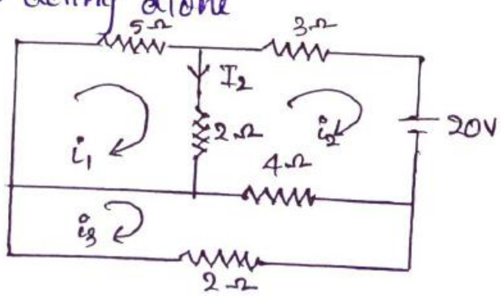
$$-4i_2 + 6i_3 = 0 \text{ ——— } (3)$$

$$i_1 = 1.5 \ ; \ i_2 = 0.4 \ ; \ i_3 = 0.35$$

$$I_1 = i_1 - i_2$$

$$\boxed{I_1 = 1.1A}$$

Case 2:- 20V acting alone



KVL to loop 1

$$5i_1 + 2(i_1 - i_2) = 0$$

$$7i_1 - 2i_2 = 0 \text{ ——— } (1)$$

KVL to loop ②

$$8i_2 + 20 + 4(i_2 - i_3) + 2(i_2 - i_1) = 0$$

$$-2i_1 + 9i_2 - 4i_3 = -20 \quad \text{—— ②}$$

KVL to loop ③

$$4(i_3 - i_2) + 2i_3 = 0$$

$$-4i_2 + 6i_3 = 0$$

$$i_1 = -0.99 \;;\; i_2 = -3.47 \;;\; i_3 = -2.3$$

$$I_2 = i_2 - i_1$$

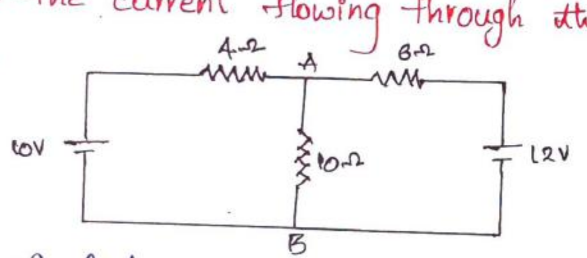
$$I_2 = 2.48$$

∴ According to superposition theorem; current through 2Ω resistor between terminals A & B is given by

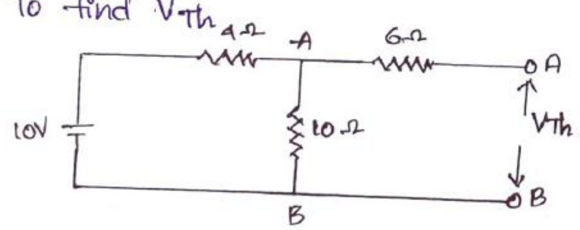
$$I_2 = 1.1 + 2.48$$

$$\boxed{I_2 = 3.58}$$

2. Determine the current flowing through the 10Ω resistor by using Thevenin's theorem.



Step 1:- To find V_{Th}



V_{th} = Voltage across 10Ω resistor

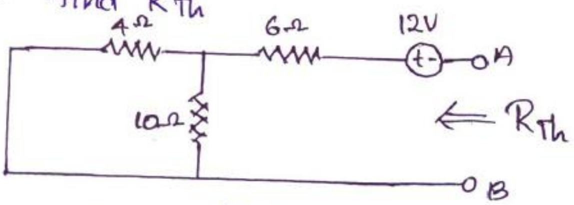
$$= I \times 10$$

$$= \frac{10}{4+10} \times 10$$

$$= \frac{10}{14} \times 10$$

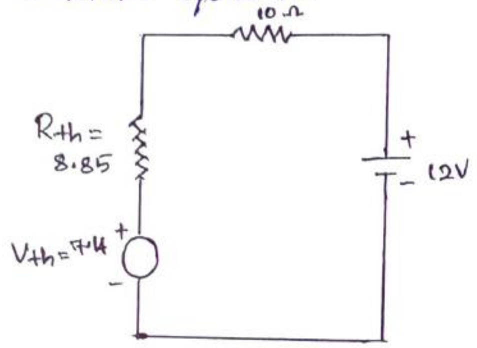
$$V_{th} = 7.14V$$

Step 2:- To find R_{Th}



$$R_{Th} = \frac{4 \times 10}{4 + 10} = \frac{40}{14} = 2.85\,\Omega$$

Step 3 :- Thevenin's equivalent



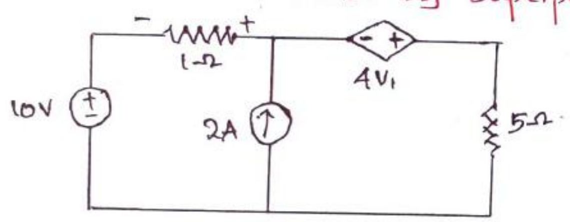
Current passing through 10 Ω is given by

$$I_{10\Omega} = \frac{12 - V_{Th}}{R_{Th} + 10}$$

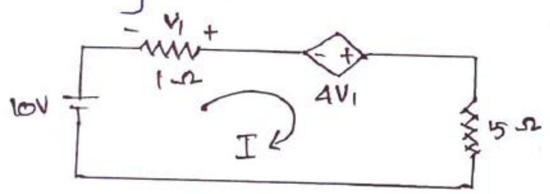
$$= \frac{12 - 7.14}{2.85 + 10} = \frac{4.86}{12.85}$$

$$\boxed{I_{10\Omega} = 0.37A}$$

· Find the power loss in 5 resistor by superposition theorem.



Case-1 :- 10V acting alone



Apply KVL

$$1I - 4V_1 + 5I - 10 = 0$$

but $V_1 = -1I$

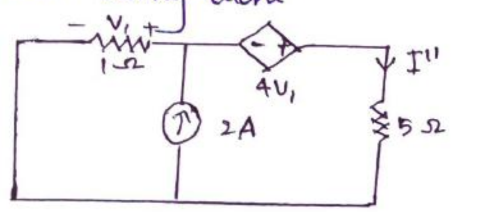
$$I - 4(-1I) + 5I - 10 = 0$$

$$10I = 10$$

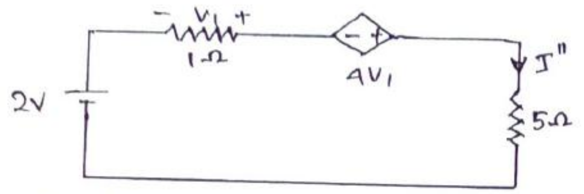
$$I = 1$$

∴ Current in 5 Ω resistor $I' = 1A$

Case-2 :- 2A acting alone



By converting current source into voltage source



Apply KVL

$$1I'' - 4V_1 + 5I'' - 2 = 0$$

But, $V_1 = -I''$

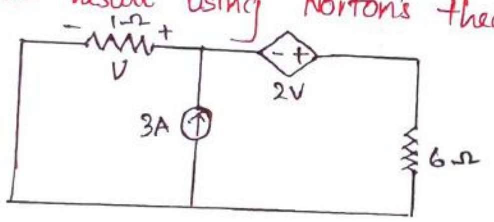
$$I'' = \frac{2}{10} = 0.2A$$

According to super position theorem,

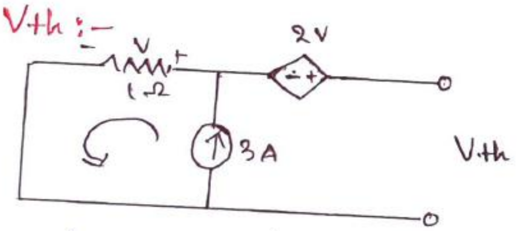
$$I = I' + I'' = 1 + 0.2$$
$$= 1.2A$$

Power loss in 5Ω resistor $P = I^2R = (1.2)^2 \times 5 = 72$

P. Find the current in the 6 resistor in the figure below, using Thevenin's theorem. Verify the result using Norton's theorem.



To find Vth :-

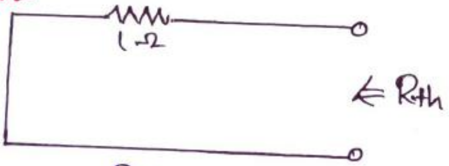


from above circuit

$$V = 3 \text{ volts}$$

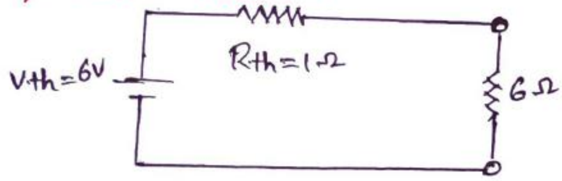
$$V_{th} = 2V = 2 \times 3 = 6 \text{volts}$$

To find Rth :-



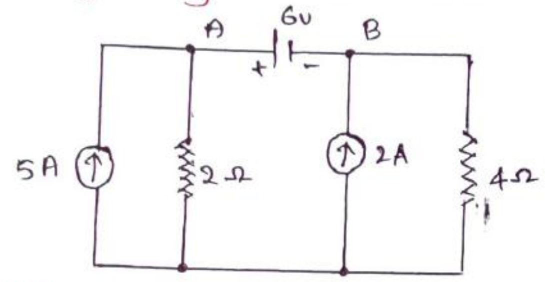
$$R_{th} = 1\Omega$$

Thevenin's Equivalent theorem :-

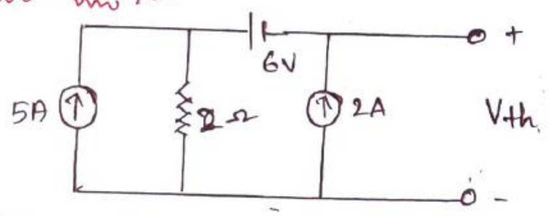


Current in 6Ω resistance $= \frac{6}{1+6} = \frac{6}{7} = 0.85$

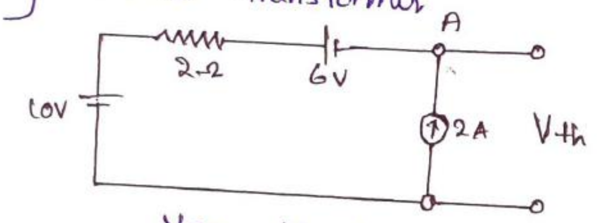
5. Determine the current in 4 ohm resistor for the circuit shown in figure given below by using Thevenin's theorem and verify the result using Norton's theorem.



To find V_{th} :-



Using source transformer

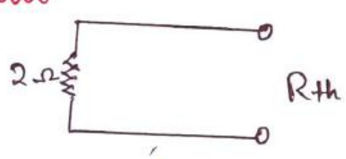


$$V_{AB} = V_A - V_B$$

$$= -6 - (-10)$$

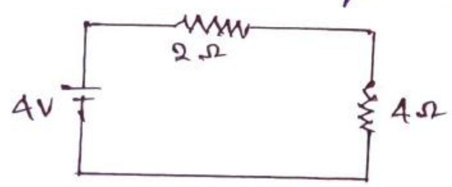
$$V_{AB} = 4V = V_{th}$$

To find R_{th} :-



$$R_{th} = 2\Omega$$

Step 1:- Thevenin's Equivalent theorem

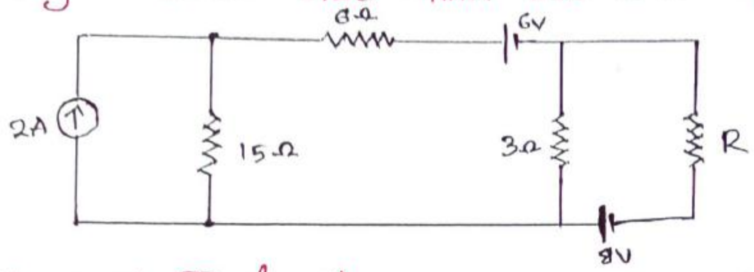


Current passing through 4Ω resistance

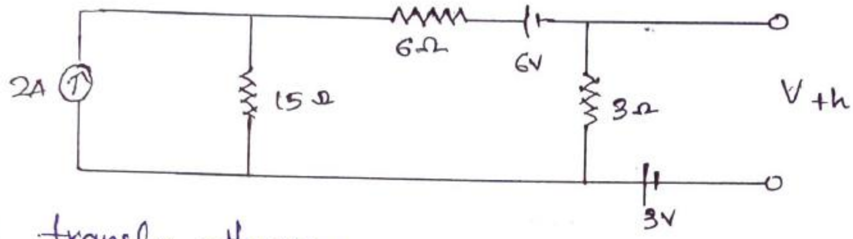
$$R_{th} = \frac{4}{2+4}$$

$$R_{th} = 0.66$$

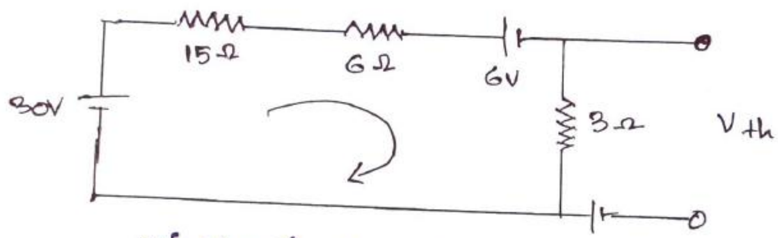
5. Calculate the value of R which will absorb maximum power from the circuit of given below. Also find the value of maximum power.



Maximum power Transfer theorem:- To find V_{th}



Source transfor theorem

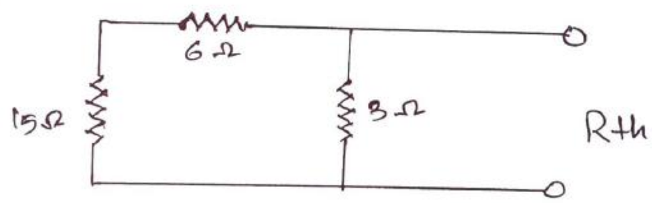


$$21i + 6 + 3i - 30 = 0$$

$$i = \frac{24}{24} = 1\,A$$

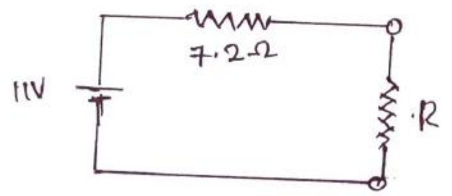
$$V_{th} = 3i + 8 = 11 \text{ volts}$$

To find R_{th}



$$R_{th} = 15 // 6 + 3 = 7.28\,\Omega$$

Thevenin's equivalent theorem:-



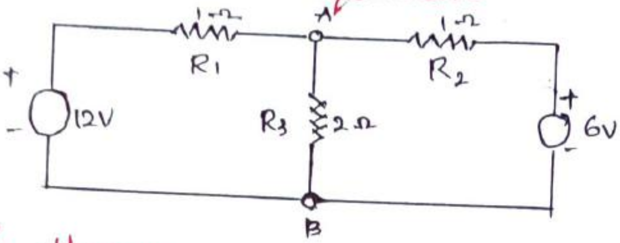
$$\text{Maximum power} = \frac{V^2}{4R} = \frac{11^2}{4 \times 7\,\Omega} = 4.2 \text{ watts}$$

According to maximum power transfer theorem

$$R_{th} = R_{th}$$

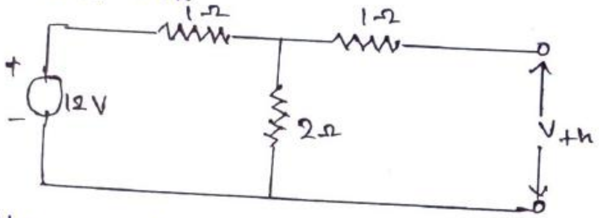
$$R_L = 7.2\,\Omega$$

7. Find the current through 2 Ω resistor in the network shown below by Thevenin's and Norton's equivalents



Nathon's ~~Thevenin's~~ theorem :-

Step 1:- To find V_{th}



V_{th} = Voltage across 2 Ω

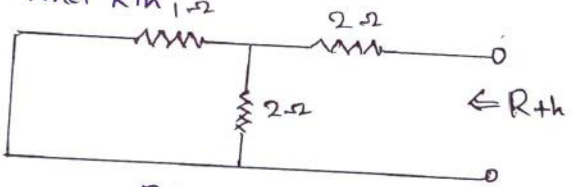
$= V_2 - I \times 2$

$V_{AB} = V_2 - \frac{12 \times 2}{1+2}$

$= 6 - \frac{24}{3}$

$\boxed{V_{AB} = -2V}$

Step 2:- To find R_{th}

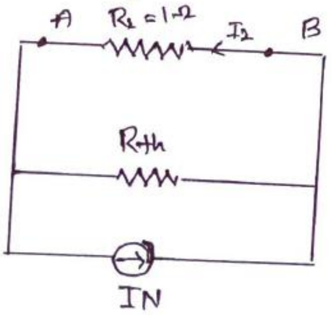


$R_{th} = 1 // 2$

$= \frac{1 * 2}{1+2} = \frac{2}{3} = 0.6\ \Omega$

$i_N = \frac{V_{AB}}{R_{th}} = -3A$

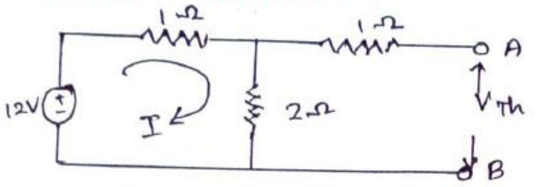
Step 3:- Norton's equivalence



$I_{2\Omega} = \frac{I_N \times R_{Th}}{R_{Th} + R_2} = \frac{-3 \times 0.6}{0.6 + 1} = -1.20A$

By using Thevenin's Theorem:-

step1:-



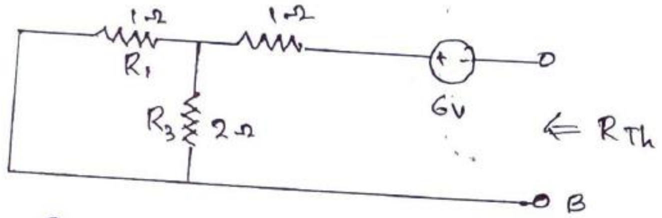
V_{Th} = voltage across 2 Ω

$= I \times 2$

$= \frac{V_1}{R_1+R_3} \times 2 = \frac{12}{1+2} \times 2 = 8V$

$\boxed{V_{Th} = 8V}$

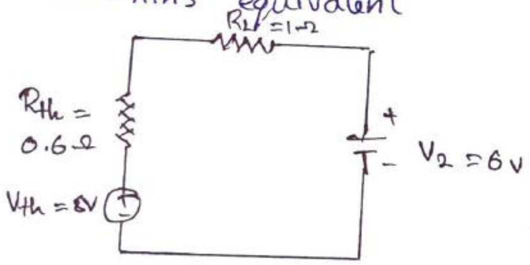
step2:-



$R_{Th} = R_1 // R_3 = 1//2$

$= \frac{1\times 2}{1+2} = 0.6\,\Omega$

step3:- thevenin's equivalent



$I_{2\Omega} = \frac{V_2 - V_{Th}}{R_{Th} + R_2} = \frac{6-8}{0.6+1} = -1.20\,A$

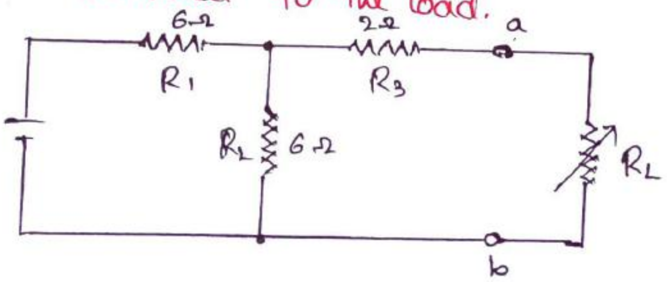
$\boxed{I_{2\Omega} = -1.20A}$

8/

In the network shown below determine:

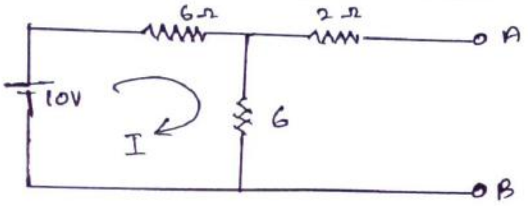
a) The value of load resistance to give maximum power transfer.

b) the power delivered to the load.



Step 1:-

Step -1 :-



$$6I + 6I - 10 = 0$$

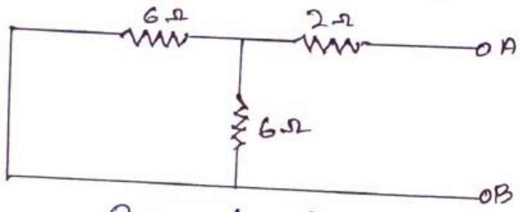
$$12I = 10$$

$$I = \frac{5}{6}$$

$$\therefore V_{th} = 10 - 6 \times \frac{5}{6}$$

$$\boxed{\therefore V_{th} = 5V}$$

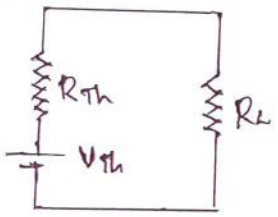
Step 2 :-



$$R_{th} = (6 \parallel 6) + 2$$

$$= 5\Omega$$

Step 3 :- Equivalent



For maximum power

$$R_L = R_{Th}$$

$$R_L = 5\Omega$$

Load Resistance $= 5\Omega$

$$P_{max} = I_L^2 R_L$$

$$I_L = \frac{V_{th}}{R_{Th} + R_L}$$

$$P_{max} = \left(\frac{5}{5+5}\right)^2 \times 5$$

$$= \frac{25}{100} \times 5$$

$$= \frac{125}{100}$$

$$\boxed{P_{max} = 1.25\,W}$$

UNIT-V
THREE PHASE A.C. CIRCUITS
****

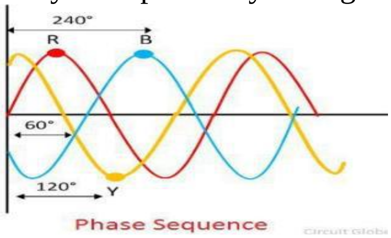
Three Phase Voltages

Phase voltage : The voltage across each phase is called Phase Voltage V_{ph} (or) Voltage between phase and neutral

Line Voltage : The voltage across two line conductors is known as the Line Voltage V_L.

Phase Sequence

In a three-phase system, the order in which the voltages attain their maximum positive value is called **Phase Sequence.** There are three voltages or EMFs in the three-phase system with the same magnitude, but the frequency is displaced by an angle of 120 deg electrically.



if phase sequence if RYB, the line voltages are

$$V_{RY} = V_m \sin(\omega t)$$
$$V_{YB} = V_m \sin(\omega t - 120^0)$$
$$V_{BR} = V_m \sin(\omega t - 240^0)$$

Advantages of Three Phase System

1. For transmission of electrical power three phase supply requires less copper or less conducting material than that of single phase system for given volt-amperes and voltage ratings. Hence 3 phase system is more economical compared to single phase system.

2. Single phase machines are not self starting machines. On the other hand three phase machines are self starting due to rotating magnetic field. Therefore in order to start a single phase machine an auxiliary device is required which not in the case of 3 phase machine.

3. Power factor of single phase machines is poor compared to three phase machines.

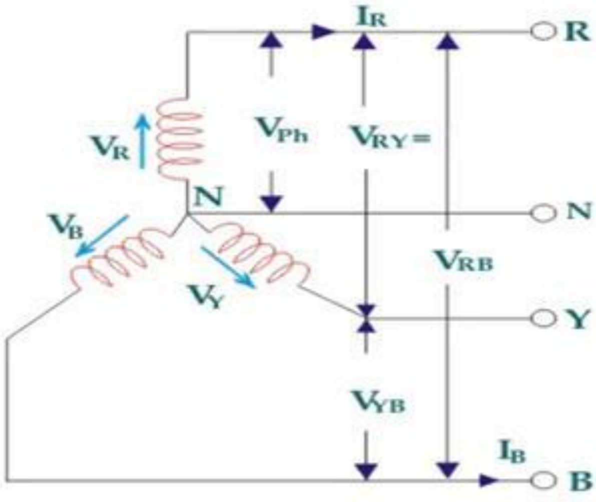
4. In single phase system the instantaneous power is function of time. Hence fluctuates with respect to time. The fluctuating power will cause significant vibrations in the single phase machines. Hence performance of single phase machines is poor. While instantaneous symmetrical three phase system is always constant

5. Three phase system gives steady output

6. Single phase system can be obtained from three phase supply system, vice-versa is not possible

7. For converting systems like rectifiers, the dc voltage waveform becomes more smoother with the increase in the number of phases of the system. Hence three phase system is advantageous compared to single phase system

Derive the voltage and current equations of a balanced star connected system
or
Relationship of Line and Phase Voltages and Currents in a Star Connected System



As we have considered that the system is perfectly balanced, the magnitude of current and voltage of each phase is the same. Let us say, the magnitude of the voltage across the red phase i.e. magnitude of the voltage between neutral point (N) and red phase terminal (R) is V_R. Similarly, the magnitude of the voltage across yellow phase is V_Y and the magnitude of the voltage across blue phase is V_B. In the balanced star system, magnitude of phase voltage in each phase is V_{ph}.

$\therefore V_R = V_Y = V_B = V_{ph}$

Voltage Relation

Now, let us say, the voltage across R and Y terminal of the star connected circuit is V_{RY}.

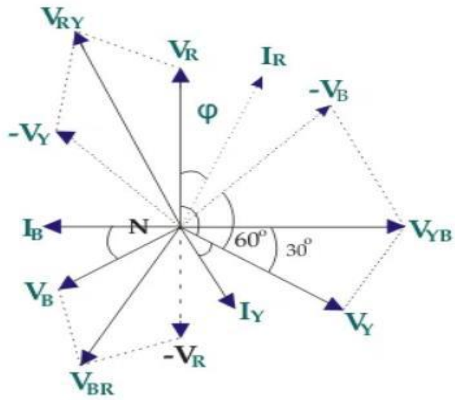
From the diagram, it is found that

$V_{RY} = V_R + (-V_Y)$

Similarly, $V_{YB} = V_Y + (-V_B)$

And, $V_{BR} = V_B + (-V_R)$

Now, as angle between V_R and V_Y is 120^0 , the angle between V_R and $-V_Y$ is 60^0



$$V_L = |V_{RY}| = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^o}$$
$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph} \times \frac{1}{2}}$$
$$= \sqrt{3}V_{ph}$$
$$\therefore V_L = \sqrt{3}V_{ph}$$

Current Relation

From diagram it is clear that line current is same as phase current. The magnitude of this current is same in all three phases and say it is I_L.

$\therefore I_R = I_Y = I_B = I_L$, Where, I_R is line current of R phase, I_Y is line current of Y phase and I_B is line current of B phase. Again, phase current, I_{ph} of each phase is same as line current I_L in star connected system.

$\therefore I_R = I_Y = I_B = I_L = I_{ph}$.

Thus, for the star-connected system line voltage = √3 × phase voltage.

Line current = Phase current

As, the angle between voltage and current per phase is ϕ, the electric power per phase is

$$V_{ph}I_{ph}cos\phi = \frac{V_L}{\sqrt{3}}I_L \cos\phi$$

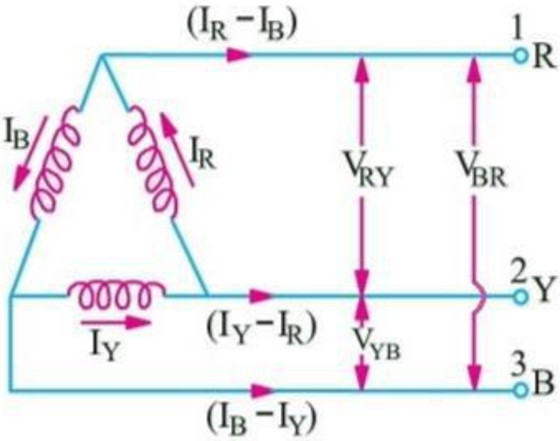
So the total power of three phase system is

$$3 \times \frac{V_L}{\sqrt{3}}I_L \cos\phi = \sqrt{3}V_L I_L \cos\phi$$

Derive the voltage and current equations of a balanced Delta connected system

or

Relationship of Line and Phase Voltages and Currents in a Delta Connected System



Line Voltages (V_L) and Phase Voltages (V_{Ph}) in Delta Connection

It is seen in fig that there is only one phase winding between two terminals (i.e. there is one phase winding between two wires).

$V_{RY} = V_{YB} = V_{BR} = V_L$ …………… (Line Voltage)

Then

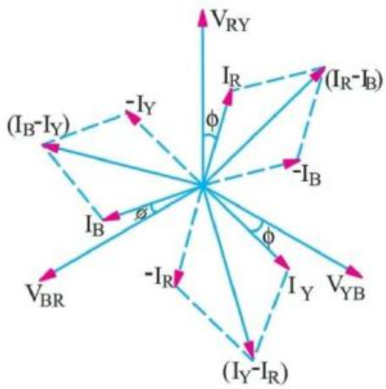
$V_L = V_{PH}$

I.e. in Delta connection, the Line Voltage is equal to the Phase Voltage

Line Currents (I_L) and Phase Currents (I_{Ph})

It will be noted from the above fig. that the total current of each Line is equal to the vector difference between two phase currents in Delta connection flowing through that line. i.e.;

- Current in Line 1= $I_1 = I_R - I_B$
- Current in Line 2 =$I_2 = I_Y - I_R$
- Current in Line 3 =$I_3 = I_B - I_Y$



The current of Line 1 can be found by determining the vector difference between I_R and I_B

we know that $I_R = I_Y = I_B = I_{PH}$ …. The phase currents

Then;

The current flowing in Line 1 would be;

I_L or $I_1 = 2 \times I_{PH} \times \cos(60°/2)$

$= 2 \times I_{PH} \times \cos 30°$

$= 2 \times I_{PH} \times (\sqrt{3}/2)$ …… Since $\cos 30° = \sqrt{3}/2$

$I_L = \sqrt{3}\, I_{PH}$

i.e. In Delta Connection, The Line current is $\sqrt{3}$ times of Phase Current.

Measurement of active power in balanced and unbalanced three phase systems

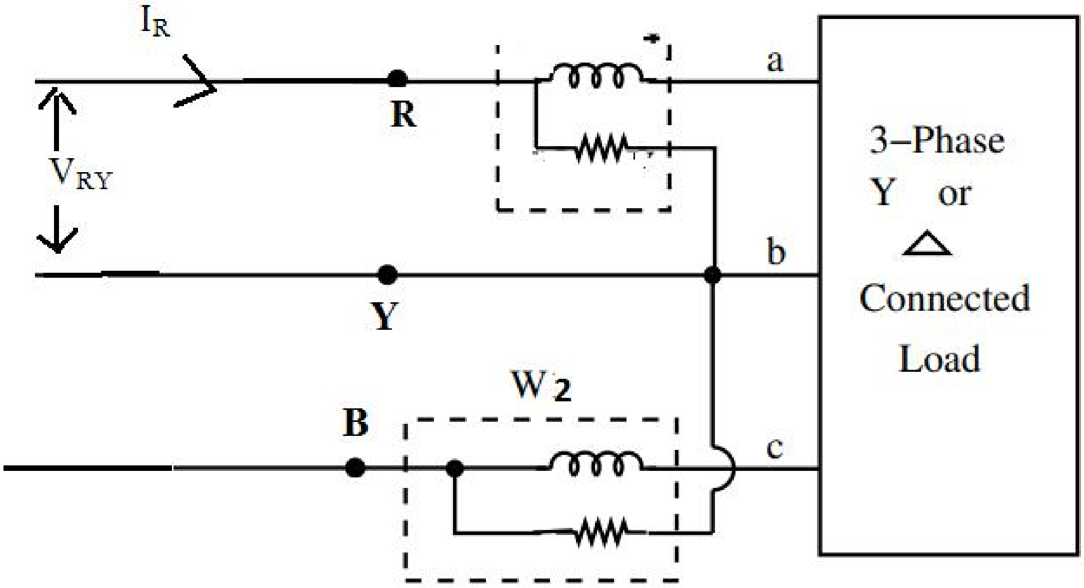
Or

Explain Two Wattmeter method of measurement of three phase power

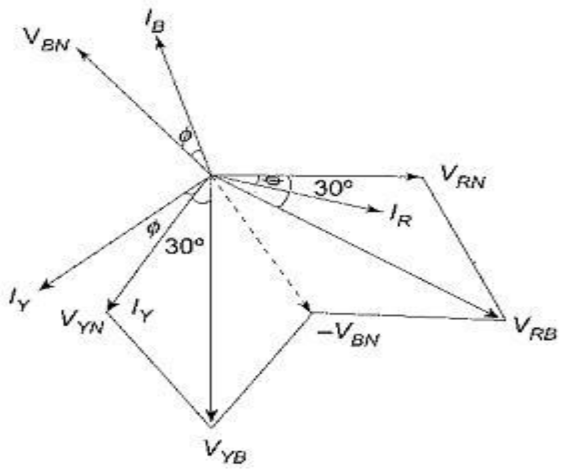
Or

Explain in detail about the measurement of reactive power in balanced and unbalanced three phase systems

Real and Reactive power in balanced (or unbalanced) 3 phase circuits can be measured by using two wattmeters connected in any two lines of a three-phase three wire system. In Fig



From the figure, it is obvious that current through the Current Coil (CC) of Wattmeter $W_1 = I_R$, current though Current Coil of wattmeter $W_2 = I_B$ whereas the potential difference seen by the Pressure Coil (PC) of wattmeter W1= V_{RY} (Line Voltage) and potential difference seen by Pressure Coil of wattmeter $W_2 = V_{BY}$. The phasor diagram of the above circuit is drawn by taking VR as reference phasor as shown below.



From the above phasor diagram,

Angle between the current I_R and voltage $V_{RB} = (30° – Ø)$

Angle between current I_Y and voltage $V_{YB} = (30° + Ø)$

Therefore, Active power measured by wattmeter $W_1 = V_{RY}I_R \, Cos\,(30° – Ø)$

Similarly, Active power measured by wattmeter $W_2 = V_{BY}I_B Cos(30° + Ø)$

As the load is balanced, therefore magnitude of line voltage will be same irrespective of phase taken i.e. V_{RY}, V_{YB} and V_{RB} all will have same magnitude. Also for Star connection line current and phase current are equal, say $I_R = I_Y = I_B = I$

Let $V_{RY} = V_{YB} = V_{RB} = V_L$

Therefore,

$W_1 = V_{RY}I_R Cos(30° – Ø)$
$= V_L I_L\ Cos(30° – Ø)$
In the same manner,
$W_2 = V_L I_L Cos(30° + Ø)$
Active Power
Hence, total power measured by wattmeters for the balanced three phase load is given as,
$W = W_1 + W_2$
$= V_L I_L Cos(30° – Ø) + V_L I_L Cos(30° + Ø)$
$= V_L I_L\ [Cos(30° – Ø) + Cos(30° + Ø)]$
$= 2V_L I_L \times Cos30° CosØ$
$W_1 + W_2 = \sqrt{3} V_L I_L CosØ$ ……………………………..(1)
Therefore, total power measured by wattmeters $W = \sqrt{3} V_L I_L CosØ$
Reactive Power
Similarly,
$W_1 – W_2 = V_L I_L \times Cos(30° – Ø) + V_L I_L \times Cos(30° + Ø)$
$= V_L I\ [Cos(30° – Ø) + Cos(30° + Ø)]$
$= 2V_L I_L \times Sin30° SinØ$
$= V_L I_L SinØ$
Hence,
$W_1 – W_2 = V_L I_L SinØ$ ………………………………(2)
by multiplying above value with $\sqrt{3}$ we will get total reactive power
Power Factor
To find the power factor of the load when individual power measured by the wattmeters are given, then we should proceed as
Dividing equation (2) by equation (1),

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin Ø}{\sqrt{3} V_L I_L \cos Ø} = \frac{\tan Ø}{\sqrt{3}}$$

Hence,

$$Ø = \tan^{-1}\left(\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right)$$

ANALYSIS OF BALANCED THREE PHASE CIRCUITS

1.3.1 Balanced delta connected load:

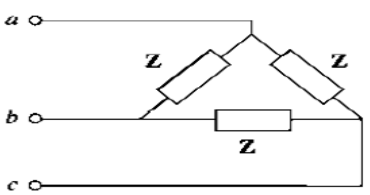


Fig.1.6

Let us consider a balanced 3-phase delta connected load

Determination of phase voltages:

$V_{AB} = V\angle 0^0,\ V_{BC} = V\angle\text{-}120^0,\ V_{CA} = V\angle -240^0 = V\angle 120^0$

Determination of phase currents:

Phase current = Phase voltage/ Load impedance

$I_{AB} = \frac{V_{AB}}{Z}$; $I_{BC} = \frac{V_{BC}}{Z}$; $I_{CA} = \frac{V_{CA}}{Z}$

Determination of line currents:

Line currents are calculated by applying KCL at nodes A,B,C

$I_A = I_{AB} – I_{CA}$; $I_B = I_{BC} - I_{AB}$; $I_C = I_{CA} - I_{BC}$

Note: Line currents are also balanced and equal to $\sqrt{3}$phase current.

Balanced star connected load

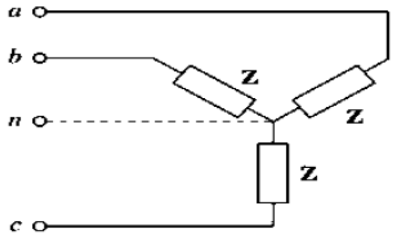


Fig.1.7

Let us consider a balanced 3-phase star connected load.

For star connection, phase voltage= Line voltage/($\sqrt{3}$)

For ABC sequence, the phase voltage is polar form are taken as

$$V_{AN} = V_{ph}\angle -90^0 \ ; V_{CN} = V_{ph}\angle 150^0 \ ; V_{BN} = V_{ph}\angle 30^0$$

For star connection line currents and phase currents are equal

$$I_A = \frac{V_{AN}}{Z} \ ; I_B = \frac{V_{BN}}{Z} \ ; I_C = \frac{V_{CN}}{Z} \ ;$$

To determine the current in the neutral wire apply KVL at star point

$$I_N + I_A + I_B + I_C = 0$$

$$I_N = -(I_A + I_B + I_C) \quad \text{(since they are balanced)}$$

5.4 UNBALANCED STAR CONNECTED LOAD WITHOUT A NEUTRAL CONNECTION:

In the case of a balanced Star connected load we have shown that the neutral current is zero, so even though there is no neutral connection,

$$V_{ab} = \frac{V_{AB}}{\sqrt{3}}\angle -30^o$$

However if the load is not balanced this equation does not apply. An alternative method should be used. In this case we will apply Kircoff's laws to the circuit. (Boylestad 2007, p. 1054)

We define the three circulating currents in the circuit to be I_1, I_2 and I_3 as shown in Figure 5.4.

It is assumed that the supply voltage is balanced and the magnitude is known. It is also assumed that the values of Z_1, Z_2 and Z_3 are known.

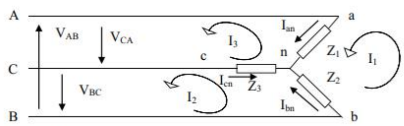


Figure 5.4

We then apply Kirchoff's voltage law to the three current loops;

Loop 1:

$$V_{AB} = (Z_1)(I_1 - I_3) + (Z_2)(I_1 - I_2)$$

From Figure 5.4 it can be seen that;

$$(I_1 - I_3) = I_{an}$$

$$(I_1 - I_2) = -I_{bn}$$

$$(I_2 - I_3) = -I_{cn}$$

And so;

$$V_{AB} = Z_1 I_{an} - Z_2 I_{bn} \qquad \text{Eqn (1)}$$

Loop 2:

$$V_{BC} = (Z_2)(I_2 - I_1) + (Z_3)(I_2 - I_3)$$

$$V_{BC} = Z_2 I_{bn} - Z_3 I_{cn} \qquad \text{Eqn (2)}$$

Loop 3:

$$V_{CA} = (Z_3)(I_3 - I_2) + (Z_1)(I_3 - I_1)$$

$$V_{CA} = Z_3 I_{cn} - Z_1 I_{an} \qquad \text{Eqn (3)}$$

In these three equations the unknowns are I_{an}, I_{bn} and I_{cn}. By solving these three simultaneous equations it can be shown that;

$$I_{an} = \frac{V_{AB}Z_3 - V_{CA}Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_{bn} = \frac{V_{BC}Z_1 - V_{AB}Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_{cn} = \frac{V_{CA}Z_2 - V_{BC}Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

1. Three loads, each of resistance 30 ohms are connected in star to a 415 V three-phase supply. Determine: i) The system phase voltage. ii) The phase current. iii) The line current.

Sol: A 415 V, 3-phase supply means that 415 V is the line voltage V_L.

i) For a star connection,

$$V_L = \sqrt{3} V_p$$

$$V_p = \frac{V_L}{\sqrt{3}}$$

$$= \frac{415}{\sqrt{3}}$$

$$\boxed{V_p = 240V}$$

ii) phase current,

$$I_p = \frac{V_p}{R_p}$$

$$= \frac{240}{30}$$

$$\boxed{I_p = 8A}$$

iii) For a star connection, $I_p = I_L$

Hence, the line current, $\boxed{I_L = 8A}$

2. Three identical coils each of resistance 30 ohms and identical 127.3 mH are connected in delta to a 440V, 50HZ, three phase supply. Determine i) The phase current, ii) The line current.

Sol: Resistance (R) = 30 ohms

Inductances (L) = $127.3 \times 10^{-3} H$

a) Delta connection

$V_L = V_P = 440V$

Inductance reactance

$X_L = 2\pi f L = 2\pi(50)(127.3\times10^{-3})$

$X_L = 40$

phase Impedance

$Z_P = \sqrt{R^2 + X_L^2}$

$= \sqrt{30^2 + 40^2}$

$Z_P = 50$

Line voltage $V_L = 440V$

i) phase current

$I_P = \frac{V_P}{Z_P} = \frac{440}{50} = 8.8$

$\boxed{I_P = 8.8A}$

ii) line current

$I_L = \sqrt{3} \times I_P$

$= \sqrt{3} \times 8.8$

$\boxed{I_L = 15.2A}$

3. A balanced, three phase, star connected load is fed from a 400 V, three phase, 50H supply. The current per phase is 25A (lagging) and the total active power absorbed by the load is 13.56 KW. Determine

i) The resistance and inductance of the load per phase

ii) The total reactive power

iii) The total apparent power

Sol: Star connection

i) $I_p = 25A \Rightarrow I_p = I_L$

$V_L = 400V$

$\Rightarrow V_p = \frac{V_L}{\sqrt{3}}$

$V_p = 230.9$

$I_p = \frac{V_p}{R_p}$

$R_p = \frac{V_p}{I_p} = \frac{400}{25} = \frac{230.9}{25}$

$\boxed{R_p = 9.2\Omega}$

ii) Total reactive power

active power $(P) = \sqrt{3} V_L I_L \cos\phi$

$13.56 \times 10^3 = \sqrt{3} \times 231 \times 25 \times \cos\phi$

$10002.5 \cos\phi = 13.56 \times 10^3$

$\cos\phi = \frac{13.56 \times 10^3}{10002.5}$

$\cos\phi = 1.35$

$\phi = \cos^{-1}(0.35)$

$\phi = 0.99$

Reactive power $\phi = \sqrt{3} V_L I_L \sin\phi$

$= \sqrt{3} \times 231 \times 25 \times \sin(0.99)$

$= 10002.5 \times \sin(0.99)$

$= 172.8$ VAR

$\boxed{\text{Reactive power} = 172.8 \text{ VAR}}$

iii) Apparent power $= \sqrt{3} V_L I_L$

$= \sqrt{3} \times 230.9 \times 25$

$= 10002.5$

$\boxed{\text{Apparent power} = 10002.5}$

4. A Delta connection load has a parallel combination of resistance 5 ohms and capacitive reactance of $-j5$ ohms in each phase. If the balanced 3-phase 400 V supply is applied between lines, Find the phase currents and the line currents and draw the phasor diagram.

Sol: Delta connection.

$$Z_p = \frac{5 \times -j5}{5-j5} = \frac{-25j}{5-j5}$$

$$Z_p = 2.5 - 2.5j$$

$$V_p = 400V$$

For delta connection

$$V_p = V_L = 400V$$

$$\therefore I_p = \frac{V_p}{Z_p} = \frac{400}{2.5-2.5j}$$

$$I_p = 80 + 80j$$

$$I_p = 113 \angle 45^\circ$$

$\boxed{\text{phase current} = 113 \angle 45^\circ}$

$$I_L = \sqrt{3} \times I_p$$

$$= \sqrt{3} \times 113 \angle 45^\circ$$

$$= 138.5 + 138.5j$$

$$I_L = 195.8 \angle 45^\circ$$

$\boxed{\text{line current} = 195.8 \angle 45^\circ}$

6. An unbalanced four-wire, star connected load has a balanced voltage of 400V, the loads are $Z_1=(4+8j)\Omega$; $Z_2=(3+j4)\Omega$; $Z_3=(15+j20)\Omega$. Calculate the i) line currents, ii) currents in the neutral wire, iii) The total power

Sol: i) $Z_1 = (4+j8)\ \Omega$

$Z_2 = (3+j4)\ \Omega$

$Z_3 = (15+j20)\ \Omega$

Star connection

$V_L = 400V$

$V_p = \frac{400}{\sqrt{3}}$

$V_p = 230.9$

$I_L = \frac{V_L}{Z_1} = \frac{400}{4+8j} = 44\angle{-63^\circ}$

$I_L' = \frac{V_L}{Z_2} = \frac{400}{3+j4} = 80\angle{-53^\circ}$

$I_L'' = \frac{V_L}{Z_3} = \frac{400}{15+j20} = 16.\angle{-53^\circ}$

ii) Currents in the neutral wire is nothing but total.

$I = I_L + I_L' + I_L''$

$I = 139.5\angle{-56^\circ}$

iii) Total power

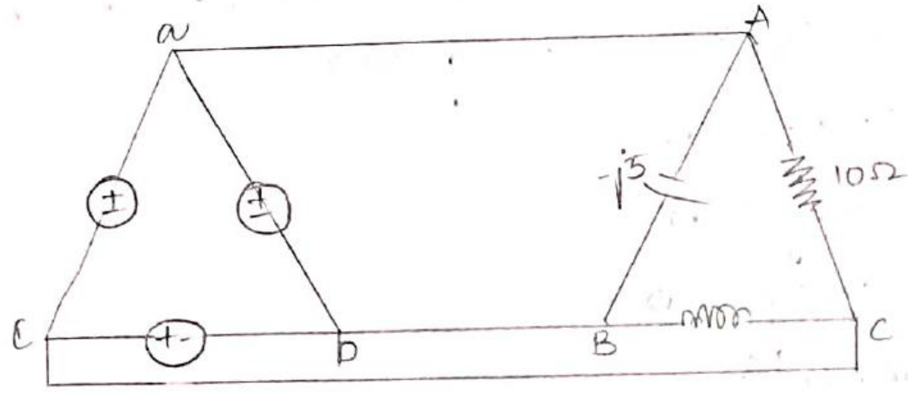
$P = I_L^2 R = 44^2 \times 16 = 30976\ W$

$P' = I_L'^2 R = 80^2 \times 9 = 5760W$

$P'' = I_L''^2 R = 16^2 \times 15^2 = 57600W$

7. Find the line currents in the unbalanced three phase circuit of figure, shown below and real power absorbed by the load.

Sol:



line currents

$$I_{ab} = \frac{V_{ab}}{-j5}$$

$$= \frac{200\angle 0^\circ}{-j5}$$

$$I_{ab} = 44\angle 90^\circ$$

$$I_{bc} = \frac{V_{bc}}{j10}$$

$$= \frac{220\angle 120^\circ}{j10}$$

$$\boxed{I_{bc} = 22\angle 30^\circ}$$

$$I_{ac} = \frac{V_{ac}}{10} = \frac{220\angle -120^\circ}{10}$$

$$I_{ac} = 22\angle -120^\circ$$

Real power $(P) = I^2 R$

$$= (22\angle -120^\circ)^2 \times 10$$

$$\boxed{P = 4840}$$

8. A positive sequence balanced delta connected source supplies a balanced delta connected load. If the impedence per phase of the load is $18+j12\Omega$ and $I_a = 9.609 \angle 35 A$. Find I_{AB} and V_{AB}.

Sol: Delta connection

$$Z = 18+j12\Omega$$

$$I_a = 9.609 \angle 35^\circ$$

$$I_{AB} = \sqrt{3} \times I_a$$

$$= \sqrt{3} \times 9.609 \angle 35^\circ$$

$$I_{AB} = 16.6 \angle 35 A$$

$$V_{AB} = I_{AB} \times R$$

$$= 16.6 \times 18$$

$$\boxed{V_{AB} = 299 V}$$

9. A 3-phase 400V system has a delta connected load with $Z_{ab} = (8+j6)\Omega$, $Z_{bc} = (12+j16)\Omega$ and $Z_{ca} = (6-j8)\Omega$. Find the currents and line currents. Determine the power consumed by each load impedance.

Sol: Delta Connection.

$$Z_{ab} = (8+j6)\Omega$$

$$Z_{bc} = (12+j16)\Omega$$

$$Z_{ca} = (6-j8)\Omega$$

For delta connection $I_L = \sqrt{3} I_p$

phase currents

$$I_p = \frac{V_p}{Z_{ab}} = \frac{400}{8+j6}$$

$$I_p = 40 \angle -36^\circ$$

$$I'_p = \frac{V_p}{Z_{bc}} = \frac{400}{12+j16}$$

$$I'_p = 20 \angle -53^\circ$$

$$I''_p = \frac{V_p}{Z_{ca}} = \frac{400}{6-j8}$$

$$I''_p = 40 \angle -53^\circ.$$

line currents

$$I_L = \sqrt{3}\, I_p$$

$$= \sqrt{3} \times 40 \angle -36^\circ$$

$$I_L = 69.2 \angle -36^\circ$$

$$I'_L = \sqrt{3}\, I'_p$$

$$= \sqrt{3} \times 20 \angle -53^\circ$$

$$I'_L = 34.6 \angle -53^\circ$$

$$I''_L = \sqrt{3}\, I''_p$$

$$= \sqrt{3} \times 40 \angle -53^\circ$$

$$\boxed{I''_L = 69.2 \angle -53^\circ}$$

power for each impedence,

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 69.2 \cos(-36^\circ)$$

$$\boxed{P = 38786\ W}$$

$$\Rightarrow P = \sqrt{3}\, V_L I'_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 34.6 \cos(-53)$$

$$\boxed{P = 14426\, W}$$

$$\Rightarrow P = \sqrt{3} \times V_L \times I''_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 69.2 \times \cos(-53)$$

$$\boxed{P = 28852\, W}$$

0. The two-wattmeter method produces wattmeter readings $P_1 = 1560\,W$ and $P_2 = 2100\,W$ when connected to a delta connected to a delta connected load. If the line voltage is 220V calculate: i) per-phase average power, ii) The per-phase reactive power, iii) The power factor and the phase impedence.

Sol: Given, $P_1 = 1560\, W$

$P_2 = 2100\, W$

$V_L = 220V$

For delta connection $V_L = V_P$

i) Per-phase average power

$$P = \frac{P_1 + P_2}{3} = \frac{1560 + 2100}{3}$$

$$\boxed{P = 1220\, W.}$$

ii) Reactive power,

Total power $= \sqrt{3} \times V_L \times I_L \cos\phi$

$$1560 + 2100 = \sqrt{3} \times 220 \times I_L \cos\phi$$

$$3660 = \sqrt{3} \times 220 \times I_L \cos(0.9)$$

$$I_L = \frac{3660}{\sqrt{3} \times 220 \times 0.99}$$

$$I_L = \frac{3660}{377.2}$$

$$\boxed{I_L = 9.7A.}$$

$$\left(\therefore I_p = \frac{9.7}{\sqrt{3}} = 5.6\right)$$

Reactive power $= \sqrt{3} \times V_L \times I_L \times \sin\phi$

$= \sqrt{3} \times 220 \times 9.7 \times \sin(0.9)$

$= 55.44$

$\boxed{\text{Reactive power} = 55.44}$

iii) $\tan\phi = (P_1 - P_2)(P_1 + P_2)\sqrt{3}$

$$= \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2} = \frac{\sqrt{3}(1560 - 2100)}{1560 + 2100}$$

$$= \frac{-935.3}{3760}$$

$\tan\phi = -0.25$

$\phi = \tan^{-1}(0.25)$

$\phi = -14.$

Power factor $= \cos\phi = \cos(-14)$

$= 0.9$

Impedence $Z_p = \frac{V_p}{I_p}$

$$Z_p = \frac{220}{5.6}$$

$$\boxed{Z_p = 39}$$

Two wattmeters are connected to measure the input power to a balanced 3-phase load by the two-wattmeter method. If the instrument reading are ~~skw~~ 8 KW and 4 KW. determine

a) the total power input and

b) the load power factor

Sol: a) Total input power,

$$P = P_1 + P_2 = 8 + 4 = 12 \text{ KW}$$

b) $\tan\phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2)$

$= \sqrt{3}(8-4)/(8+4)$

$= \sqrt{3}(4/12)$

$= \sqrt{3}(1/3)$

$= \frac{1}{\sqrt{3}}$

$\tan\phi = \frac{1}{\sqrt{3}}$

$\phi = \tan^{-1}(1/\sqrt{3}) = 30^\circ$

Power factor $\Rightarrow \cos\phi = \cos 30^\circ$

$\boxed{\phi = 0.866.}$

12. Three identical coils, each of resistance 10 ohm and inductance 42 mH are connected (a) in star, (b) in delta to a 415 V, 50 Hz, 3-phase supply. Determine the total power dissipated in each case.

Sol: a) Star connection,

Inductive reactance,

$$X_L = 2\pi f L = 2\pi(50)(42 \times 10^{-3}) = 13.19$$

phase impedance,

$$Z_p = \sqrt{R^2 + X_L^2}$$

$$Z_p = \sqrt{10^2 + (13.19)^2}$$

$$Z_p = 16.55$$

line voltage, $V_L = 415\,V$

and phase voltage,

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240V.$$

phase current,

$$I_p = \frac{V_p}{Z_p} = \frac{240}{16.55} = \underline{14.50}\ A$$

line current $I_L = I_p = 14.50$

power factor $= \cos\phi = \frac{R_p}{Z_p} = \frac{10}{16.55} = 0.6042$ lagging

power dissipated,

$$P = 3I^2R = 3(14.50)^2(10)$$

$$\boxed{P = 6.3\,kW}$$

b) Delta connection

$$V_L = V_p = 415\,V$$

$$Z_p = 16.55$$

$$\cos\phi = 0.6042 \text{ (lagging)}$$

phase current,

$$I_p = \frac{V_p}{Z_p} = \frac{415}{16.55} = 25.08\,A$$

line current, $I_L = \sqrt{3}I_p = \sqrt{3}(25.08)$

$$\boxed{I_L = 43.44A}$$

power dissipated

$$P = 3I^2R$$

$= 3(25.08)^2(10)$

$\boxed{P = 18.87\,kW}$

3. A 400 V, 3-phase star connected alternator supplies a delta connected load, each phase of which has a resistance of 30 and inductive reactance 40. Calculate (a) the current supplied by the alternator and (b) the output power and the KVA of the alternator, neglecting losses in the line between the alternator and load.

Sol: a) Considering the load,

phase current $I_p = \dfrac{V_p}{Z_p}$

$V_p = V_L$ for a delta connection.

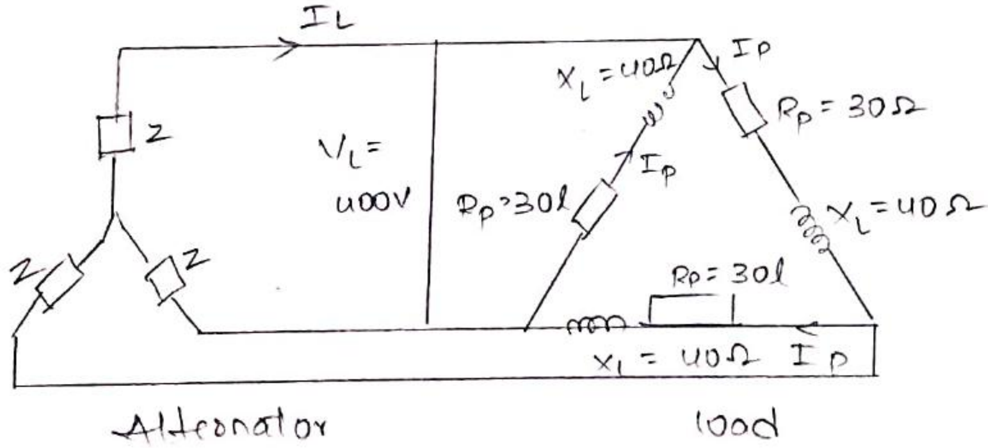
Hence, $V_p = 400V$

phase impedence,

$$Z_p = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{30^2 + 40^2}$$

$$Z_p = 50$$



Hence $I_p = \frac{Z_P}{V_p} = \frac{400}{50} = 8A$

For a delta connection,

line current $I_L = \sqrt{3} I_p$

$= \sqrt{3}(8) = 13.86A.$

Hence, 13.86A is the current supplied by the alternator

(b) Alternator output power is equal to the power dissipated by the load

ie; $P = \sqrt{3} V_L I_L \cos\phi$, where $\cos\phi = R_P/Z_P = 0.6$

$P = \sqrt{3}(400)(13.86)(0.6)$

$\boxed{P = 5.76 \, KW}$

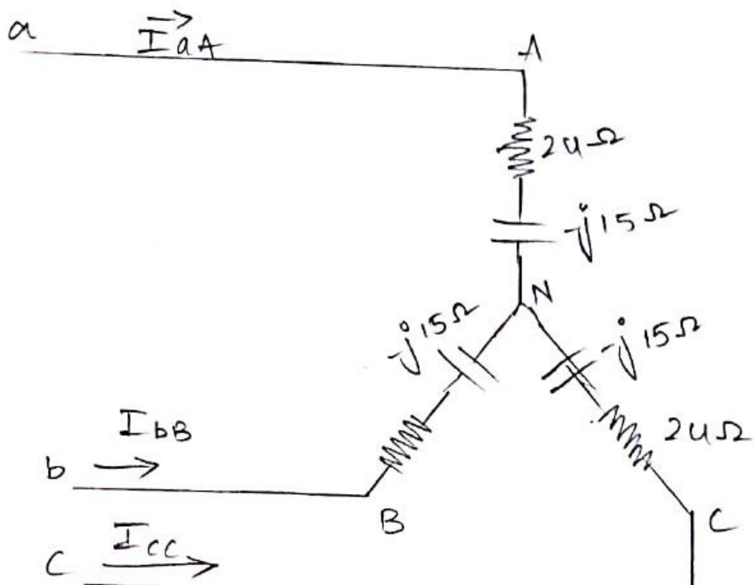
Alternator output KVA

$S = \sqrt{3} V_L I_L = \sqrt{3}(400)(13.86)$

$\boxed{S = 9.60 \, kVA}$

14. For the balanced circuit below, $V_{ab} = 125 \angle 0° V$, Find the line currents I_{aA}, I_{bB} and I_{cC}.

Sol:



Transform the source to its equivalent

$$V_{ab} = \frac{V_p}{\sqrt{3}} \angle -30^\circ = 72.17 \angle -30^\circ$$

Now, use the per-phase equivalent circuit

$$I_{aA} = \frac{V_{ab}}{Z}$$

$$Z = 24 - j15$$
$$= 28.3 \angle -32^\circ$$

$$I_{aA} = \frac{72.17 \angle -30^\circ}{28.3 \angle -32^\circ} = 2.55 \angle 2^\circ A$$

$$I_{bB} = I_{aA} \angle -120^\circ$$
$$= 2.55 \angle -118^\circ A$$

$$I_{cC} = I_{aA} \angle 120^\circ = 2.55 \angle 122^\circ A.$$